Wicksell's cumulative process subject to criticism by Cassell.

Cassell: Long-run banking policy may determine natural rate.

2-3 further points.

Doctrine of Vested Savings developed by Robertson, Henry Hayek, Schumpeter. Robertson independent of Wicksell & the Austrians.

Wicksell: money rate = natural rate
increase d for producers' goods, increase d for factors, increase d for consumers' goods.

Doctrine of Vested savings maintains that this process implies a C in aggregate d for commodities in favor of producers' goods and against consumers' goods.

Increase in m of money put in hand by entrepreneurs, they command a higher 7% of capital. Money there are consumers shun of money than previously.
100 billion
70 " consumers.
30 " productive entrepreneurs.
10 billion increased.
110 billion
70 " consumers.
40 " Entrepreneurs

Entrepreneurs bid away part of the resources formerly employed in producing consumer goods.

Consequently, prices of consumer goods rise relatively to prices of consumer goods more in proportion to their money income.

Result is an increased rate of capital accumulation, called forced saving.

Theory of forced saving: if money rate > natural rate, abortive saving.

Robertson considered a more special case where the natural rate falls.
Cassell argued that this forced saving results in a natural rate + money rate.

The forced saving doctrine assumes full employment.

All Wicksellian analysis starts with full employment.

Modification by Schumpeter: concerned with a particular type of discrepancy that which occurs with technological progress.

\[ \sqrt{\text{cannot obtain natural rate: only export factors.}} \]

Schumpeter points out what happens with natural rate - rate of profit - is that rate is not increased uniformly in all industry but in only in certain innovating 'rates'. Industries is natural rate increased.

If a discrepancy between industry and firms in the same industry in regard to the natural rate. Assume if a credit expansion these innovating firms will utilize expand their loans & demand for factors.

Expansion channelled into innovating firms & industries fixed bid away.
Factors of productivity:

These expenditures then will increase. Hence
saving in regard consumers a forced reduction in demand
by other entrepreneurs

Tried saving a way of financing innovations.

4/ Anticipations.
5/ Unemployment.

Wicksell mentions an accelerating factor of cumulative process.
Entrepreneurs expectant accelerate influence of consumers cumulative process.
Expectations of continuing rise in prices reinforces effect of divergence between natural rate and bank rate.

Lindahl = not actual natural rate but anticipated or expected rate which is important, substitute for natural rate, if expected natural rate becomes in a way a psychological phenomenon.
Unemployment: Previous Wicksellian
Doctrine assumed Full Employment.

Assume rigid wages:

- money rate above real rate.
- Fall in employment of factors making capital goods
- total income falls
- consumption goods demand falls. Employment falls.

until money rate = natural rate.

When money rate = natural rate 
employment of factors in production goods increases.

Cumulative process of rising employment until full employment is achieved.

Then Wicksellian rise in price occurs.

With assumption of rigid factor prices cumulative process of rise or fall in quantity of employment.

Keynes picked up this idea. Keynes showed that with cumulative process occurs with flexible wages.
Wicksell has been achieved a theory of what causes so in climate level.

Keynes: Discuss Keynes's General Theory.
Read Keynes by Next Week.

Tuesday: Discuss effort of Wicksell and Mach Ervin: Sequence Analysis.
Wicksell: Cumulative Process:
level of prices rise & fall:
definite time pattern rise & fall
predictable: possible to indicate by
how much aggregate demand
aggregate supply is in a
discrepancy between the
two rates.

Does beyond traditional
Equilibrium analysis

Stability of Equilibrium concept.

Wicksell thought of cumulative
process as quite deterministic.

Wicksell: monetary equilibrium
an unstable Equilibrium

Cumulative process
- absence of monetary equilibrium
- deviation
- definite time pattern
- expectations change.

led to development of
Process analysis
Essentially concerned with changes in time
patterns of disequilibrium

Process analysis: Sequence Analysis
Come from neo-Wicksellian Swedes.

Lindahl Lindberg.
Mathematical Economics: Attempt to establish a Dynamic Theory

Assume D schedule of ordinary type: e.g. demands to price change of ordinary type as immediately

Assume Supply is f of previous price: e.g. this year's output is f of last year's price.

Supply curve S is a relationship between present price + future output.

D = new demand comes.

Get a year to year fluctuation of price to output if you know the nature of curves & nature of time.
Patterns can indicate a time sequence of transient quantities.

Damped pattern as indicating on p. 2.

Approximately a non-stable equilibrium.

Decaying pattern.
Sequences which are, as to speak, cyclical or oscillatory.

Sequences which are not oscillatory.

Classification of time series:
- Oscillatory
- Non-oscillatory
Cobweb Theorem:

Question of divergent fluctuation. Do we have something which can stop them?

1. Supply curve
   somewhere becomes extremely inelastic.

2. In long run markets constant no divergent fluctuation because curve changes its position. Each year it makes a loss. The loss is greater than the profit it makes.
   This is an assumption that free entry and exit, etc.

3. Based upon specific expectation assumptions self-feeding expectations.
Condition for constant, convergent, or divergent
convergent

\[ \frac{d}{dt} \left[ \frac{1}{\eta_s} \right] = \frac{d}{dt} \left[ \frac{1}{\eta_0} \right] \]
constant.

convergent \( \eta_s < \eta_0 \) at \( t \to 0^+ \) \( \text{interior} \)

divergent \( \eta_s > \eta_0 \)
constant. \( \eta_s = \eta_0 \)

\[ L \cdot \frac{\partial \cdot B}{\partial x} = \eta_0 \]

\[ \eta = \left( \frac{2\pi}{L} \right) - \frac{1}{k} \]

\[ E = \left( \frac{2\pi}{L} \right) \cdot \frac{1}{k} \]

\[ \frac{\eta}{E} = \left( \frac{2\pi}{L} \right) \]
At pt. of intersection: ratio of elasticities = ratio of slopes

At pt. of intersection:
\[ \frac{\text{ratio of elasticities}}{\text{ratio of slopes}} \]

Algebraic Formulation of Cobweb Theorem:

To find logical foundation of sequence analysis and indicate why we can say move by means of time:

\[ D(t) = f(p(t)) \]
\[ S(t) = \Phi[p(t-\theta)] \]

Rewrite:

\[ D(t) = f[p(t); a \cdot \theta] \]
\[ S(t) = \Phi[p(t-\theta)] = b \cdot \Phi[p(t-\theta)] \]

At each moment of time:
\[ D = S \]
\[ a \cdot p(t) = b \cdot \Phi(p(t-\theta)) \]
\[ p(t) = \frac{a}{v} p(t-v) \]

\[ p(t+\theta) = \frac{1}{a} p(t) = \left(\frac{b}{a}\right)^2 p(t-\theta) \]

\[ p(t+\theta) = \frac{1}{a} p(t+\theta) = \left(\frac{b}{a}\right)^3 p(t-\theta) \]

\[ p(t+\theta) = \left(\frac{b}{a}\right)^{n+1} p(t-\theta) \]

As long as D t b cune
remain the same you
can knowing last yps.
price compute grnmet
price
Algebraic formulation of Coomb's doctrine.

\[ D(t) = a \cdot p(t) \quad t: \text{time} \]
\[ S(t) = 6 \cdot p(t-\theta) \quad \theta: \text{time lag} \]
\[ D(t) = S(t) \quad \text{between price and appearance of supply on market} \]
\[ a \cdot p(t) = b \cdot p(t-\theta) \]
\[ p(t) = \frac{b}{a} \cdot p(t-\theta) \]

by successive substitution, can get price at
\[ t, t+\theta, t+2\theta, \ldots \]

... \[ t+n\theta \]

\[ p(t+n\theta) = p(t) \cdot \left( \frac{b}{a} \right)^n \cdot \left( p(t-\theta) \right) \]

obtain a determinate pattern of time sequence of quantities demanded and supplied

\[ p(t) = -2 \cdot p(t-\theta) \quad \text{let } p(-\theta) = 1, \theta = 4 \]

\[ p(t-\theta) = 1 \]
\[ p(t) = -2 \]
\[ p(t+\theta) = 4 \]
\[ p(t+2\theta) = -8 \]
\[ p(t+3\theta) = 16 \]
\[ p(t+4\theta) = -32 \]
D\(t\): \(-4p(t)\)
S\(t\): \(-2p(t)\)
\(p(t)\): \(-1/2\) \(p(t-\theta)\)
\(1(\theta)\): \(=1\)
Prices: \(1, -1/2, 1/4, -1/8, 1/16, -1/32\)

Convergent fluctuations
Prices tend to approach 0.

\(D(t)\): \(-2p(t)\)
S\(t\): \(-2(\theta)\)
\(p(t)\): \(-p(t-\theta)\)
\(1(\theta)\): \(=1\)

1, -1, 1, -1 ... constant fluctuations

Equilibrium analysis:
all variables taken as
undated: either all at
same moment of time
or we aren't interested
in time

Frisch: relations between
variables at
different moments
if time: called
dynamic:
diff between static
dynamic theory
Undated variables: leads to
equilibrium analysis
Dating variable leads to times sequence analysis.

Hicks book on value & capital Chapters 4 & 10.

\[ D(t) = \frac{1}{(t e)} \]
\[ s(t) = \frac{1}{(t e)} \]
\[ \frac{1}{(t e)} = \frac{1}{(t e)} \]
\[ p(t) = \frac{1}{(t e)} \]

Cobweb theorem I & simplest case of dynamic relations. Not all sequence analyses is this simpler type.

Cobweb Theorem: Harvard
Eckstein: 1939 Q J E.
Buchanan: Reconsideration of Cobweb Theorem: April 1939

Length of cycle of time Day
CoWeb theorem an extremely limited type of sequence analysis.

Assume that \( S, D \) are functions of current price only.
Bring in Expected price.
Need an Expectation function.
Hicks takes Expected prices as date.

Sequence analysis depends upon a relationship between future prices and present prices.
Pr. Evans Mathematical Int. to Economics Ch.5.
Allen: Math. Int. to Economics Ch.16.

Evans: Set with following assumptions:

\[ D(t) = f[p(t)] \]

assumes expected price is based upon past experience.
Fram assumed that they would project trend along the tangent of actual development.

\[ D(t) = \int \left( \frac{p(t)}{\delta t} \right) \]

called dynamic D. function

\[ s(t) = \Phi \left[ \int \left( \frac{p(t)}{\delta t} \right) \right] \]

\[ \int s(t) \frac{e^{Bt}}{\delta t} = \Phi \left[ \int \frac{p(t)}{\delta t} \right] \]

\[ p(t) = \Phi \left[ \frac{t}{\delta t} \right] \]

\[ v(t) = \Phi \left[ \frac{t}{\delta t} \right] \]

2 types of sequence analysis:

1. Based upon time lag corrobore theorem.
   Other types based upon study of quantities and their rates of change.

2. More important than first?

\[ \frac{\Delta (2) \text{ capital}}{\text{unit of time}} \]

Investment = \[ \frac{\Delta (2) \text{ capital}}{\text{unit of time}} \]
All relationships between stocks and flows are of the relationship.

If you can establish
relationships between these, you can have a
regressive analysis.

Cobweb time lag and
empirical relationships

Cobweb cobweb relationship
based upon empirical
analysis.

The study between
stocks and flows can be handled
theoretically.

Cycles need some type of
dynamic relationship.
Equilibrium position only if we know certain dyadic relations.

If it is stable a new equilibrium will be established if D + S functions are undergoing historical change. We obtain patterns of a through time.

\[ \begin{align*}
S_1 & \leftrightarrow S_2 \\
D_1 & \leftrightarrow D_2 \\
\end{align*} \]

\text{comparative statics}

State with relationships at same moment of time but the equilibrium position change.
Have studied sequence analysis and dynamic theory. Deal with relationships between variables at different moments of time.

E.g., Evans 2.3, functions. Relationships between stocks and flows.

Essentially limited our discussion of sequence analysis.

Cobweb theorem: Prominent example of sequence analysis.

1. Don't take too seriously.
2. Don't actually explain some cycles: E.g., autonomous cycle: Hog cycle - potatoes - Mint - Tree growing - etc - Building, Ship building. All explained by a cobweb mechanism.

Present price determines decision to breed future supply.

Autonomous cycle can be explained by means of cobweb theorem and modifications thereof.
1. Supply and Demand curve for whole economy has no meaning.

2. Take cycle as consisting of autonomous cycles in each commodity.
   Different autonomous cycles in different industries have different periods and amplitudes.
   How to combine?
   Why should they be synchronized?
   Necessary for finding a synchronizing factor.
   Some claim the working of the monetary system does this.

Except in agricultural production autonomous cycles have not been established.

No autonomous cycles established in non-agricultural industry.

Look into cobweb theorem. We will find answer as to why they have not been established for other industries.
assumption of direct
relationship between today's
price and future production.

Time-lag

Adjustment of supply to
change in price occurs in jumps.

Today - 1 yr

Despite the need for
steel mills due to
depair in returns instead.

1 yr to construct furnace.
During entire period
conditions in industry remain
the same.
Output increases by steps - price falls by steps. possibility of an investment cycle but the cobweb production cycle does not occur.

Essential basis of cobweb theorem: no continuous adjustment is possible.

Fields where continuous output adjustment is possible output would not undergo can autonomous cycle.

Continuous adjustment envisioned by Marshall can interpret Marshall
A Q instantaneous supply

Q market price

If short run supply becomes according to time given for firm to adjust itself.

Professor Knight: Cost of Production in short & long periods

May interpret diagram as a succession of short run equilibrium
1] Stage 1: Step the 
long the journey allowed 
for adaptation. Follow 
directly from laws 
diminishing marginal 
utility?

Price sheds down the 
demand curve from A to B.

Get obviously a time
sequence. For each
moment of time the
sequence leads a short
period supply curve
of a price.

Mashallian Adaptation
His Theory was a Way of
Sequence Analysis
Many less patterns of
Sequence analysis

Relationship between
Equilibrium analysis
& Sequence analysis

Nothing else than a # of 
short period equilibrium.
Can consider time sequence as a succession of short run equilibrium.

Does there any difference between dynamic comparative statics and statics as a process of variation figure?

In sequence analysis there 0 data is explained by economic analysis.

Marshall's short run and long run equilibrium.

Exactly how we obtain 0 in position 0 in analysis explained historically e.g. just happened.

Essential distinction between comparative statics and sequence analysis.

Essentially all he wants to say is about sequence analysis.
Obviously all business cycle is a type of sequence analysis.

The business cycle is a pattern of change in time.

We must find a sequence which is a pattern of changes in time.
Factors which determine the level of Employment.
Mr A.P. Lerner Article:

Mr Keynes' General Theory of Employment,
Interest and Money -
The International Labour Review.

The problem of unemployment.
Classical doctrine: unemployment is incompatible with equilibrium.

Keynes: Involuntary unemployment caused by real wage inrespective of money wage.

Keynes accepts empirical fact of sticky money wages.
What would cut in money wages result in: effect of wage cut upon costs and upon demand.

Psychological assumption: people whose income is increased tend to increase their purchases by the increase in income.

Cut in money wages does not result in increase in employment.

Determination of the level of employment: Income Y, Consumption Goods C, Investment Goods I.

\[ Y = C + I \]
S = saving = gross aggregate income over expenditure on consumption goods.

\[ S = Y - C \quad \text{for} \quad Y = C + I. \]

\[ S = I \]

If I is a change in I then it is a change in S.

\[ \text{Keynes' note on difference between society and individual in regard to decisions to save and invest.} \]

If individual savages S, he cuts C, diminishing \( Y = C + I \) leaving \( Y - C = I \) the same as before.

Society 'hoarding'...

"... an attempt by people to save more than they desire will diminish consumption and incomes and employment etc., but will never seek in making saving greater than investment." Determine size of margin and consumption.
Call Employment + Aggregate output level of economic activity. May change generally in the same direction. Interested in variables which indicate level of employment. Study the factors which determine the short period equilibrium level of output and employment.

Short period level of output Employment and Keynes

Keynes's theory a pure equilibrium theory is not a process analysis. Its results are highly relevant to business cycle analysis. Its results do not contain a theory of the business cycle.

Short period static theory Sense in which it is a short period analysis. Total stock of capital in the community is a datum. Changes which take place render a constant quantity of capital. Walking out by H H Robinson of a in results will be long run change in quantity of capital.
Relation between stock of capital and annual rate of change in stock of capital will lead to a theory of the cycle.

Keynes theory summarized in a few words as follows.

1. Review the level of factor prices and money wages, the employment of labor (and other factors); their employment depends upon the aggregate demand for commodities.

\[ \text{Agg} \cdot D \text{ composed of} 1 \text{ pt} \]

\[ D \text{ for consumption goods and investment} \]

Consumption Demand depends upon habits of community.

\[ D \text{ for investment goods depends upon Rate of Return + Money rate of interest} \]

\[ \text{e.g.} \text{, upon expected rate of return} \]

\[ \text{r} - \text{if int} = \text{x} \cdot \text{r} \text{at } D \text{ for investment goods, max} \]
Aggregate D. for all commodities is determined.

Each factor of aggregate demand results in certain D. for factor of production.

Wages rigid - D in aggregate demand results in D in level of employment.

So far it is nothing beyond the post-Wicksellian theory.

Discussion of relationship between expected rate of profit + money rate of interest to D. for investment goods.

Keynes developed a technique in which he mapped how much employment changes.

Rate of interest - Wicksell takes it as institutionally determined price - elastic supply does not decline.

B money
2. The willingness of banks to hold the quantity of money

3. The Keynesian shows that under certain conditions a rise in factor prices will not affect the demand for factors and consequently their employment.

4. Monetary systems of such nature that a rise in price causes a rise in money

5. Unemployment not due solely to rigidity of factor prices

6. Prices result in a rise in money which neutralizes effect of a rise in prices

7. Level of prices a numerical fact

8. Economic activity independent of factor prices
Technical formulation of Keynes's work

Basic concepts of Mr. Keynes:

\[ AD \quad \text{money value} \]

\[ C = D \quad \text{for consumption goods} \]

\[ I = D \quad \text{for investment goods} \]

\[ C = \text{expenditure on consumption} \]

\[ I = \text{rate of investment} \]

\[ Y = \text{national income} \]

\[ Y = C + I \]

holds always by \( DF \):

real values

\[ Y_r = C_r + I_r \]

If meaning of real value of national income.

Keynes took an index of wage for deflation: measured in terms of wage.

\[ \text{numeraire of Walras} \]

in wages of labor in Keynes.
2/3 to 2/4 of national income spent on wages as labor becomes the most important commodity.

Keynes' wage unit already an index concept. Take an index of wages: called Labor Standard by Edgeworth.

Keynes' inability to escape the index problem.

\[ Y = C + I \quad \text{all flows per unit of time} \]

Saving as a flow:

\[ S = Y - C \quad C_2 = C \]

\[ S = I \quad \text{always equal, not only in equilibrium} \]

Concept of Income + Investment.

Gross + Net money investment.

As we have written it we mean gross income.

Wet income:

\[ Y_{NW} = Y - R \quad I = E - R \]

R = Replacement
\[ Y_n = C + I_n \]
\[ S_n = Y_n - C \]
\[ S_n = I_n \]

**Source + Investment = Where**

**Gross Income + Investment**

**that part is prime cost due to**

**used equipment.**

**Ent: Receipts A.**

\[ A = (U + S) - F \]

\[ F = factor\ cost \]
\[ U = labor\ cost : S = what\ rent\ from\ source \]
\[ S = \text{Supplementary cost:} \]

\[ H = (u + S) - F \]

added not the community;

**Intermediate products**

**cancel out:** \( F \) ent remain

**as wages.**

\[ E = H - U - S - F \]

\[ Y_n = E_n + F \]
U + F = Prime Cost.

DF. Income.

\[ Y = (A - U - F) + F = Y_w + S \]

\[ Y_S = A + S \]

Results are 5 regardless of what we call Income.

Provided we know what we are calling it.
\[ Y = C + I \]
\[ S = Y - C \]
\[ S = I \]

Types of income + types of investment and saving.

gross and net income.

relationships hold for both gross and net concepts.

Keynes Hybrid if: between gross and net income.

\[ A_i = \text{wage} + \text{entrepreneur} \]
\[ U_i = \text{user cost} - \text{wage} \]
\[ V_i = \text{time depreciation} + \text{maintenance cost} + \text{factor cost} \]

\[ E_i = \text{entrepreneur net income} \]

\[ A - U - V - F = E \text{ entrepreneur net income} \]

\[ E_n + F = Y_n = A - U - V \]
\[ E_m = A - U - V - F \]

\[ Y_e = A + F \]

\[ Y = E + F \text{ where } E : A - U - F \]

\[ Y_k = A - U - F + F = A - U \]

Mr. Keynes does not subtract supplementing cost:

\[ Y_k = A^* - (u + F) - F \]

\[ U + F \]
Y_n affects Entrepreneur propensity to save.

Income in temp. = \( Y_w = A - U - V = E_N + F \)

Identity between Saving and Investment true only for a closed economy.
Can be df as it holds with International Trade.

Following relationship with Int. Trade:

\[ Y = C + I + (E - T) \]

\( E - T = \text{foreign balance} \)

\[ Y = \text{Saving} + \text{Investment} + \text{foreign balance} \]

Can always redefine terms so as to get these relationship.

Keen's results that \( S = I \)

Difference in life between Keynes General Theory + Theory of Money.

Item left out of income:

\[ I = E + F \]

\( E_n = A - U - \text{windfall profits} \)
Mr. Keynes: normal profit in white economy kept economy at given level of output.

\[ Y_t = Y - P = C + I - P \]

Keynes's Treatise on Money:

\[ S_t = Y_t - C = C + I - P - C \]
\[ S_t = I - P + \text{windfall profit} \]
\[ P = I - S_t \]

\[ S_t \text{ in Treatise only when windfall profit is zero.} \]

Windfall profit depends on contracts employment

Professor Robertson gave a different meaning to saving and consumption that \( S \neq I \)

Robertson: Df. Income as

\[ Y = C + I \]
\[ S = Y_t - C_t \]

\[ Y_t: \text{Yesterday income} \]
\[ S_t: \text{Today saving} \]
\[ Y_{t-1}: \text{Yesterday income} \]

S. Today Saving
\[ S_t = \gamma_0 - \gamma_1 \]
\[ \gamma_t = \gamma_0 + I_t \]
\[ \gamma_0 = \gamma_0 + I_0 \]
\[ Y_t = \gamma_1 + I_t \]
\[ S_t = Y_t - \gamma_1 = \gamma_0 - \gamma_1 + I_t \]
\[ I_t - S_t = Y_t - \gamma_0 \]

Diff between today's income & yesterday's income.

\[ Y_t - \gamma_0 \] depends upon relative between today's investment and today's savings.

Boxterra: if \( S = I \) then income has not changed.

Keese theorem dealt with motives for expansion of industry.

\[ S_t - I_t = \gamma_0 - \gamma_1 \] Hoarding

Hoarding as the diminution of income.
Approach a matter of convenience.

Robertson's people thought differences were chemical rather than terminological.

Casual claim of Robertson.

Robertsonians thought they had a causal explanation. They obviously were wrong.

But as for sequence analysis.
Keynes's Cost
\[ S = Y - C \]
\[ \therefore S = I \]

Keynes's Treatise Def.
\[ s_r = y_r - C \]
\[ I - s_r = P \quad \text{(windfall profit)} \]

Robertson:
\[ S_1 = Y_0 - C_1 \]
\[ S_1 - s_r = Y_0 - Y_1 \quad \Delta \text{normal} \]

Robertson's claims to supremacy:
1. Explaining value of his def. Values cannot explain facts by mere verbal def.
2. Usefulness: More appropriate for sequence analyses than Keynes; because it is based upon distinction between variables at different moments of time.

Whether we can express changes in income from one period to another in Keynes' symbols:

Robertson
\[ I_1 - s_r = Y_0 - Y_1 \]
\[ Y = C - I, \]
\[ Y_0 = C_0 - I_0. \]

\[ Y_1 - Y_0 = (C, C_0) - (I, I_0) \]

1. Change in income is a in consumption - a in investment.

2nd Sense in which saving can # be investment.

3rd Sense in which saving can # be investment.

\[ S = Y - C_0 \] called ex ante.

Saving planned saving based upon what they think will happen.

Ex post saving: df like Keynes e.g. \[ S = I \]

Ex ante saving need not be equal to investment.

Ex ante - planned.

Ex post - realized.

One simplification do drop the distinction between ex post & ex ante consumption.
Expected $L$: Ex ante L
in Swedish Writing.

\[ Y_a > Y_c \]

do distinguish between ex ante and ex post investment.

- Ex ante = ex post investment due to accumulation of stocks.
- Unintentional investment (sale of stocks) possible due to accumulation or sale of stocks.

Difference between ex ante + ex post saving

\[ S - S_a = Y - Y_a = I - I_a \]

be for:

\[ Y = C + I \]

\[ Y_a = C + I_a \]

\[ S - S_a = I - I_a \] unintentional investment.

\[ Y - Y_a \] explained by $S - S_a$

\[ Y - Y_a = I - S_a \]

Similar to Robertson:

ex post has the difference is between anticipated and realized incomes.
Why realized income is either $ or than effective income. Obviously, Swedish ifs = Robertsons.
If people would affect income of proceeding period to continue.

Swedish D: can say same thing about them as about Robertsons.

3 ways $ I by mean.
1. Old type Keynes.
2. Robert.

Fourth type $ I

Concept of hoarding: Had in connection with Roberton called hoarding.

Hoarding a flow concept in Roberton.

All savings here are flows.

Keynes says the most natural way of hoarding is if the stock of money means...
Community cannot hold any more if quantity of money is constant.

Cash balances reflecting stock of money

Best of all: Hoarding as a change in the desire to hold money. No change if the community desires to hold more money.

Kames calls this a change in liquidity preference.

If you speak of a desire to save which may change independently of investment.

4) Hoarding: Change in composition of cash balances. Connected with a diminution of cash balances in form of active balances instead of idle balances / active balances.
Mises
active balance - current trans acting idle balances - other purposes

Mrs. Romaine:
Economic Journal 1937
Concept of Hoarding

Use Keynes Terminology
Start exposition of Theory
Based upon Y, relationships between C, S, I, S
Saving may be left out when discussing the whole economy

Have both I + S which are always same except identity equation.
S, I, C + Rate of Interest (money rate in the Wicksellian sense)

As a measure of the interest rate, bonds, long-run gilt edge bonds, the yields
in them.

Differentiate between
S.R. & L.R. rates

Keevil: Preference Rate of Interest, Short-term Investment + Rate of Interest: Long Run

C, I, C - preference to consume

C, I, I - preference to consume
司政令 B

\[ Y = \phi(X) \]
\[ C = \phi(Y) \]

consumption is \( 5 \) francs.

Efficiency of consumption, efficiency of \( C \) in francs by less than increase in income.

Taken by Keynes as a generalization of efficiency.

\[ \frac{C}{Y} = \frac{70}{1} \]

\[ \frac{OC}{OY} = 20 ; < 1 \]

\[ 0 < \frac{OC}{OY} < 1 \]

\[ \frac{OC}{OY} \] marginal propensity to consume.

Graph of propensity to consume.
Propensity to consume $\Phi$ as a function of relationship between expenditure on consumption and income:

$C = \Phi(Y)$

\[ \frac{dC}{dY} \Phi(Y) = \text{marginal propensity} \]

$0 < \frac{d\Phi}{dY} < 1$

Propensity to save $\Psi$ as relation between saving and income:

$S = \Psi(Y)$

\[ \frac{dS}{dS} = \text{marginal propensity} \]

\[ \Psi(Y) = \frac{d\Phi(Y)}{dY} \]

\[ Y - C = \Phi(Y) \]

\[ \frac{dS}{dY} = 1 - \frac{dC}{dY} = \Psi(Y) \]
\[\frac{ds}{dy} + \frac{dc}{dy} = 1\]

Obviously follows from Keynes' df. of saving as \(Y - C = S\).

Propensity to consume \(< 1\) means that propensity to save is \(> 1\).

An increase in income leads to an increase in saving.

\[\frac{dc}{dy} < 1\]

Additional empirical assumptions which can be made but which are not necessary for the development of Keynes' theory.

\[\frac{d\xi}{dy} < 1\]

\[\frac{d^2\xi}{dy^2} < 0\] not needed by Keynes.
Average propensity to consume

General relationship: average curve has no upward slope.

Average propensity to consume decreases when income increases.

Where marginal propensity decreases 

dy < 0 then

Average propensity to consume decreases.

Empirical analysis:

Mr. Ezekiel

C = 0.9, 1.5 in current

Business study

Not only the level of income but also how much income
Eqs. 332
May 21, 1942
J E P

\[ C = 10.256 + 0.712Y - 0.09Y^{-1} \]

This dynamic relation means..."factors"

It takes time for people to adapt themselves.

Propensity to save \( D < 1 \)

in studies of the Natural Resources Committee.

Relationship between \( C, S + I \)

\( S \) increases constantly as income increased.

People with low income have negative savings.

They spend more on consumption than they earn.

Hoists for families with incomes of \(< 1,000 \) per year.

\( 1,200 \) per year. The break even point.
Propensity to consume refers both to an individual and a household.

Take whole economy.
Not only level of net of income but also distribution of income. I constant fuel changes propensity to consume changes.

Price & Total Income
Survey of current Business
April 1942

Economy March 1942 A.E.R

Keynesian formulation of propensity to consume.

\[ C = f(y, i) \] we know that \( i \) is interest rate may have some offset upon saving.

Need not be the effect which Marshall assumed.

\[ \frac{dC}{di} \] and \[ \frac{dC}{dy} \]
The Multiplier

The Multiplier

\[ Y = C + I \]

If \[ C = aI \]

then \[ Y = (a + 1)I \]

\[ Y = \varphi(Y, i) + I \]

given \( i, I \) we can determine \( Y \)

\( \frac{\Delta Y}{\Delta I} \) called the Multiplier

\( \frac{\Delta Y}{\Delta I} = \frac{\partial \varphi \partial C + \partial \varphi \partial I}{\partial I} \)

\( \frac{\Delta I}{\Delta Y} = \frac{\Delta I}{\Delta I} + 1 \) consider

\( \frac{\Delta I}{\Delta Y} = \frac{\Delta I}{\partial C + \partial I} \)
\[ \frac{\Delta y}{\Delta I} = \frac{1}{1 - \frac{\Delta \bar{y}}{\Delta I}} \]

quantity by which you have to multiply an increase in investment in order to obtain an increase in income.

\[ \Delta y = k \Delta I = \Delta \bar{y}, \Delta I \]

Marginal rate of change in income corresponding to a change in investment.

slope of curve is multiplicity.
\[ \Delta Y = \frac{1}{\Delta T} \]
\[ \Delta T = \frac{1}{\Delta Y} \]

Multiplici grows rather important econominc role.

Caution needed in using the multiplici.

\[ 1 < \frac{\Delta Y}{\Delta T} < \infty \]

the multiplici

2 Further point
\[ \frac{\Delta Y}{\Delta I} = 1 - \frac{\phi}{\delta} \]

\[ Y = (1 + I; \quad Y = \phi Y + I) \]

\[ C = \phi Y \]

\[ \Delta Y \text{ total increase in investment in the community not the increase in particular investment.} \]

limitation of multipliers: if a slope of curve connecting income and investment

\[ \frac{\Delta Y}{\Delta I} \]

multiplier is tangent not along curve cannot use the multipliers for large changes in income.

"Same reasons why we cannot use the elasticity of demand for large changes."
If \( Y = Y_i + Y_f \) for a straight line, then tangent and slope are identical.

Concept of arc elasticity of demand vs. point elasticity of demand.

Multiplier for large scale vs. investment in as an arc concept. Relation to such a multiplier to the Keynesian elasticity of demand.

Multiplier in a closed economy. If international trade the multiplier will be smaller.

\[
Y = I + C_o + C_F
\]

\[
\frac{\Delta C}{\Delta Y} = \frac{\Delta C_o + \Delta C_F}{\Delta Y} \frac{\Delta F}{\Delta Y}
\]

\[
\frac{\Delta C}{\Delta F} = \frac{\Delta C_o - \Delta C_F}{\Delta F} \frac{\Delta Y}{\Delta F}
\]

\[
\frac{\Delta Y}{\Delta F} = \frac{1}{1 - \Delta C_o / \Delta F} = \frac{1}{1 - \Delta C + \Delta C_F / \Delta F}
\]
Multiplier can be used under conditions of full employment if we express them in terms of money income.

It indicates how much given the propensity to consume money, investment will increase money income.

Under full employment, the multiplier will evaluate the inflationary effect of an increase in investment.

Multiplier is useful mostly in discussing money terms.

Derivation of multiplier by means of an infinite series. Such a derivation is found in Mr. Robinson's book.

\[
\frac{dX}{dI} = \frac{1}{3}
\]

Hence, investment is

\[
3 = 1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^n
\]
Concept of leakage in this analysis:

\[
1 = \frac{1}{3} + \frac{1}{3} \left( \frac{2}{3} \right) + \frac{1}{3} \left( \frac{2}{3} \right)^2 + \ldots
\]

Multiples first introduced in the phase of:

\[
1 + \frac{1}{3} + \frac{1}{9} + \ldots
\]

Mrs. Robinson speaks of x's making rounds. Does not mean a time sequence, not dated, regarded as simultaneous or conceptual decomposition of changes in time.

Not understood as sequence analysis.

Multiples may be used in sequence analysis.

Can build type dynamic multiplier by dating the various appendile of the appanage:

\[
1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots
\]

One of practical interest.
Using the sequence approach,

\[ \frac{dF}{dE} + \frac{dE}{dF} \left( \frac{dE}{dF} \right)^2 = 0 \]

\[ \frac{dF}{dE} + \frac{dE}{dF} \left( \frac{dE}{dF} \right)^2 + \frac{dE}{dF} \left( \frac{dE}{dF} \right)^3 + \ldots \]

Total increment in

\[ \frac{dF}{dE} = 1 + \frac{dE}{dF} \left( \frac{dE}{dF} \right)^2 + \frac{dE}{dF} \left( \frac{dE}{dF} \right)^3 + \ldots \]

Some progress summary was

\[ \frac{dF}{dE} \]

Obtain formula without using directly.
Factors determining rate of investment or its change.

Keynes' doctrine a direct development of the Walrasian doctrine of the two interest rates.

Keynes finds a criterion which indicates the exact rate of investment with a given money rate.

Marginal efficiency of capital deferred in the same way as rate of return on capital was defined by earlier authors. Especially saving theor.

\( g_1, g_2 \ldots \) for returns.

Rate of interest vs. rate of discount which adjusts present value of return to cost of product.
\[ Q = \text{cost of product} \]
\[ Q = \frac{a}{U + c} + \frac{a_2}{U + c} + \ldots + \frac{c_n}{U + c} \]

Solve for \( a \) which gives the discounted value of returns = cost of product

Visit developed by Kevin Fisher.

Time patterns of returns:

- \( \text{time} \)
- \( \text{return} \)
- \( \text{constant} \)
- \( \text{constant} \)
- \( \text{average} \)
- \( \text{standard deviation} \)

The concept of the average is the role of returns which is constant over the lifetime of the investment and what can be taken as equivalent to stability of returns.
\[ S = S_{\text{given price of the instrument}} \]

\[ S = \frac{\alpha}{1 + \gamma} + \frac{\alpha^2}{(1 + \gamma)^2} + \cdots + \frac{\alpha^n}{(1 + \gamma)^n} \]

\[ q \text{'s expected rates are subjectively} \]

\[ \text{affected actions rather than actual returns.} \]

\[ \text{How expectations about future returns are formed.} \]

\[ \text{Rate of return. rate of offering.} \]

\[ \text{Marginal rate of efficiency is the lowest rate.} \]
Econometric Business Cycle Research:

1. Int.

   2.1 The Arrow scheme and Elementary Equations
   2.2 Variables: Coefficients, Unsystematic Terms
   2.3 The Determination of the System's Moment: The Elimination Process