Chapter II: Models of Income Determination

We shall now examine how the level of Income and Employment is determined in a free enterprise economy. We shall (a) isolate the pertinent variables, and indicate what elements are considered to be outside the problem, (b) divide the pertinent variables into exogenous and endogenous variables, and (c) construct relations between these variables. The set of endogenous and exogenous variables and the relations among them constitute a model. We shall construct a number of different models and indicate what observations enable us to reject a particular model. In the following chapter, we shall discuss a number of tests which have been made of the models: some by interpretation of the observations of history, others by rather sophisticated statistical analysis. We shall attempt to evaluate this evidence, so that we can choose among these models.

The models to be presented fall into two general classes which will be called Keynesian and Classical. The essential difference between the two is in their equilibrium characteristics. The Keynesian models do not exhibit a unique equilibrium level of employment, whereas the Classical models do exhibit one equilibrium level of employment, namely full employment.

Since both classes of models deal with the same economic system, different conclusions should be immediately testable by reference to the facts. If value judgments about the existing economic system center around its ability to achieve full employment, the Keynesian models' demonstration of stable equilibrium at less than full employment in a competitive system can be the foundation
of a critique of the existing system.

All of the models to be discussed are "aggregative"; that is, they deal with the behavior of households as a whole and with firms as a whole, not with individual households or firms. Consequently, we shall not be interested in the demand for particular goods, but in the demand for all goods, which we shall call "aggregate demand"; and we will deal with the level of prices, not particular prices.

Definition of the Pertinent Variables

The problem is to construct models in which the level of income and employment may be determined. We shall first list and define the variables which different writers have suggested to be related to the level of income and employment.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment</td>
<td>N</td>
<td>The number of workers employed in the community.</td>
</tr>
<tr>
<td>National Income</td>
<td>Y</td>
<td>Net value of goods and services produced after maintaining intact the community's capital = sum of income payments to individuals = sum of Net Value added by each firm in the community.</td>
</tr>
<tr>
<td>Consumption</td>
<td>C</td>
<td>Value of goods and services used up by households in a period of time. These goods and services do not appear on the market in the future (for simplicity, no consumer durable goods appear in this model).</td>
</tr>
<tr>
<td>Investment</td>
<td>I</td>
<td>Difference between goods and services purchased by firms and goods and services sold by firms, i.e., net additions to inventories plus net additions to plant and equipment.</td>
</tr>
<tr>
<td>Variable</td>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Output</td>
<td>O</td>
<td>The net physical output of the economy, after allowance is made for attrition of the community's capital.</td>
</tr>
<tr>
<td>Savings</td>
<td>S</td>
<td>The unconsumed part of household income.</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>r</td>
<td>The cost of borrowing money.</td>
</tr>
<tr>
<td>Prices</td>
<td>P</td>
<td>The level of prices of final products.</td>
</tr>
<tr>
<td>Wages</td>
<td>W</td>
<td>The level of wages.</td>
</tr>
<tr>
<td>Money</td>
<td>M</td>
<td>Currency plus bank deposits.</td>
</tr>
</tbody>
</table>

As defined above, there are two relations which always hold among the variables. These are:

1. \[ Y = C + I = C + S \quad \therefore S = I \]
2. \[ Y = pO \]

1. The first relation is an identity which states that the Total Income of the community is used for consumption or investment purposes. This is equal to the total consumption plus saving of households. Therefore, the total investment of the community is always equal to the unconsumed portion of household income.

2. The second relation is also an identity which states that the money value of the National Income is equal to the physical output times the price level.

It should be noted that \( Y, C, I, \) and \( S \) as defined above are money values. Corresponding to these money values are physical counterparts, i.e., real values. We shall write the real value of Consumption, Investment and Savings as \( c, i, s \). It should be obvious that just as \( Y = pO \), so must

\[
\begin{align*}
C &= pc \\
I &= pi \\
S &= ps
\end{align*}
\]
A Simple Classical Model

In the model of the economy which is implicit in the writings of the pre-Keynesian economists, the level of output was determined by the equilibrium in the labor market. The supply curve of labor in the classical formulation, is an upward sloping function of the real wage. The demand curve for labor is derived from the theory of the firm under competition: each firm combines additional workers with its capital equipment until the value of the marginal product is equal to the wage rate. In order to analyse the change in output attributable to a change in employment we must specify a production function for output as a whole. Such a function shows us the relation between the output of the economy and the number of workers employed, for a given stock of capital instruments (at a given time). Stated symbolically we have:

(1) Supply function of labor

(1) \( N_s = f_1 \left( \frac{W}{P} \right) \)

(2) Production function for output as a whole

(2) \( \sigma = \delta(N) \)

NOTE THAT: \( \frac{d\sigma}{dN} \) = marginal product of labor (depends upon \( N \) above)

(3) Demand function for labor. The firms add labor until

\( P \cdot \frac{d\sigma}{dN} = W \): therefore, the demand for labor is a function of the real wage

(3) \( N_D = f_2 \left( \frac{W}{P} \right) \)

\[ \frac{W}{P} \]

\[ \left( \frac{W}{P} \right)_e \]

\[ N_0 \]

\[ N \]
The intersection of the Demand schedule (Equation 3) with the Supply schedule (Equation 1) give us the equilibrium real wage, \( \frac{W}{P} \), and level of employment, \( N_e \). If we substitute \( N_e \), the equilibrium level of employment, into the production function, we get the equilibrium level of Output, \( Q_e \). This solution is a "full employment" solution, because the equilibrium level lies on the supply curve of labor. That is, all laborers who wish to work at the equilibrium real wage are employed. Unemployment is defined to be the difference between the quantity of labor demanded and the quantity of labor supplied at a particular real wage. As such, unemployment can only exist if the real wage is above the equilibrium real wage. In a free market, if the real wage is above the equilibrium level, there exists an excess supply of labor. The excess supply will initiate a process by which the real wage is lowered.

The system of three relations described above determines the equilibrium level of employment, output and the real wage. However, we must still determine the absolute level of prices. To do this, an additional relation is needed; this is the quantity theory of money. One presentation of the quantity theory is the Cambridge equation: \( M = KpQ = KY \). This states that the quantity of money individuals wish to hold is some fraction of their money income. This simple quantity theory contains the assumption that money is used only for transaction purposes. If \( M \) is taken as an exogenous variable, \( K \) as a parameter, then the model has four equations and one identity which determine the equilibrium quantity of the 5 endogenous variables: employment, output, real wage, level of prices and money income.
(1) \( N_s = f_1 \left( \frac{W}{P} \right) \)

(2) \( O = \bar{M} \ (N) \)

(3) \( N_D = f_2 \left( \frac{W}{P} \right) \)

(4) \( M = KPO \)

(5) \( Y = PO \)

In this model, the equilibrium values in the labor market are stable with respect to a change in the exogenous variable, the quantity of money. If we increase the quantity of money, the price level will rise; this reduces real wages so that the quantity of labor demanded is greater than the quantity supplied. This results in an increase in money wages, until the old equilibrium real wage and level of output is restored.

The variables in our list which have not been determined are \( C, I, S, \) and \( r \). We can find the equilibrium rate of interest by means of a supply and demand analysis for "investment resources". The supply of resources are the savings of individuals out of income. Savings are considered to be a function of the interest rate. The assumption that saving takes place only for income is typically made: therefore, we have a savings function which rises with increases in the rate of interest and is zero when the rate of interest is zero. Investment, the demand for investment resources, is also a function of the interest rate.

The return upon an investment is the marginal physical productivity of investment times the expected price of the product during each "period" of the investment's expected life minus the sum of the amortization allowances. If, when this sum of expected values is discounted back to the present at the prevailing
"interest rate", it is greater than the cost of the investment, the percentage yield on the investment is greater than the interest rate. This aspect of "looking into the future" in terms of both the yield and the life of the investment introduces a fundamental uncertainty in investment decisions, so that the amount of investment forthcoming is dependent upon the subjective valuations by businessmen of these future values.

If we assume a constant "subjective state" on the part of investment decision makers, we know that the lower the interest rate the greater the quantity of investment. As a numerical example: a perpetual yield of $50 at 5% is worth $1,000; at 4% it is worth $1,250. If an investment originally cost $1,100 which yielded $50 in perpetuity, it would not be made at 5%, it would be made at something more than 4%. We therefore have that the amount of investment is determined by the interest rate. This investment function has the property that as r decreases, I rises; e.g., it slopes downward and to the right.

We therefore have the following set of relations:

(6) \( S = S(r) \)

(7) \( I = I(r) \)

(8) \( S = I \)

(9) \( Y = S = C \)

This model has 4 equations and 4 endogenous variables, \( S, I, r, C \).
Note that $Y$ is exogenous to this model since its value is determined elsewhere. Graphically, we have

![Graph showing the intersection of the Savings and Investment schedule](image)

The intersection of the Savings and Investment schedule gives us the equilibrium rate of interest and the equilibrium quantity of production which is devoted to Investment. If this value is substituted into (9), we find the volume of consumption. Changes in the quantity of money have no effect on the quantity of resources devoted to investment, nor upon the equilibrium rate of interest. Further, if the Investment schedule should shift upwards due to a change in investor's expectations of profit, there would result a higher interest rate, a higher level of savings, a lower level of consumption, but no change in the level of output or prices.

**Conclusions**

This "classical" model of 9 equations has very interesting properties: Equations (1) through (3) determine the equilibrium level of output and of employment. Equations (4) and (5) determine the price level and consequently the level of money income. Equations (6) through (9) determine the equilibrium rate of interest and how the economy divides the social product between consumption and investment. Changes in the quantity of money can only alter the price level, while the interest rate is determined solely by
conditions of productivity and thrift as represented by the savings and investment schedules.

In this model, any unemployment is transitional. For example, were the quantity of money suddenly reduced, the price level would fall. A fall in the price level, money wages remaining unchanged, means an increase in real wages. This in turn implies that the quantity of labor supplied is greater than the quantity of labor demanded; workers willing to work at the prevailing wage are not hired. A process of wage cutting begins which lowers real wages and therefore eliminates the unemployment. Thus the system is returned to the level of output and real wages which existed prior to the decrease in the quantity of money.

The classical system always provides equilibrium at full employment. Any unemployment is due to obstacles which prevent the real wage from falling to its equilibrium level. The obstacle may arise simply because it takes time to readjust wage contracts to the new situation. Another obstacle may be the existence of strong trade unions which fix a money wage and prevent the equilibrating process from operating.
A Modified Classical Model

An interesting modification of the classical model is due to Knut Wicksell, a Swedish economist of the late 19th - early 20th century. Wicksell was not satisfied with the inherited statements of the quantity theory of money; he asserted that the problem was to detail the process by which a change in the quantity of money affects the price level. As a result of this effort he derived a model in which a cumulative process occurs. This model, or a slight modification of it, has proved useful in analysing the rise and fall of prices associated with the business cycle.

In the classical model, the quantity of money is an exogenous variable. Wicksell modified this by making the interest rate an exogenous factor; the interest rate being determined by the banking system. According to Wicksell, when the interest rate, fixed by the banking system, is below some "natural" or "equilibrium" rate, the quantity of money, as created by the banking system, will increase; when the interest rate is above the natural rate, the quantity of money will decrease. The change in the quantity of money is an endogenous variable. Therefore, using the quantity equation as a basis, we can write the change in the price level as determined by the change in the quantity of money. [the Greek symbol $\Delta$ will always mean change of $\cdot$].

$$\Delta P = \frac{\Delta M}{ko}$$

The demand schedule for loanable funds is a schedule relating the amount of investment and the interest rate; $D_L = I(r)$. The supply of loanable funds is made up of two parts; 1) the quantity of savings and 2) the quantity of money created (or destroyed) by
the banking system. The amount of savings is determined by the interest rate \( S = S(r) \). The supply of loanable funds is \( S(r) + \Delta M \) (\( \Delta M \) may be positive or negative). We have that \( S(r) + \Delta M = D_L = I(r) \).

Graphically:

```
  r

S(r)----------I(r)
  r_o

S(r_o)  I(r_o)
  I,S
```

The interest rate for which \( \Delta M = 0 \) is what Wicksell called the equilibrium or natural rate of interest. You will note that no matter what the interest rate may be, the supply of loanable funds is equal to the demand for loanable funds, for the change in the quantity of money varies so as to keep them equal.

Beginning from a full employment equilibrium, let us assume that the banking system sets the interest rate below the equilibrium rate (for example at \( r \) in the diagram above). We now have that \( \Delta M = I(r) - S(r) > 0 \); this causes an increase in the price level. The rise in the price level means that real wages have fallen; therefore the quantity of labor demanded is greater than the quantity supplied. Money wages will rise, but as the quantity of money keeps on increasing, wages will continuously lag behind prices, \( (\Delta M \) being the change in the quantity of money during a given time period). This process continues: an increase in money supply produces an increase in prices and an increase in wages. As long
as the interest rate set by the banking system is below the equilibrium rate, the quantity of money will increase, and the above process will continue.

If the country is on a gold standard, a unilateral rise in prices will result in a loss in gold, and the banking system will raise the interest rate to protect its gold reserves. If the country is on an inconvertible paper currency, the currency will depreciate in the international markets, but there is no limit to the expansion of the money supply. If the central bank raises the interest rate so that the quantity of money contracts, a similar downward cumulative process would result. If the central bank raises the interest rate so that the quantity of money remains constant, the price level will remain constant.

We get another interesting result by making the investment schedule shift upward if prices have been rising, and shift downward if prices have been falling. \( I = I(\bar{r}, \Delta P) \) such that if \( \Delta P > 0 \), the quantity of Investment is greater for each \( r \) than if \( \Delta P \) were zero. This is a reasonable hypothesis, because businessmen's money profits rise when prices rise, and investment decisions are based upon anticipated money profits. It is reasonable to assume that anticipations are determined by recent experience. This dependence of the quantity of investment upon the change in prices means that businessmen expect the rate of change in prices that has occurred to continue in the future.
In the diagram $I_1(r)$ is the investment scheduled when prices are fixed, $I_2(r)$ is the investment schedule when prices are rising. With prices constant, the banking system sets the rate of interest at $r_1$. As a result the quantity of money will increase, leading to a rise in prices. This means that the investment schedule shifts to the right to $I_2(r)$. If now the banking system sets the rate of interest high enough to stop an increase in the money supply when the schedule is $I_2(r)$, the schedule will shift back and the quantity of money will decrease, leading to a fall in prices (we can even include a third investment schedule, to the left of $I_1(r)$, to represent investment when businessmen expect the price level to continue falling.) As a result of the disturbance of the savings and investment process when the interest rate set by the central bank deviates from the natural rate, we shall observe periods of expansion and contraction in the price level.

The Wicksellian process operates because investors are trying to use more resources than households are willing to give up at the given market rate of interest. The banks satisfy the investors' demand by creating new deposits. The investors, in turn, using the new deposits, attempt to bid resources away from the consumers.
industries by paying higher prices. However, the investors' excess demand is never fully realized, consequently the process continues.

The Wicksellian analysis focused attention on the relation between fluctuations in the price level (or income) and the saving-investment process, rather than upon the supply and demand for labor and the quantity theory of money. As such it is a link from the classical to the Keynesian models where the labor market relations are dominated by the saving investment process. The Wicksellian process explains unemployment in the depression phase of the business cycle by pointing to the lag in wages behind falling prices; and as long as prices keep on falling, wages never catch up, resulting in unemployment. This is a more useful explanation than that of the pure classical model; but it does not explain the existence of unemployment with a constant price level. For this we must turn to Keynes.

Simple Keynesian Model

The simplest way of looking at the Keynesian models of income and employment is to note that whereas the classical models determined income and employment in the labor market, and the interest rate in the savings-investment market, Keynes developed a savings and investment theory of income. It is obvious that if savings and investment determine income, they cannot also determine the interest rate; Keynes used the asset characteristics of money to determine the interest rate. However, the simplest Keynesian model ignores the interest rate, and we shall consider that model first.

The simplest Keynesian theory:

1. \( Y = C + I \) is one equation relating three unknowns. In order to determine \( Y \) we need 2 additional equations.
2. The first additional equation. Consumption is a function of income... \( C = C(Y) \): consumers spending is determined by their income, this holds for all consumers individually and for the community as a whole. This consumption function has the additional characteristic that when income increases consumption increases, but not by as much as income increases. By definition \( Y = C + S \), and therefore \( Y - C(Y) = S \), so we can now write \( S = S(Y) \). This represents a fundamental divergence between Keynesian income and employment models and classical models. The Consumption (or Savings) function is a major innovation of the Keynesian analysis. The savings function may be such that at a high (full employment) level of income a large amount of savings takes place, even though the rate of interest be zero or negative. The contrast between the Keynesian \( S = S(Y) \) and the classical \( S = S(\bar{I}) \) is fundamental.

3. Second additional equation. Investment is determined by firms independently of income or consumption... that, so far as the income determining system is concerned, investment is a "given", i.e., investment is an exogenous variable.

\[ I = \bar{I} \text{ a constant} \]

4. The simplest theory. Substituting \( Y = C(Y) \) and \( I = \bar{I} \) into \( Y = C + I \) we get

\[ Y = C(Y) + \bar{I} \]. This can be plotted as follows:
A digression on the multiplier: Y

Now that we have a simple income determining system, we can "play" with it. We can ask the following question: if investment changes, by how much will income change?

a. The consumption function has the following properties:

1. \( dC/dY \) is greater than 0, that is, an increase in income increases consumption.

2. \( dC/dY \) is less than one, that is, an increase in income results in an increase in consumption which is smaller than the increase in income.

3. \( \frac{d^2C}{dY^2} \leq 0 \): as income increases the rate of increase in consumption decreases or is constant.*

b. Let us assume that the consumption function is linear:

\[ C = a_0 + a_1 Y, \]

where \( a_0 \) is the amount of consumption at zero income, and \( a_1 \) (the marginal propensity to consume) is > 0, and \( \leq 1 \). We can write the simplest system of paragraph 4 as follows:

1. \( Y = C + \bar{I} = a_0 + a_1 Y + \bar{I} \)

therefore \( Y = \frac{a_0 + \bar{I}}{1 - a_1} \)

c. In the above equation we know that \( \frac{dY}{d\bar{I}} = \frac{1}{1 - a_1} \).

*This is the strong form of the "marginal propensity to consume". Consistent with the findings of budget studies; not consistent with the findings of time series studies of consumption-income relation.
and this is called the multiplier. It says that if investment increases by $1, income will increase by 
$\left( \frac{1}{1 - a} \right)$. For example if $a$ is $1/2$, a $1$ rise in investment results in a $2$ rise in income.

d. If we do not use a linear form we get that

$$\frac{dY}{dI} = \frac{1}{1 - \frac{dC}{dY}}$$

where $\frac{dC}{dY}$ is the marginal propensity to consume.

e. By devising other relations, where income is again a function of the parameters ($a$'s) and the 'exogenous' variables (such as $I$) we can get other multipliers. For example, if we use

$$Y = C + I + G$$

$G$ being government expenditures

$$C = a_0 + a_1 Y$$

as above

$$I = \overline{I}; G = \overline{G}$$

(Investment and government expenditures being exogenous)

we get

$$Y = \frac{a_0 + \overline{I} + \overline{G}}{1 - a_1}$$

and the government expenditure multiplier is

$$\frac{dY}{dG} = \frac{1}{1 - a_1}$$

the same as the investment multiplier.

f. The multipliers given above are means by which we can compare two (or more) levels of income. They do not describe the manner in which a change in the rate of investment (or of government spending) raises or lowers income over time to the level indicated by the multiplier.
The Stability of the Simple Model

To investigate the stability of this equilibrium we have to set up a time process of changes in $Y$ due to an initial deviation from the equilibrium values of $Y$. The simplest way to set up such a process is to write today's consumption as a function of yesterday's income; and today's income as the sum of today's consumption and today's investment. The subscript $t$ will denote today, and $t-1$ will denote yesterday. When $t = 0$, we are at the start of our time process. Let us suppose that we start with a situation in which $Y_0 = 100$, $C_0 = 60$, $I_0 = 40$, and further that $C_t = .4Y_{t-1} + 20$ so that the marginal propensity to consume is .4. Let us imagine that investment rises by $10$, and trace out the time path by which income rises to a new equilibrium level.

<table>
<thead>
<tr>
<th>Day</th>
<th>Today's Income</th>
<th>Yesterday's Income</th>
<th>Today's Consumption</th>
<th>Today's Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100.0</td>
<td>100</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>110.0</td>
<td>100</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>114.6</td>
<td>110</td>
<td>64</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>115.6</td>
<td>114</td>
<td>65.6</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>116.24</td>
<td>115.6</td>
<td>66.24</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>116.30</td>
<td>116.24</td>
<td>66.30</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>116.52</td>
<td>116.30</td>
<td>66.52</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>116.61</td>
<td>116.52</td>
<td>66.61</td>
<td>50</td>
</tr>
<tr>
<td>n</td>
<td>116.67</td>
<td>116.67</td>
<td>66.67</td>
<td>50</td>
</tr>
</tbody>
</table>

Thus the increase of investment has produced an increase of income and consumption. Income has risen by $16\frac{2}{3}$ or $10 \times 1\frac{2}{3}$. $1\frac{2}{3}$ is the multiplier, i.e. $1.67 = \frac{1}{1 - .4}$. An increase in income in response to an increase in investment will continue until enough new saving is generated to equal the extra investment. Income has risen by $16\frac{2}{3}$, consumption by $6\frac{2}{3}$, therefore saving has risen by $10$. Let us now construct a situation in which a
change in investment increases income, but the system can find no new point of equilibrium. Again, suppose, at the start, that \( Y_0 = \$100, \ C_0 = \$60 \) and \( I_0 = \$40 \), and further, suppose that \( C_t = 1.5 \ Y_{t-1} - 90 \), so that the marginal propensity to consume is 1.5. Again let investment rise by $10, and be maintained at $50 per period in succeeding periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>Today's Income</th>
<th>Yesterday's Income</th>
<th>Today's Consumption</th>
<th>Today's Invest</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>100</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>110</td>
<td>100</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
<td>110</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>147.5</td>
<td>125</td>
<td>97.5</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>181.25</td>
<td>147.5</td>
<td>131.25</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>231.87</td>
<td>181.25</td>
<td>181.87</td>
<td>50</td>
</tr>
</tbody>
</table>

It is clear that we have an "explosive" model: no new equilibrium position is approached. The equilibrium at period 0 was unstable, in the sense that a displacement from equilibrium did not either (a) permit a return to the old equilibrium; or (b) bring the system to a new equilibrium position. The model was unstable because the marginal propensity to consume was here greater than one. The existence of stable equilibrium positions in the world leads us to conclude that the only relevant models are those in which the marginal propensity to consume is less than one.

If we had to stop at this stage our achievement would not be great, for we are in effect saying that the determination of investment cannot be explained. In an economy such as the Soviet Union, or in a War Economy, we could leave it there, for the level of investment is determined by the authorities. But, in an economy where the government's role is relatively restricted, investment is determined by the actions of individual businessmen.
We must now widen the system so that it also explains the level of investment. That is, we must make $I$ an endogenous variable and build a larger model. This is done by accepting the essentials of the classical formulation of the relation between the rate of interest and investment, e.g., $I = I(r)$. Let us expand the Keynesian model to include an investment behavior equation. We now have a theory with the following relations:

\[(10) \quad Y = C + I\]
\[(11) \quad C = C(Y)\]
\[(12) \quad I = I(r)\]

We have 3 equations and 4 unknowns: the system is not determinate. Another relation is needed.

Another way of saying that the system is incomplete is to show that any interest rate which might set by the central bank becomes an equilibrium rate, because of appropriate adjustment in the level of output. Let us graph the relations $I = I(r)$, and $S = S(Y)$, for a level of Income $Y_0$. Further let us suppose that the central bank sets an interest rate $r_b$ which is not the rate at which $S(Y_0)$ and $I(r)$ intersect. What happens?

In the Wicksellian model, because $r_b$ was below the "natural" rate, the banks supplied investors with new deposits and the price level rose. In this model, however, $S(Y_0)$ is not "frozen". If investment is greater than saving at $r_b$, the level of Income will rise until
enough saving is generated so that $S = I$. That is, if at the rate set by the banks, $S < I$, income will rise from $Y_0$ to $Y_1$, and, at the higher level of income, $S = I$. The curious student may legitimately ask, "Why does the saving function shift as a result of the discrepancy between $S$ and $I$, whereas in the Wicksellian model, both schedules remained firm?" The answer is that the Wicksellian analysis presupposes that the level of output is determined in a free labor market. Output being exogenous to the saving and investment relations, the excess demand of Investors can produce only a price rise. The Keynesian model, however, assumes that the level of output will respond to changes in the level of effective demand. Therefore, an excess of Investment demand over the quantity of savings is met by an expansion of output, and a shift of the Savings schedule. This is the rock upon which the Keynesian analysis is based. Exactly how the level of output responds to changes in the level of demand will be seen later when we return to the labor market. Here, it is sufficient to recognize that this is the foundation of the Keynesian system, the reason why in this system savings and investment determine the level of income.

Because Income is endogenous to the Savings-Investment model contained in equations 10 to 12, some outside force must determine the rate of interest. We shall now turn our attention to the manner in which the Keynesian model determines $r$.

**The liquidity approach to Interest:**

a. When an individual receives income which he chooses not to spend -- that is, to save -- he has to decide in what form he will keep this increase in his net worth. He has
a "portfolio" problem; he has to choose between assets. Now the
different assets can be ranked along two scales: one is an income
scale and the second is a "liquidity" scale. The income scale is
obvious (being the expected return from the different assets): the
liquidity scale deals with the ease (cost) with which you can
transform an asset into the form in which it can be used to
discharge a debt.

Money has the characteristic of being universally acceptable
to discharge a debt. One who has a debt to pay and who possesses
sufficient money has no problem: the offering of the money
discharges the debt. Suppose an individual possesses a different
form of asset, say a consol,* and had to sell it, obtain money,
and use the money to discharge the debt. Now if he paid $100 for
a bond yielding $3 a year, and today a $100 bond yields $4, he can
sell his $3 bond for only $75 — that is, if the interest rate
rises, he takes a capital loss upon liquidation: a gain if the
interest rate falls. Assets can be ranked in terms of the ease
with which their cash value can be realized: and the cost of
realizing such cash values. Those assets for which the ease is
greatest and the cost least are said to be most liquid. Money is
the perfectly liquid asset, and its yield is zero. Saving deposits,
government bonds are highly liquid and they yield but a small
return. Corresponding to higher rates of return we find increased
uncertainty about market values and increased cost and trouble of
selling. The least liquid assets are "physical goods" and therefore
the return from owning real capital goods is normally highest.

*Consol: Consolidated government stock: Bonds issued in perpetuity
by the British Government. Used for example, because there is no
problem about the "due" date limiting the fall in selling price of
the bond.
If the return on income earning assets of all types, rises, the relative advantage of holding cash will fall; if the interest rate falls, the advantage from holding cash will rise.

b. The above analysis deals with the asset characteristic of money. But money is a "means of payment" in addition to being an asset [its unique asset position results from its being a means of payment]. Therefore the amount of money that people have to hold depends upon the amount of payments that they have to make. The quantity of money needed to make payments also depends upon institutional characteristics such as the "time gaps" between income and outgo: we assume that these "time gaps" do not change during the periods under consideration.

c. If we combine (a) and (b) we get a relation, for each individual and for the economy, between the amount of money and the interest rate, for a given level of income. When the interest rate is low people hold some money as assets, and some money for transaction purposes; when the interest rate is high people hold money only for transactions.

With the quantity of money $M_0$, we could have an interest rate
If income is \( Y_0 \) or \( Y_1 \)
\[ r_0 \]
if income is \( Y_2 \)
\[ r_1 \]
if income is \( Y_3 \)
\[ r_2 \]

The quantity of money in a modern economy is determined by the policies of the government either directly or by the Central Bank which is always on arm of the government.

We now have
\[ r = L(M,Y) \]

where \( M = M_0 \); an amount given by "institution" or by "government" action. We now have the following income determining system.

(10) \[ Y = C + I \]
(11) \[ C = C(Y) \] (or equivalently \( S = S(Y) \))
(12) \[ I = I(r) \]
(13) \[ r = L(M_0,Y) \]

4 equations with 4 unknowns: a determinate system. We can deal with it graphically as follows:

We can begin with graph 9. A quantity of money \( M_0 \) is given; this quantity of money is consistent with \((r_0, Y_0) \) \((r_0, Y_1) \) and \((r_1, Y_2) \) \((r_2, Y_3) \), etc., note that the higher interest rate is associated with a higher income level. We can graph it as follows:
Graphs 10 and 11 give us respectively, the income level associated with each level of investment, and the amount of investment associated with each interest rate; we therefore have a relation between the interest rate and income. If the interest rate falls we get more investment and a higher income level: we can graph it as follows:

\[
S(Y) = I(r)
\]

If we combine graphs 12 and 13 we get:

\[
S(Y) = I(r)
\]

which determines \( r \) and \( Y \).

With a negatively sloped "demand" curve, \( I(r) = S(Y) \) and a positively sloped "supply" curve the equilibrium is stable.
The above is a framework: the simplest system within which income is both determinate and variable and where the major "exogenous" factor (the quantity of money) is one that is conventionally determined by factors not directly dependent upon the actual operations of the economy. (If we wished to assert that the central bank determines the interest rate, we can then have the quantity of money determined by the interest rate and income and introduce the Wicksellian modification of the classical system into the Keynesian system.) We can vary the system by analysing elements such as wage rates, government spending, exports, imports, and movements of prices.