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# Acceptable Title Pending: Probing the Limits of Precision Measurement and Academic Assessment

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*Bard College*

# Acceptable Title Pending: probing the limits of precision measurement and academic assessment

A Senior Project submitted to
The Division of Science, Mathematics, and Computing
of
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by
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SMART
WHELP

# Abstract

This is a project in two parts. The first is an attempt to impart onto the reader the necessary mental models required to understand a scientific experiment related to the improvement of gravitational wave detectors. Part one is illustrated in collaboration with Simone River Wilding, Sohpie Foley, Roma Taitwood, and Cam Goldberg.

Part two is a technical description of efforts made to reduce speckle in measurements of scattered light. Gravitational wave detection requires extremely high precision measurement, and one source of noise in the detectors is scattering off of defects and surface roughness in optical coatings. Research into the development of new coatings is ongoing, and a primary method of characterization and evaluation of these coatings is the study of scattered light. Due to the coherence of laser light, speckle inhibits accurate imaging by distorting the intensity distribution. This study uses a spatial light modulator in a preliminary attempt to reduce the speckle in studies of laser scattering.

# Contents

# Dedication

The Unknown:
Althea, Helena, Hugo, Simi, Rafi, Raph, Zohar, etc. etc.

*A child is like a clock*
*Resets your own sense of time*
Rebecca Elson

# Acknowledgments

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# Part I
## Warning: Jokes and opinions

In the following chapters I may occasionally digress, externalizing my internal reflections.

Sincerest apologies.

# 1
# Prologue

This is a literary experiment: an attempt to tell a story about a scientific experiment. I do not know if I can do it well, but I am resolved to make an attempt.

If you don't want to believe all this, I understand. In this writing, I will not tell a complete history of the experiments and theories that build up the physical models that motivate this project. That endeavor would no doubt occupy me for centuries, and I am not so optimistic about my life expectancy. I choose to accept these models as useful approximations of reality. They seem to work well, considering the effectiveness of medicine, phones and the like. Also, it is more fun to learn to speak in that language and see what you can see, than abandoning the pursuit of truth, which—I want to believe—is out there.

Many things that are said about the universe make no sense to me. I don't understand how a thing can have no boundary, no end, not exist within any larger entity, and yet still be expanding. I don't understand the Big Bang—how everything can have started off so small and hot and gotten so big cold. Compared to what? I'm told the universe is disappearing from view. Where's she off to, I wonder, if there is nowhere to go? I am not upset by my lack of understanding. It's like the best part of a crush. Mystery and imagination. The thrill of the chase.

In my youth I was a sheep, born to and herded by believers, taught to have faith in science. As a child of Jesus might begin to question the God/Son/Spirit conundrum, I too began to desire more rigorous explanations of the grand claims that science men[1] espoused. I heard rumors about "electrons" and "black holes" and I wanted to know what I could know. I began studying physics with a pure curiosity that was weakened and lost and found in the academic industrial context. I found some understanding and amazement, and I came to believe that awe is a gift, and a practice, and a responsibility [1].

When I wasn't learning about the natural world, I was learning about my self and life, which I eventually realized, is another part of the natural world. Humans are just clumps of stuff, wiggling around and flying through a little part of a big space. I have found the study of self and others to be another source of awe. Amazing how different we are! How lovely and annoying we can be. I also came to the belief that we the people cannot be divorced from the science. In my *subjective* opinion, there is no such thing as objectivity, and for this reason, I shall be present in this writing. I will not pretend that anything represented in these pages is True. I will try my best to honestly represent my current understanding of physical principles, and the methods by which I conducted my science quest, but I might be wrong or make a mistake, and later in life I might draw different conclusions or change my mind.

This study is broader in scope than one particular science experiment. It is also an investigation of communication: the research questions: how is information moved from one mind to others? What of one person's understandings are even commensurable to another mind? You know when you are talking to someone and it feels like the words they are stringing together have no apparent meaning in relation to the words you strung together? This is one of the most agonizing and distressing experiences for me. It makes me pessimistic. This document hopes to pass along some of the perspectives I gathered in my time away, not as a demonstration or record of knowledge acquisition, but as an act of optimism. I do hope that awe can be transmitted through a detailed, somewhat technical and (hopefully) entertaining explanation. Science is

[1] Watch *Cunk on Earth*

useful, but if the good people of this earth cannot understand it, and wield it's power, if they are expected to believe in it on an inherited faith, or in political allegiance, or reverence for an intellectual elite, then it may as well be propaganda. (Not that propaganda isn't effective, though it is expensive.) I understand why people don't believe in science. It is shrouded in vocabulary and scientific communities are exclusive to many, but it is incredibly powerful, and important, and I want people to believe, because it is a useful tool in efforts to decrease world suck[2]. My experiment is in the pursuit of accessible and comprehensible communication. It is not for political reasons that I want people to experience and be convinced by the power of scientific learning. I also think that there is something beautiful and magical about understanding. There is a lot that is awesome, and I want people to enjoy it.

The moments of my education in which I experienced the most awe, have been those I spent working on research projects related to a huge experiment conducted by the Laser Interferometer Gravitational-Wave Observatory (LIGO) (pronounced *lie - go.*) I will explain what all of these words mean in due course. The project I will describe in the following pages is a very very small piece of LIGO's efforts to understand what is going on in deep space. Detecting gravitational waves turns out to be a bit of a tough job, but nicely, doable. I hope that I will be able to explain it in a way that is awesome and not entirely boring or annoying...

A final note: I have met some people who experience acute distress and fear when faced with the magnitude of the cosmos and the smallness of the earth. I have one friend who calls this phenomenon, "having outer space thoughts." Outer space thoughts usually arrive in the night, when local objects become difficult to see, and distant ones shine through the darkness, so far away, floating around in a nothingness. (I personally find this experience, of shrinking away into a vast expanse, to be comforting. It is a great relief to me, to remember that the consequences of my actions or idleness are not at all significant on a cosmic scale of time and space.) I suggest that people experiencing outer space thoughts should skim the introduction in the light of day. This sets the scene: space, and the motivation: knowing what's going on out there. The chapters

[2]Credit to the Green Brothers for popularizing this phrase

that follow focus on the models physicists use to describe physical phenomena, how those models are used to plan a science quest, and the adventure itself, which takes place in a very grounded setting: a basement!


Figure 1.0.1: Basememnt of the Reem-Kayden Center for Science and Computation

# 2
# Introduction: where, who, what, why, how

Our story begins above our skies, deep in space, where fantastically enormous events occur, causing a cosmic ruckus that echos all around the universe. Millions and billions of light years away mysteries abound.[1] Dust floats in gaseous clouds and stars burn with ferocious exuberance. Black holes swallow their surroundings, and things of middling size float about too, all in one great improvisational dance.[2] Tales from these far off places are carried on light, to our eyes, and in more detail, to our cameras. We look and listen and learn what we can, trying to see what we can see, and come closer to the mystery.

## 2.1 Where: spacetime

Before I begin to describe gravitational waves, you must first have an understanding of the medium through which they travel. Ocean waves are made of water. Sound waves are compressions of air, which enter our ears and beat on our ear drums. Gravitational waves propagate through *spacetime*, which is the stuff out of which the universe is made. It is the environment in which everything occurs. Humans exist in oxygenous environments, and jellyfish exist in the ocean, (or in aquariums I guess,) and everything exists in spacetime. Planets, stars, and

[1] For a sense of how much a million is, I recommend checking out the book *How Much Is a Million* by David M. Schwartz. Coming soon to a Bard Stevenson Library near you.

[2] Or is it choreographed...

asteroids and your crush. It is space, plus time—turns out they are stuck together, in holy matrimony; not even in death shall they part. Something to aspire to... Maybe.

In empty space, humans don't work, for there, we cannot breath, for there, there is not air. A clock in empty space though, does work (at least so long as it's battery does.) The brave little hearts tick on, and from this we intuit that wherever there is space, there is also time, thus we name of all the space where there isn't stuff, space*time*. The intricacies of spacetime are not clear to me, (or anyone, I'm pretty sure. Though Einstein had ideas, and people are thinking.) but for the purpose of this experiment it is enough that we say that spacetime is the setting in which our story (and all other stories) take place. A *gravitational wave* is the event of spactime itself momentarily changing shape. Don't panic, dears.

## 2.2 Who: Albert, LIGO

Albert Einstein, heavy hitter in $20^{\text{th}}$ century physics, predicted the existence of gravitational waves in 1915, in his General Theory of Relativity. His equations said that the coalescence and collision of super dense objects like black holes would result in perturbations of spacetime itself. 100 years after Einstein published his prediction of gravitational waves, people measured them. Those people were members of the LIGO collaboration. That's the Laser Interferometer Gravitational Wave Observatory, in case you forgot.

In the 1970's people started to think about how they might go about confirming Einstien's theory experimentally. They spent a while doing math and materials science, and eventually, in 1999 finished building a few special measuring tapes that would be able to detect changes in spacetime itself. LIGO is a global collaboration of scientists who work on the project of gravitational wave detection [13].

## 2.3 What: Gravitational Waves

Gravitational waves displace the very stuff that the universe is made of. This is strange. Parcels of spacetime get squished and stretched, causing points in space to move closer together and farther away as the wave passes, like the rings of a slinky when it's stretched and squished. Figure 2.3.1 shows an exaggerated visualization of the effects of a gravitational wave, as it passes through the ever unlucky Charlie Brown. Above his heads, are representations of two massive objects engaged in a pas de deux, in the relative orientations to Charlie that would produce the corresponding effects on him. Below his heads are schematics of how the LIGO detectors would react to the movement of celestial bodies.

Gravitational waves are often called "ripples in spacetime," but I prefer "the little squish and stretch," or simply, "gravitational waves." They are very small. The displacement of space that is measured by gravitational wave detectors is on the order of $10^{-18}$ meters at most. This is smaller than I can imagine. But not smaller than Einstein could imagine.

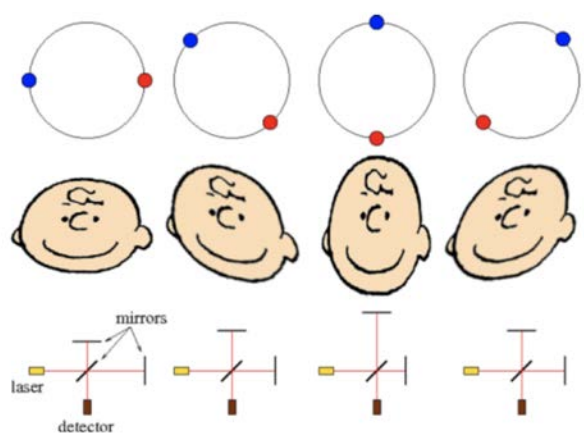



Figure 2.3.1: A Charlie Brown Gravitational Wave

## 2.4 Why: well, for fun

People were fairly well convinced by Einstein's theories, but thought it would be nice to prove him right. Gravitational waves also provide us a new way to map distant space. It is hard to see things that are very far away from us because there is other stuff blocking our view. Gravitational waves pass through all of this, enabling us to have a better understanding of the geography of the spaces we can't see, and opening up the possibility for multi-messenger astronomy. If a gravitational wave is detected that is expected to have come from an astronomical source that will let off light, such as a supernova, LIGO can inform telescopes of the place in the sky where they can expect to collect that light. Telescopes in general, look around at whatever astronomers are interested in, and it is unlikely that they would be pointing toward the exact sky location where a supernova was taking place, at the moment of the event. With notice from LIGO, they may be able to see the aftermath of the explosion.

By studying the physics of extreme astronomical phenomena, we can learna lot about the fundamentals of how things work on earth[3].

The detection of gravitational waves probably isn't going to save anyone's life, but the research that goes into developing the technologies that are required to accomplish such a measurement, can be applied to problems in other fields. The project to be explained in these very pages aims to mitigate a hindrance to gravitational wave detection, as well as advances in medicine (cool) and phones (boo.)

## 2.5 How: detectors

The LIGO detectors are made out of lasers, lenses and mirrors that are housed underground in vacuum sealed rabbit holes. Just kidding. The LIGO detector tunnels are a lot cleaner than rabbit holes. The detectors first ran for 8 years, from 2002 - 2010, without detecting any gravitational waves. In 2015, at the start of the second observing run, LIGO made their first detection of a binary black hole merger (two black holes orbiting and then eating each other[`?ligo`].

The experimental technique used to measure the squish and stretch is called interferometry: the measurement of interference. Waves are combined, resulting in a different wave which is detected, decoded and interpreted. In the case of LIGO, the waves are made of light, and the resulting signal is interpreted as distance traveled by the light—the length of the underground corridors. Details to follow.

[3] Also by watching *Cunk on Earth*

# 3
# Intermission: it's electric, fields, waves, light, and interfereometry

There are some things you should know.

## 3.1 Subatomic particles

You may have heard of *atoms*, famously small "fundamental building blocks" of stuff. *Molecules* are little groups of atoms, holding hands to form larger globs of stuff, also known as "matter." Atoms and molecules are bound together by love. And the *electromagnetic* force, which you will hear about later. The arrangement of atoms within a molecule depends on the ratio of *subatomic* particles that make up the constituent atoms. There are 3 ingredients that make up atoms: *electrons*, *protons* and *neutrons*. The relative abundances of these ingredients in the atom is what determines it's name, and behavior. For example: Magnesium is made of 12 electrons, 12 protons, and 12 neutrons, and it can help you feel calm. Gold is made of 79 electrons, 79 protons and 118 neutrons, and can help you get married to your crush. Arsenic is made of 33 electrons, 33 protons and 42 neutrons, and it can help you die in the event that your crush is unrequited.

Sub-atomic particles are massive (though their mass is minuscule), and they take up (a very small amount of) space. Electrons and protons also have a property called "electrical charge."

## 3.2 Electrical charge

There are two flavors of charge: *negative* and *positive*. These are arbitrary names, chosen to denote the binary nature of charge. We could have named them yellow and blue, but the choice of positive and negative is convenient, because we can think of amounts of charge as values on a number line, with uncharged particles having zero charge. Oppositely charged objects are attracted to one another, and like charges repel. We know this because science men did experiments. Neutrons, which cohabitate with protons in the center of atoms, have no charge. Protons and electrons carry the same amount of charge, of opposite signs, protons being positively charged and electrons negatively charged. When together, they create an electrically neutral object.

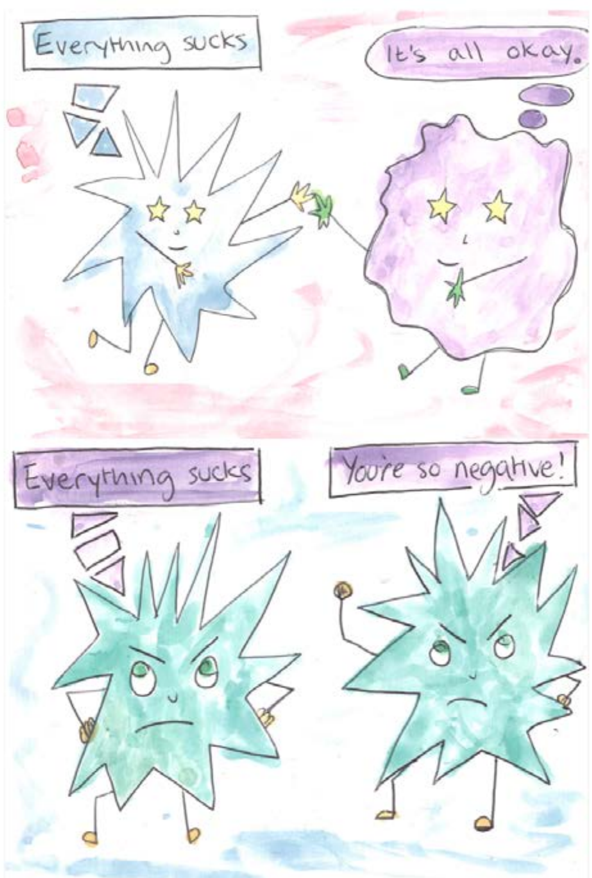



Figure 3.2.1: Positivity neutralizes negativity. Negativity exacerbates negativity.

## 3.3 Fields: visible and not

For clarity, in this section I will italicize the physical concept of a *field*, and use upright letters to refer to the familiar wide and grassy patches of earth.

A *field* is like a database. It holds information. In the language of physics, a *field* holds a piece of information for every point in space. Imagine a field of goldenrod. At each point in the field, a plant grows out of the soil.

Figure 3.3.1: Bard meadow, late September.

Two goldenrod plants are alike in many ways. They each have yellow flowers and whisker-like leaves adorning flexible branches, growing from strong green stems. There are slight differences between plants though. The width of the stalks, the number of flower buds per branch, and the length length of leaves all vary from plant to plant. We might describe the field as a nice place to go on a walk with your crush, or if we wanted a quantitative description of the field, we might describe it using *fields*. For every point in the field of goldenrod there are numerical values that describe different aspects of the environment. These values can be sorted by kind, collected, and referred to all together, as a *field*. One *field* to represent the height and thickness of stalks, a *field* that records the number of flowers and buds growing on each plant and a *field* that contains the concentration of pollen at each point in the air above the meadow. Thinking about one *field* at a time can help reveal patterns across the whole space. Are the height of the stalks changing with some regularity? Is there any relationship between the temperature in different areas of the field and the color of the leaves? How much sunlight reaches the different quadrants? What is the topology of the land? Where does rainwater flow and collect? *Fields* help us to understand spaces by compartmentalizing information, describing it efficiently, and illuminating patterns that were not visible when all the details were expressed together all at once. This is akin to sorting your rock collection by color, size, texture, or opacity. As you rearrange your rocks, different stories may reveal themselves to you that were not visible when you beheld the whole. Hopefully you have a sense of what the physics people mean when they say *field*. I will stop italicizing the concept and you may assume moving forward that every field is in reference to the physical concept, and not the physical object that sits outside. Good to go outside though. Good to rest, to take breaks. You should spend some time in a meadow. I invite you to invite your crush.

## 3.4 Electromagnetic field

Above a meadow, the field representing the amount of pollen present in a small cube of space will take on larger values than it would, for example, in the house of a person who is deathly allergic to pollen (or one should hope.) It is true that the pollen field exists everywhere, so we could say that an object "gives value to" the fiels around it, but this language is cumbersome, so more often we say that an object "creates" or "sets up" a field around itself. In our hearts though, we know that the fields are everywhere, just stronger in some places. Massive bodies create a gravitational field around them, and as a result, apples to fall into it. Likewise, charged objects set up electric fields around them, and cause other charged objects to fall into them or zoom away from them, depending on the flavor of the charge. Like space and time, electricity and magnetism are also in love and inseparable. Together they create the *electromagnetic* field.

## 3.5 Electric force

Maybe you have noticed that there are some forces in this universe that cause things to move around. Gravity, love, and maybe you haven't really thought about it, but the electromagnetic

Figure 3.4.1: Apple on earth.

force also exists. The electromagnetic force is much stronger than the gravitational force, despite us experiencing much stronger effects from the earth's gravity than we do from the subatomic particles that are everywhere. The reason for this discrepancy is that we are enormous and subatomic particles are tiny baby little imagined mathematical models. Luckily, most things such as bananas, roller skates, fish, and your crush are composed of communities of molecules, which, when convened tend to drown each other out, resulting in electrically neutral objects. This should come as a relief, as it would make human life more complicated if objects were flying hither and thither due to excessive quantities of rogue charge. Occasionally this does happen in a small way. People set up situations in which streams of charged particles move large objects. This is called electricity. Blenders, for example, liquefy your breakfast by directing river of electrons through your walls. How does this motion occur? Maybe you have seen the famous expression: $F = ma$ Or maybe you have not. It means, "how an object moves is related to how you try to move it, and how much mass it has."

Often in order to exert a force on an object, we must come into contact with the thing, and give it a nudge. The gravitational and electromagnetic forces are special because they exercise their power over other objects through telekinesis. That is not true. But they do exert forces on objects which they are not physically touching. The information stored in the electromagnetic field is the amount of force an object would feel due to the force field if it were sitting at that point in space.

The electric field falls off to zero as you get farther from charged particles, so when objects distance themselves from one another, the forces they exert on one another weaken, and as they get closer together, the force between them gets stronger. Interesting question to pose to people who are in love.

The direction of motion depends on the flavor of the particle creating the field and the particle being placed in the field. Both objects do create fields around them, but we imagine one of them to have much less charge than the other, so that the field created by the smaller guy does not have a significant impact on the field it is placed inside. (The apple has mass, but the field it creates does not move the earth very much.) A positively charged particle placed in a positive field will run away. And a negatively charged object will move toward the source. As a dear professor once put it in an embodied demonstration of electrical interactions: "I could be attractive, or I could be repulsive." And that's how it always is with a new crush I guess[1].

## 3.6 Vectors

A *vector field* holds a pocket full of[2] information at each point in space. The field is like a filing cabinet and the vector the folders. A vector looks like a list of numbers: $[1, 3, 12, 4, 25, 10, 28, 43, 88...]$ If we know the significance of each slot in the vector, we can interpret the meaning of the vector on the whole. In general, vectors can hold any amount of quantities, and even extend infinitely. This is horrible and painful to think about, so you don't have to try. I just want you to know that the world of math is a dark and dangerous place where infinities can break your brain if you aren't careful; see Figure 3.6.1. But despair not. The electric field has only three pieces of paper per folder in the filing cabinet, corresponding to the three dimensions of space.

[1] :/
[2] sunshine

Figure 3.6.1: Using helmet is a wise choice.

The pieces of paper tell us how a particle would move when placed in some. The first tells us how a particle would move side to side, the second how it would move forward and backward, and the third describes the vertical motion. If the value of the electric field at some location were represented by the vector $[1, 0, 2]$, a charged particle placed there would be pulled twice as strongly in the upward direction than the rightward direction, and it wouldn't move forward or backward at all. Vectors can also be represented by arrows. For example: in a field describing wind patterns the vectors would point in the direction the wind is blowing and the length of the vector would be longer where the wind flow is stronger.

## 3.7 Waves

Waving is what you do when you see your crush. Swoosh your hand back and forth, back and forth. The level of enthusiasm with which you wave could be interpreted as desperation or disinterest, and it could be quantified by the frequency of the wave:the number of times per second your hand moves back and forth, and the *amplitude*: the distance traveled by your hand left to right. Small, quick, casual? Like you are trying to help a plane land safely on tarmac?

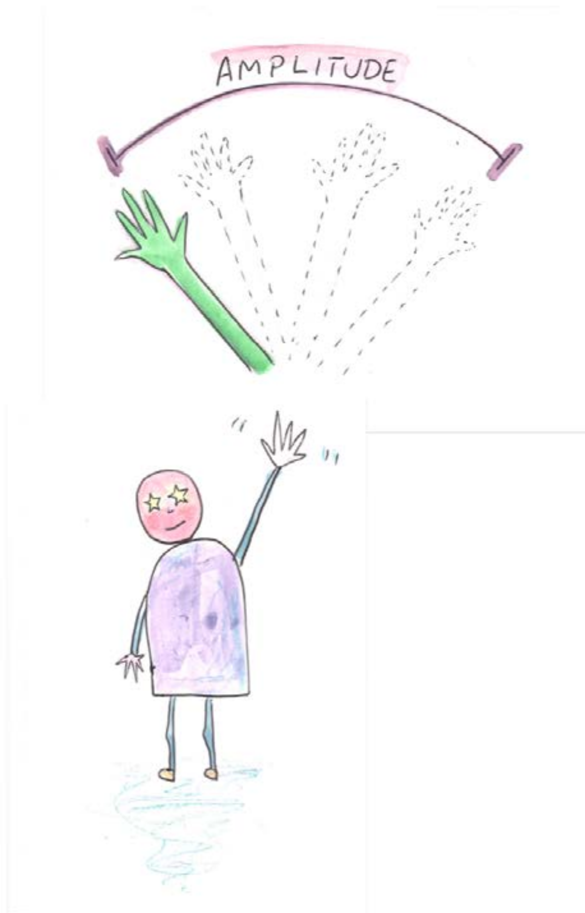



Figure 3.7.1: It is nice to wave.

The term "wave" takes on many meanings in the language of human people and in the language of science men. Let us understand oscillatory waves. Notice in Figure **??** that the waves go up and down, in an oscillatory fashion, hence the name, oscillatory. Imaging this wave in motion, zooming off the page. The distance from peak to peak is called the wavelength and the height from the center to the peak is the amplitude. By the relationship: $distance = rate * time$ we can find that the time it takes for one peak to replace the position of the next, the frequency $f = speedofthewave \div wavelength$.

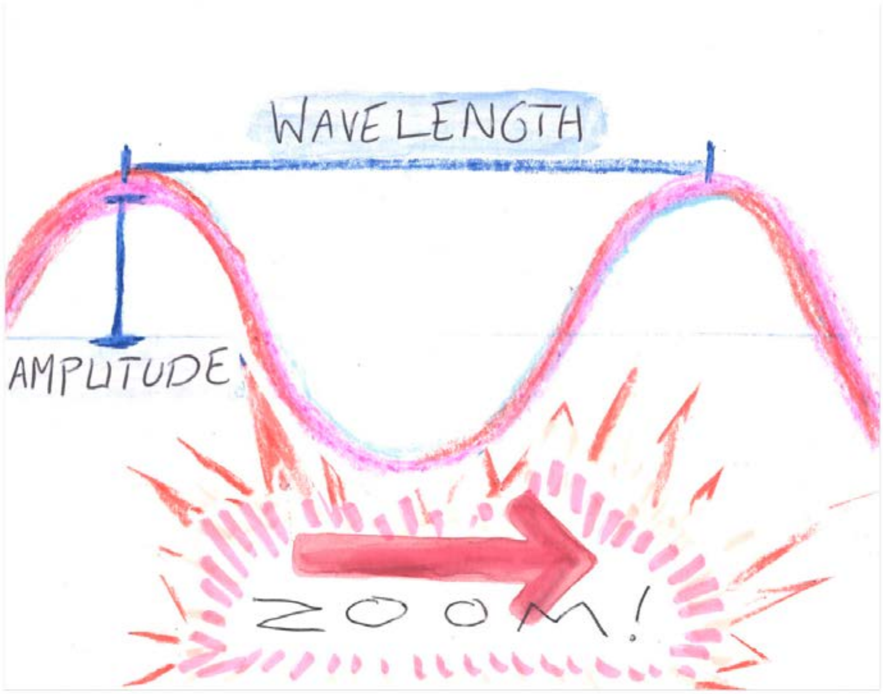



Figure 3.7.2: An oscillatory wave.

## 3.8 Light waves

Light is called an *electromagnetic wave* because it is the product of oscillations in the electromagnetic field. All you need to create light is move charged particles. Since charged particles set up an electromagnetic field in the space around them, moving charges change the field. Remember that the electric field is a vector. Light is a waving of the electric field vectors. Just like the air moves when you wave your arm waves through it, the magnitude of the electric field vectors at each point in space change as the particles move, and those perturbations have to go somewhere.

### *3.8.1 Light speed*

Light travels at $3 * 10^8$ meters per second, which is 671,080,887.616 miles per hour, which is fast. The fastest, in fact. We know this because two science men called Michelson and Morely attempted to prove the presence of the "luminiferous ether." Somewhat disappointingly, to

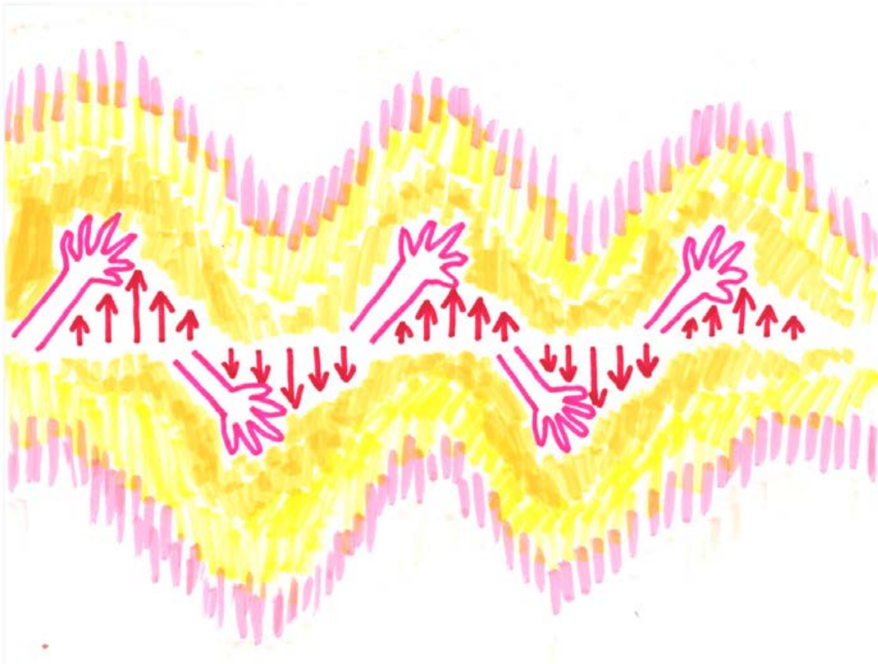


Figure 3.8.1: Waving in the electromagnetic field.

Michelson and Morley, they proved that it does not exist, and that light always moves at the same speed.

You can try to measure this speed using the same relation above: $distance = rate * time$. By stationing lamps atop mountains a few miles apart and having very good clocks, you can measure the amount of time that passes between the time of light on and the time of light seen. You may not succeed due to the limitations imposed by eyeball to brain to hand communication being slower than light speed.

Conversely, if angler fish had a wrists, better eyesight, highly accurate wristwatches, and knew the speed of light in saltwater, they could measure the distance between a special rock and the dinosaur under the sea. One of them would alight its esca, and and another would record the moment it sees the light. They would reconvene, compare and compute.

LIGO uses the same technique that the angler fish to measure the length of their rabbit holes. They send light down two corridors, and compare the time it takes for the light to return. As a gravitational wave passes through the earth, the space in which the detector sits will undergo a

Figure 3.8.2: Measuring the speed of light.

little stretch and squish. The difference between the length of the arms of the detector tells us something about the source of the gravitational wave.

## 3.9 Electromagnetic spectrum

if a wave must travel at the same speed, then as the peaks become more frequent in time, the distance between them, the wavelength, must become shorter in space.Charged particles are restricted in the ways they can wiggle and jiggle, depending on the molecules in which they live, and the different movements produce waves of different shapes. This results in the electromagnetic spectrum[3].

[3] Similar to the Kinsey Scale, but with light and not lesbians. The electromagnetic spectrum represents what qualities or properties the different frequencies of light exhibit

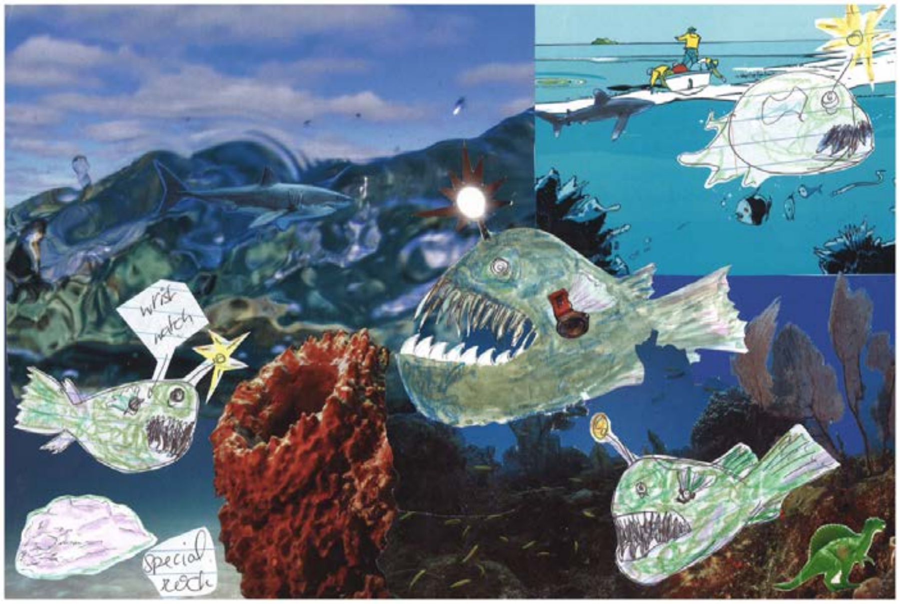



Figure 3.8.3: A gaggle of angler fish measure the distance between the special rock and the dinosaur under the sea.

What we call color is our eyes' interpretations of different wavelengths of light. If you could travel at any speed, a ray of light could appear to be any color. Human eyes can only see a sliver of the electromagnetic spectrum. Some light waves have a wavelength of $500 * 10^{-9}$ meters, and these we call "blue." Others have a wavelength of 1 meter and these we call "WRTI 90.1 FM member-supported public radio in Philadelphia." When a radio wave comes into contact with an electronic device, charged particles in the device begin to move, due to the changing electric field. The radio is designed to convert the electromagnetic waves into sound waves. Pretty neat, I think.

Much of the light we detect from space is invisible to us. We see it by arranging charged particles in objects (telescopes) and putting them outside to collect it. The sensors in the telescope oscillate in a characteristic way depending on the wavelength of the impinging light, and astronomers use computer math to interpret the signals that result. Many of these telescopes have nice names like "Very Large Telescope."

[poem about the little boy and the light?]

## 3.10 Interference

Light can be described as a ray, or a line, starting from one point and ending up somewhere else, and I will sometimes draw it that way, but is important to understand as a wave because much of it's behavior can only be explained using waves as a model. One important behavior of waves is their ability to interfere with eachother. To interfere is to meddle, like you do when your friend has a crush[4]. Waves interfere when they are *superimposed* with one another—that means, put in the same place. When waves superimpose, they add up.

Two extreme examples of wave interference:

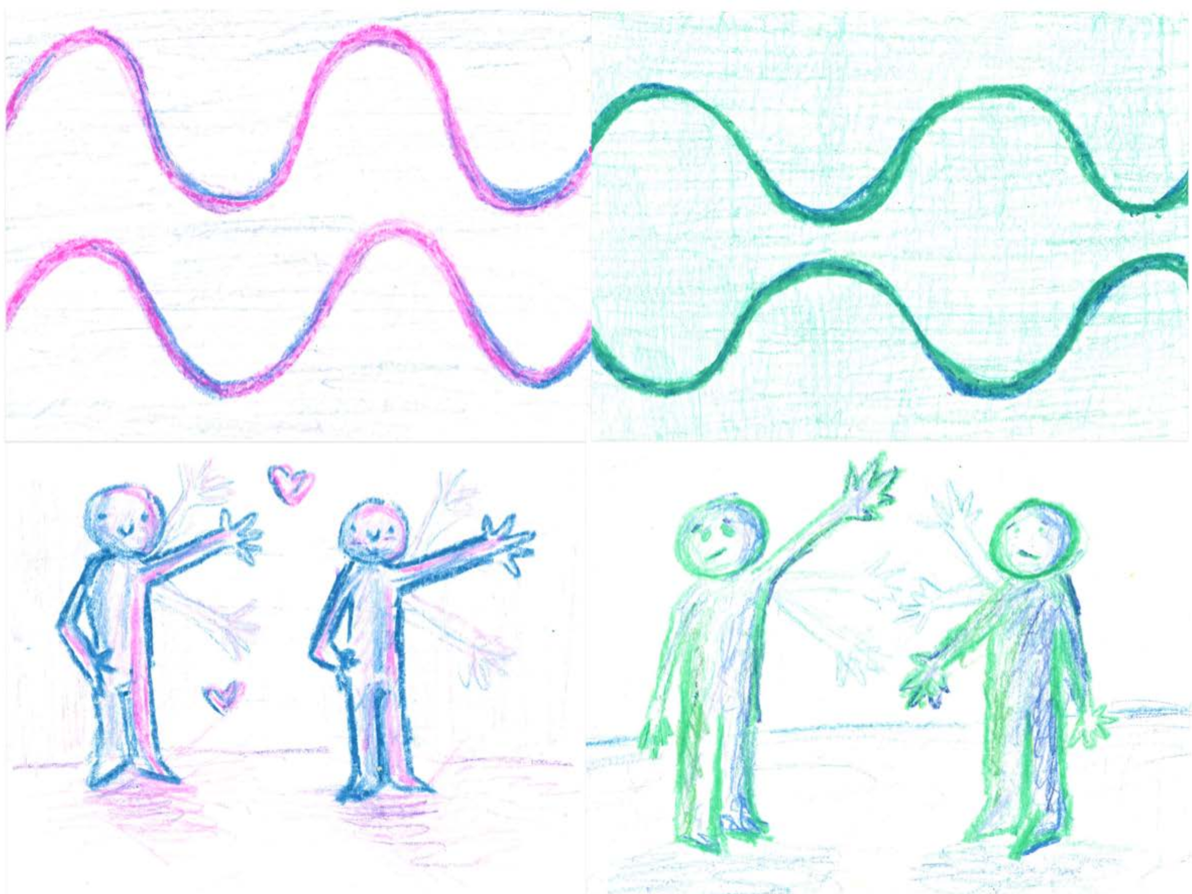


Figure 3.10.1: When waves are in phase, they constructively interfere, and when they are out of phase, they deconstructively interfere.

[4]If you are a good friend

In Figure 3.10.1 the two identical waves are *in phase*, the peaks and troughs are perfectly aligned and if the two waves were stacked on top of each other the amplitude of the resulting wave would be double that of the starting waves. The two waves would be *constructively* interfering. Figure **??** shows two identical waves that are perfectly misaligned. If they were brought together, the heights of the wave in each location would add up to a flat line. This is called *deconstructive* interference.

For a felt sense: You know when you are chatting with your crush and you are on the same wavelength, simply perfectly in sync, and it dramatically amplifies your feeling of well being? This is constructive interference. Conversely, when you are completely missing each other and you feel confused and dead inside, you are having a lived experience of deconstructive interference.

Most social interactions do not fall into these two categories, and likewise, physical phenomena outside of highly controlled lab settings do not often exhibit these extreme effects. Most often in the wild, the superposition of waves results in a new wave with a more complicated pattern of peaks and troughs. When we encounter these more common and complex waves in the wild, we can deconstruct them into a cohort of constituent of waves of singular frequencies and amplitudes.

The readout of the detectors actually comes as a wavy signal, that describes the interference of two beams of light. Mirrors are placed at the ends of the arms of the detectors, the laser travels down the corridors and bounces back to meet again, and combine. The result is one signal that can be decoded into the difference between the lengths of the arms.

## 3.11 Interferometry

Interferometry is the measurement of interference. Interferometers are devices that cause waves to interfere. Many interferometers are made with mirrors and lasers. A laser beam is split, sent in different directions, redirected to recombine, resulting in a more complicated wave which is collected and interpreted. There are many kinds of interferometers that are constructed by arranging mirrors in different configurations. The one used in LIGO is called a Michelson interferometer, named after our same science men who discovered the constancy of the speed of light

The LIGO interferometers look like this:

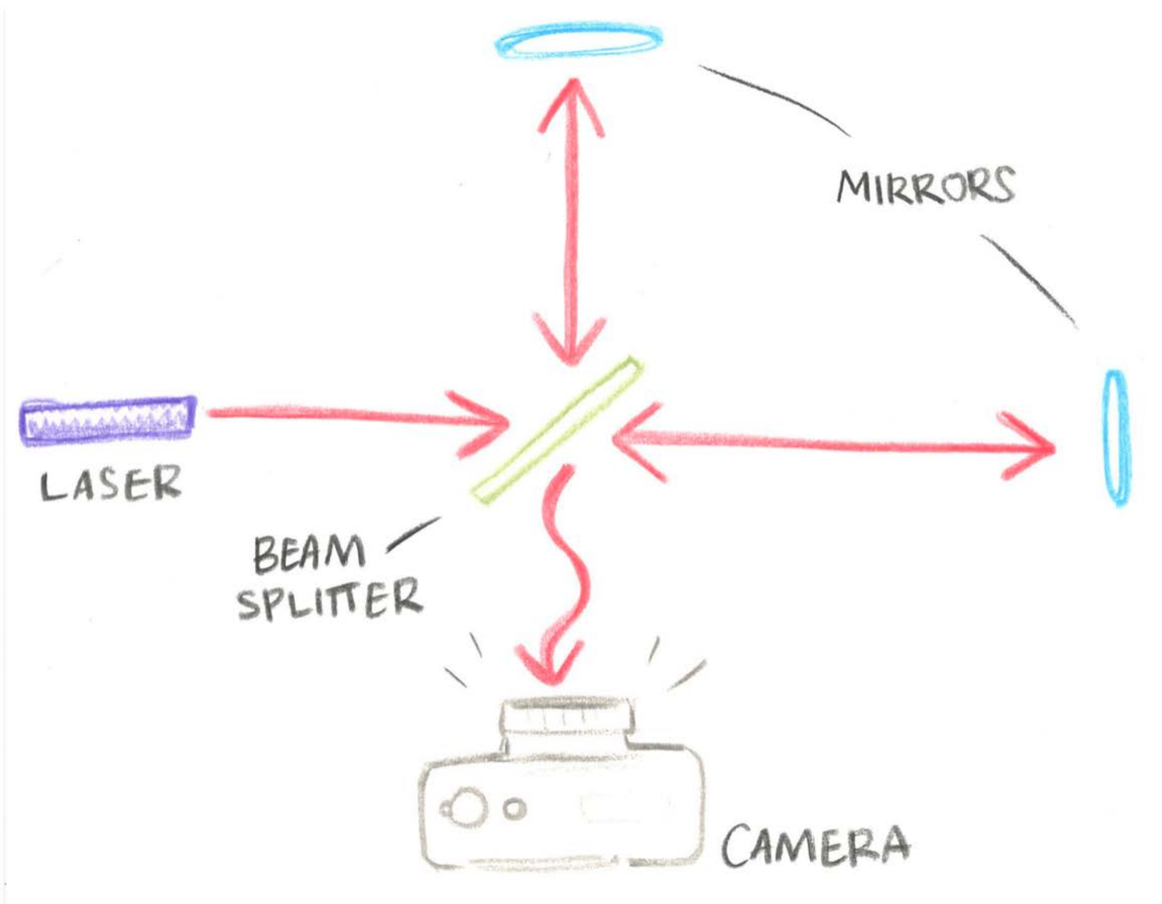



Figure 3.11.1: A laser hits a beam splitter, which reflects half of the beam toward one mirror and transmits the other half to the other. The beams return and are reflected and transmitted such that they recombine, produce an interference signal which is detected by a camera.

# 4
# More introductions: noise, mirrors, speckle

## 4.1 Noise:

Key components of the LIGO detectors are the laser, mirrors, beam splitters, lenses, the vacuum chamber in which the entire experiment is housed, and the electronics that control all of the systems. Many of the elements that make the measurement possible, also add *noise* into the data. Noise can be thought of like the noise at the bar. It is difficult hear what your crush is saying over the raucous Bard students. A cacophony of sound waves obscure their sweet sentiments. This happens in the LIGO experiment as well. Signals that we are not interested in muddy the signal we are looking for. Noise sources in the LIGO detectors are motors, temperature fluctuations, and imperfect mirrors. Any motion of objects in the apparatus will add noise into the data. The pure unadulterated readout can be described as the sum of various waves, some noise, and the waveform expected to result from gravitational wave.

Nicely, noise sources can be accounted for. People have spent a long time building mathematical models of different noise sources, and calculating their expected impact on the measurement. Different noise sources add a characteristic waveform to the LIGO readout. If the waveforms are understood, they can simply be subtracted from the output signal to reveal the shape of the gravitational wave.

Figure 4.1.1: A noisy signal.

The frequency of a gravitational wave is determined by the mass of the orbiting bodies, so if we really want to know who is out there, we have to quiet down some of this noise so that the detectors will be sensitive to more gravitational waves. The *noise budget*, shown in Figure **??** is a graph that shows the contributions of noise from various sources, at different frequencies. The noise budget is what limits the gravitational waves we can detect.

### *4.1.1 Scattering Noise*

One prominent noise source in LIGO is *scattering noise*. It is the erroneous signal that results from laser light that has scattered off of blemishes on the end mirrors. Due to the sensitivity of the measurement, the tiny amount of light that doesn't perfectly reflect back into the heart of the detectors makes a significant effect on the readout. People are studying these mirror defects to understand where they come from and how to prevent them, in the hope that in

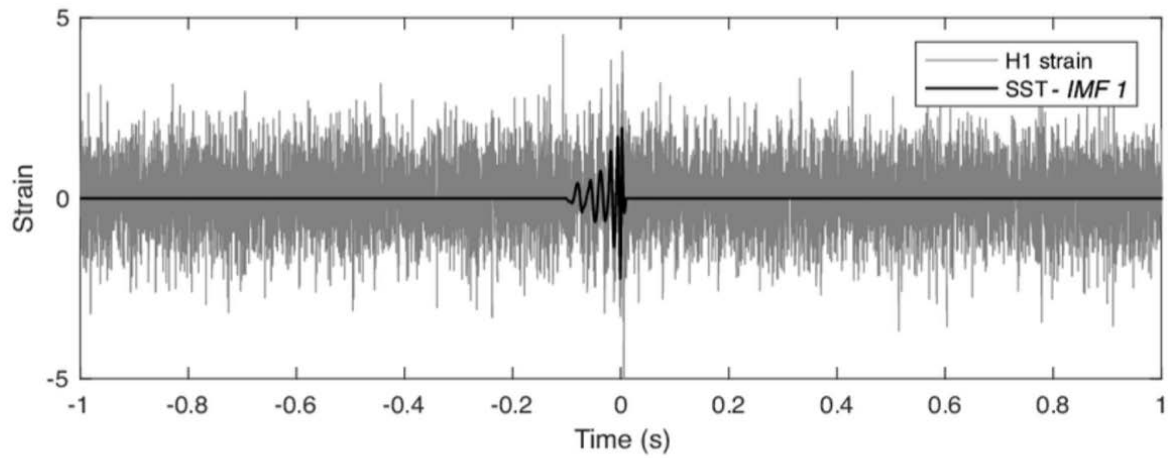



Figure 4.1.2: The noisy LIGO signal, with the gravitational wave hidden underneath [20]

future iterations of the project scattering noise can be mitigated. The experiment I will explain in the following chapter is an effort to improve those studies.

### *4.1.2 Mirrors*

The mirrors used in the LIGO experiment are designed to perfectly reflect one wavelength of light—1,064 nanometers, which is about the width of a human hair as exemplified in Figure 4.1.4. 1064nm light is outside of the visible range, in a region of the electromagnetic spectrum called "near infrared."

LIGO mirrors are covered with "interference coatings," layers of material designed to nearly[1] perfectly reflect specific wavelengths of light. When light of the intended wavelength shines upon the surface, the coating leverages the power of wave interference to maximize reflection and minimize transmission. As light passes through a material, some light is always reflected and some transmitted. Figure **??** illustrates the behavior of light in materials. The angle of reflection matches the angle at which the light came toward the mirror—the "angle of incidence." The transmitting ray does not pass straight through the material, but bends slightly inward. This is called *refraction*. As light moves through a substance, the charged particles inside cause

[1]Nobody is perfect.

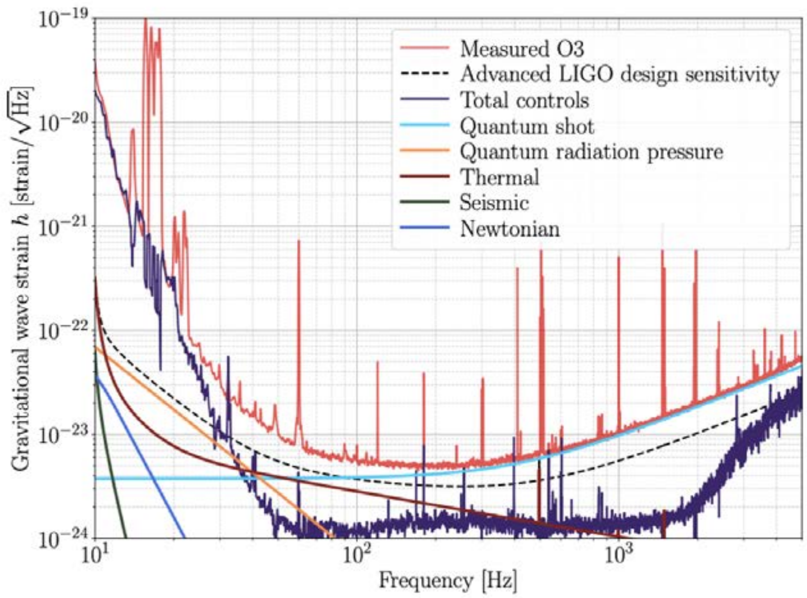



Figure 4.1.3: LIGO noise budget [13].

Figure 4.1.4: Human hair  $10\mu m$. Glasses for scale.

it to slow down. The *index of refraction* is a quantitative value, characteristic of a material that describes how light will move through it.

Different materials reflect and transmit light of particular wavelengths with varying efficiency. As an electromagnetic wave illuminates a material, the charged particles in the material begin to oscillate. This movement produces new electromagnetic waves and light is emitted by the material that may be of a different wavelength than the impinging wave. The shapes of molecules, and the crystalline structures in which they can be arranged determine how light will behave when it comes in contact with those materials.

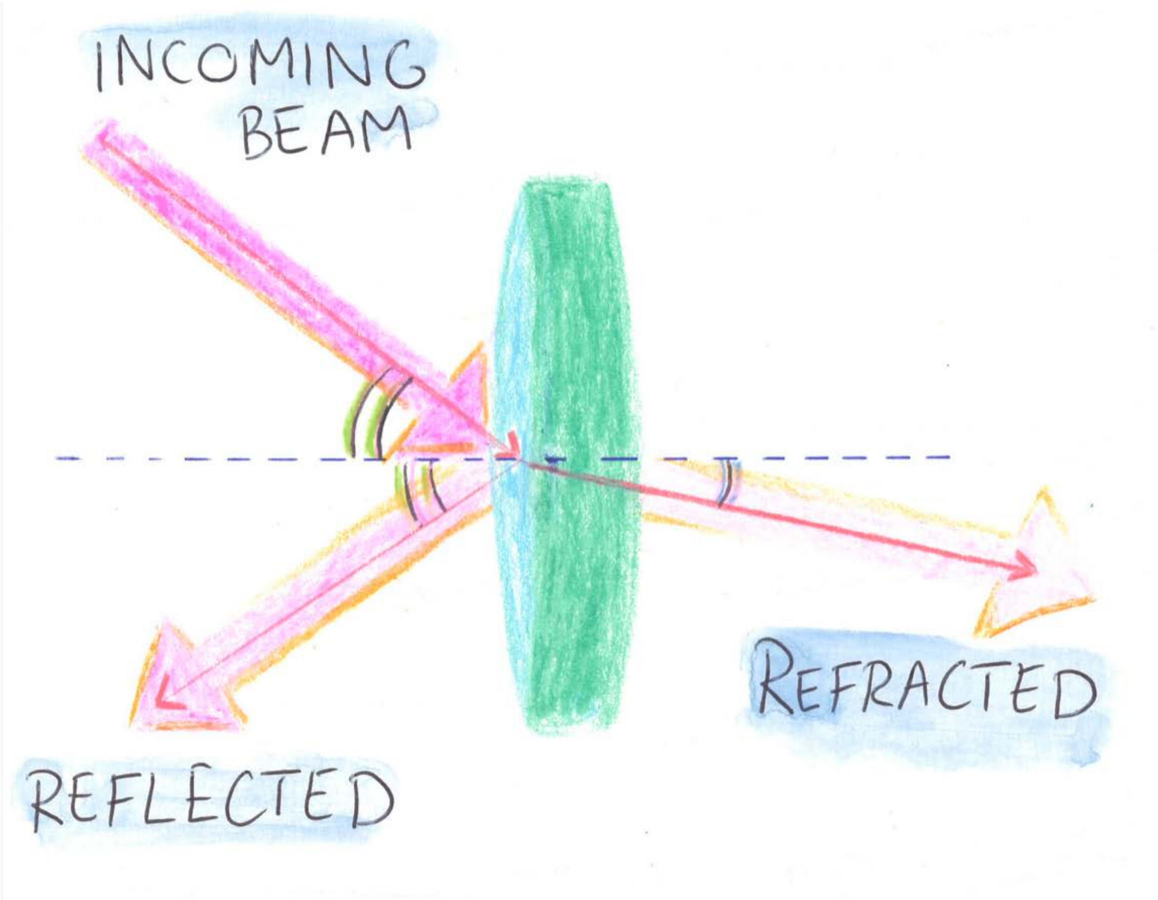



Figure 4.1.5: Light reflecting off of a surface, and transmitting through. Reflecting with equal angle as incidence, and refracting inward when transmitted.

There will always be some transmission through a material, but highly reflective coatings can be designed by layering different materials. As light passes through the top layer, some is reflected and some is transmitted through to the second layer, which is chosen such that the light reflected back will deconstructively interfere with the light that continues to pass through the first layer. Layers are stacked to establish many lines of defense against transmission through the mirror.

## 4.2 GOLAB

In the Gravitational-Wave Optics Lab (GOLAB), at Bard, Antonios Kontos and his motley crew of undergraduates study and characterize the quality of mirror coatings that are being

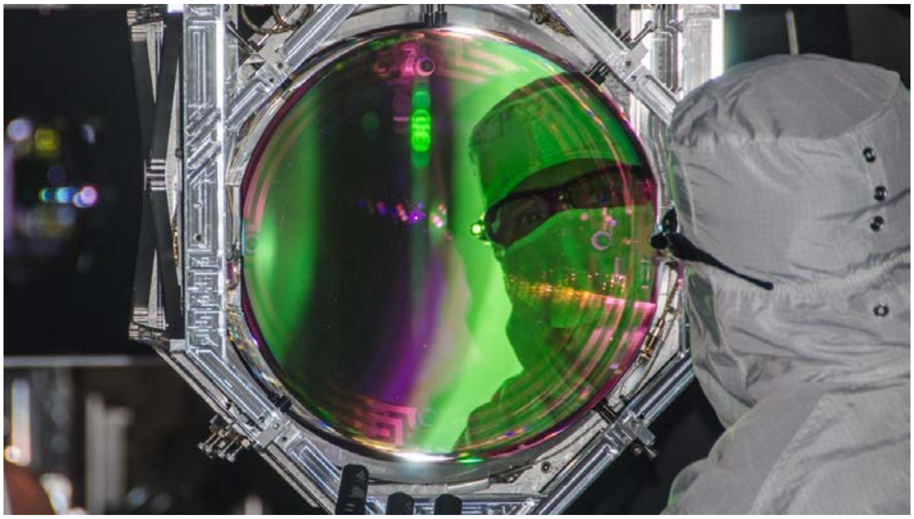


Figure 4.1.6: LIGO mirror under inspection [21].

developed for use in LIGO's future iterations. The primary features of the mirrors they are trying to understand are the defects that appear in coatings. The method of study is the measurement of scattered light

Scattering is a type of reflection. It is an example of the dissonance between simplified physical models and the imperfection of real things. Scatter is light that does not perfectly reflect off of mirrors and instead bounces around in other directions. Though the mirrors are nearly perfect, defects such as bubbles and crystals do appear. Studying how they scatter light is what allows us to characterize them.

These measurements are done by photographing the scattered light and comparing these images. We collect images and try to interpret the intensity of scattering at different angles. From this distribution of light we deduce the shape and structure of defects. One major challenge in this endeavor is the presence of an interference effect called "speckle."

## 4.3 Speckle

Speckle is an effect that occurs when highly organized or *coherent* light scatters off of a smooth surface. Coherence is a property that describes the relationship of the phase of one part of a light source to another part. Light is coherent when each part of a wavefront is in phase with

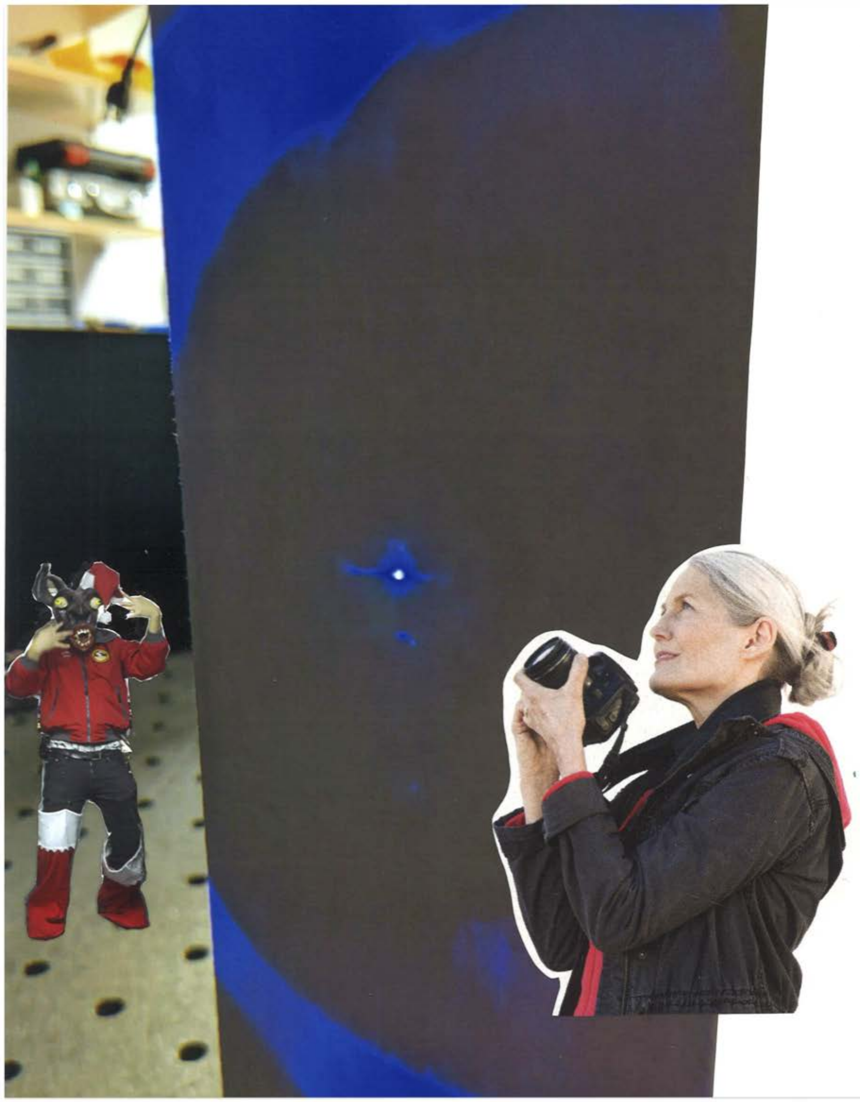


Figure 4.2.1: Scattering is frustrating, so we study it.

each other part. Instead of overlapping coherent waves to produce an amplified wave, rays of light are traveling next to each other, perfectly in phase[2].

Surprisingly (to me), it is the organization that results in disorder. Different parts of a scattered beam can interfere with each other and produce aberrations in an image. The interference causes patches of constuctive and deconstructive interference in the image that create a speckled effect. If the sections of the wavefront didn't have a special phase relationship, all the random phases would average out and you would see the image intensity as it actually is.

[2] I'm imagining synchronized swimmers.

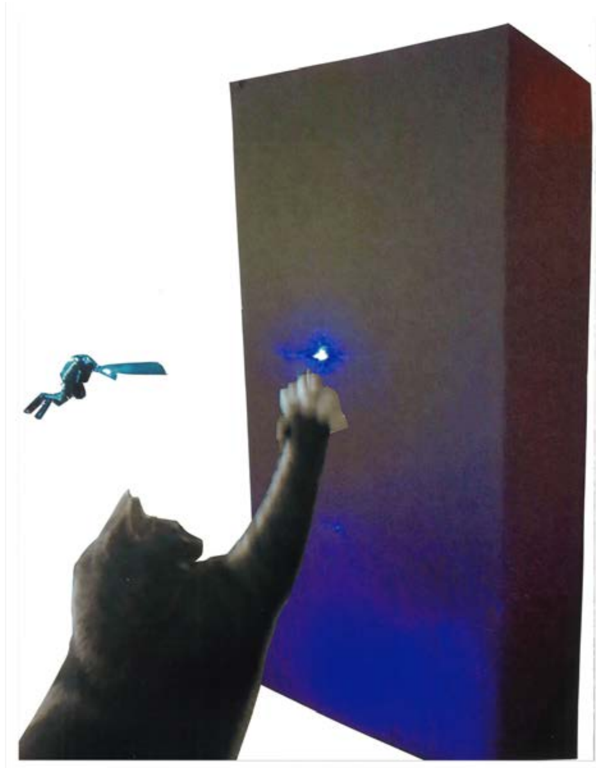


Figure 4.2.2: The cat does not approve of speckle.

We can model scattered light as an array of points, each emitting a spherical wave—one that propagates out in all directions from the point. Like a star.

A *wavefront* is the shape that is created by all of the parts of a wave that are in phase. A spherical wave would releacse spherical wavefronts that eminate from the center. The cross section would look like circles as in Figure 11.

We will think of each point on the scattering surface as a star, and model that as an array of stars. Since each star sends light in every direction, each pixel on the camera's sensor will receive a ray from each star.

Because each star is a slightly different distance from each pixel on the camera, light from one star will have to travel slightly farther than another to reach the same pixel. This means that two rays beginning with the same phase at the scatter surface will be in a slightly different phase relationship by the time they meet at a pixel. These rays will interfere and the amplitude of the combination of waves will be affected. It will cause *speckle*.

Figure 4.3.1: A star emits light in all directions around it, not just in 5 directions. This is an urban legend.

Speckle is another kind of noise. As scattering was a source of noise in the LIGO experiment, speckle is a source of noise in the measurement of scattering. The GOLAB needs to image the scattered light of a coherent laser off of a smooth surface and we want these images to accurately represent what is present in the mirror coatings. If the defects in the mirror are of a comparable size to the speckle in the image, we cannot accurately characterize them. The speckle may obscure them or produce phantom defects.

Since speckle is a result of coherent light, we have one strategy with which to battle it—confuse the army of wavelets. Unfortunately, randomizing the phase map just creates a different speckle pattern. An organized randomization though, can disrupt the phase enough that the speckle is reduced.

## 4.4 Another brief intermission.

Note on the scientific method:

It is often a good idea to have a prediction of what you expect to see before you set out on a science quest, or a date for that matter. Predictions can help you prepare. Do you think it will

Figure 4.3.2: Cross section of spherical wave. Circular wavefronts propagate out from a star.

rain? You could confer with a weather app before departing. Typically, before a science quest one reads stories of past explorations. Since science is big and useful, and many people want to know the answers, but it takes a very long time to do science, people invented a thing called "Academia."

Academia is uhh, a global system of educational institutions that claim to learn for the benefit of all. Science people in Academia make science and publish it so that everyone can read it and know about the science they made, instead of having to do it themselves. This is actually a lie, because not many people have access to the journals in which science is published. But I am currently being educated by an institution that makes them available to me. So I read some papers to learn about the strategies that other people have used in the fight against speckle.

Typically calculations are done using theory and models like the ones introduced above. These models all come with math that you can use if you spend some time learning what Greek letters mean to physicists. Calculations will inform the equipment you use, the amount of time you set timers for, and allow you to compare your measured results to what the math men expected. If the findings are not aligned with predictions, one must make checks to try to track down the causes of discrepancies.

I will not tell you about math. If you want to, you can read Part II. Anyway, I did not do much math before setting out on my quest, because I spent a lot of this year in a panic.

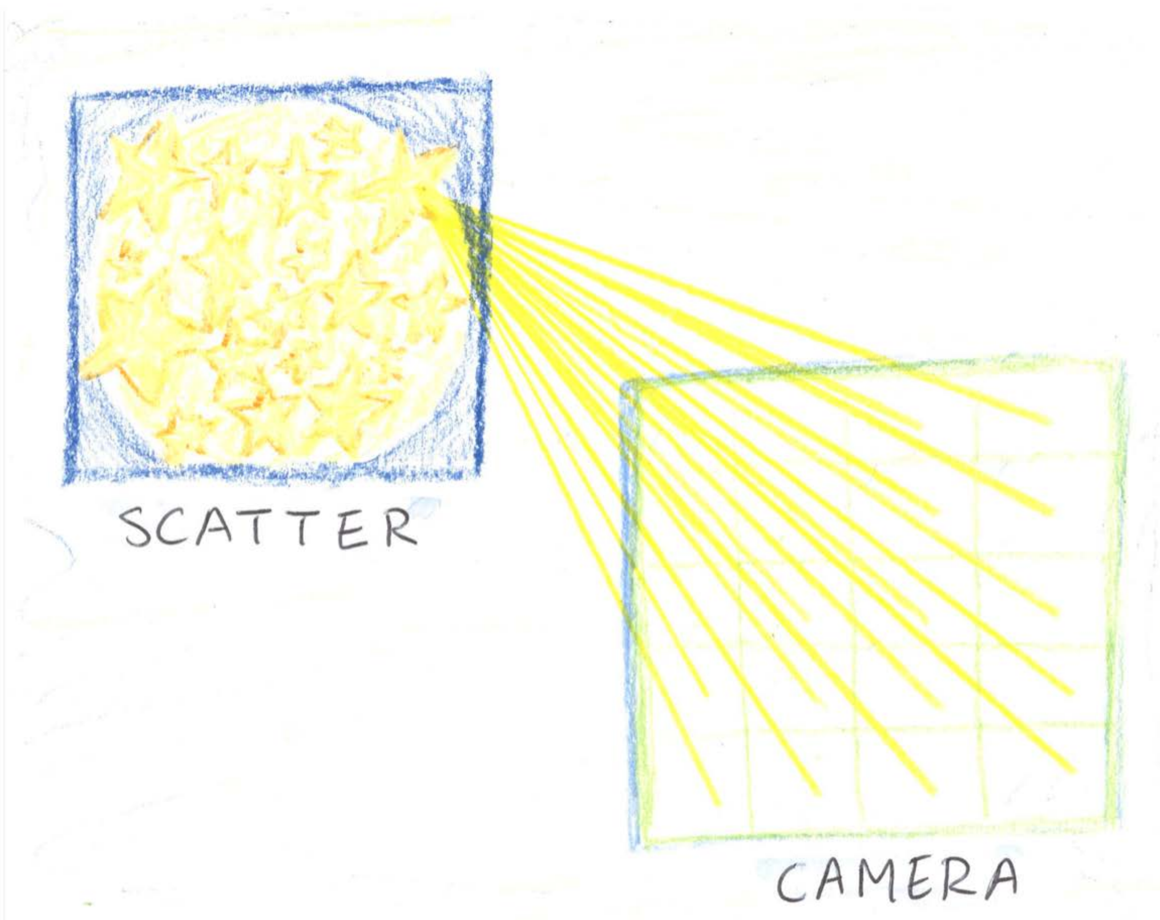



Figure 4.3.3: Each star emits rays that will reach each pixel of the camera.


Figure 4.3.4: When speckle interferes, things are not what they seem.

# 5
# Science quest

## 5.1 Despeckle

Finally we arrive at the science quest. We have learned that the problem is speckle, the perpetrator is coherent light, and the antidote is controlled chaos. The weapon of choice is a gadget called a Spatial Light Modulator (SLM.) An SLM is a pixelated mirror, in which each pixel is connected by a wire to a computer that can be programmed to change the way light is reflected off of each pixel.

Certain materials have molecular structures that will change orientation when the material is placed in an electric field. A commonly accessible source of an electric field is a wall socket. When an electrical signal is applied to the pixels in the SLM, the molecules in each pixel reorient, changing the index of refraction of each pixel, and consequently, the phase of each parcel of the wavefront.

SLMs can be used to create holograms, improve ultrasound imaging, and, they have also been shown to be effective tools in the despeckling of laser light. People have used SLMs to despeckle, often applying patterns called "Hadamard matrices" to the pixelated surface. A matrix is a series of numbers that can sometimes be interpreted as an image. The numbers encode instructions.

Hadamard matrices are special because the patterns they form follow a particular mathematical relationship. They are made of values 1 and -1, represented visually by square blocks that are combined to form a characteristic pattern. Figure **??** shows what would happen to the pixels of the SLM when different matrices are applied.

## 5.2 Set up

I will scatter light off of a smooth surface, use a lens to collect the scattered light into a concentrated point (remember incinerating bugs with a magnifying glass?) and then use a beam splitter (an object that reflects part of a beam, and allows the rest to pass through) to separate the light, half landing on the surface of the SLM and half flying into the abyss. The light then bounces off of the SLM and passes back through the beam spitter, transmitting through to the camera.

After arranging the equipment, I will take pictures: without any pattern applied to the SLM and then with various matrices applied. In order for the despeckling to occur, the phase pattern applied to the SLM must change in the time that the light is being collected. If it did not there would just be a consistent, though altered phase pattern. To facilitate this I will take the Hadamard matrices and switch the -1's and 1's. The SLM can take these two matrices and alternate between them at a variable rate.

## 5.3 Experiment

I generated Hadamard matrices of varying sizes, and applied them to the SLM. Each block of the matrix represents a section of the wavefront that will be phase shifted. The size of the blocks needed to reduce speckle is related to the wavelength of the scattered light, and the imaging system used to capture the light. I didn't take this into consideration before setting up my experiment, and instead experimented with matrices to see what would happen.

## 5.4 Analysis

Speckle is characterized by the *speckle contrast ratio* (SCR.) It is the ratio of the *standard deviation* of the intensity to the mean intensity. Standard deviation is a measure of how far most values in a distribution are from the mean, which is the average. The ration of the standard deviation to the mean always gives a value between 0 and 1. A speckle contrast of 0 means there is no speckle, and SCR of 1 corresponds to a maximally speckled image.

I calculate the speckle contrast by selecting an area of the image that has an approximately uniform intensity, and then I take the average of the pixel values accross that image, and the standard deviation of the distribution of pixel values, and compute the SCR. I do this for the same regions on the images in the moving and nonmoving (SLM and no SLM) cases, and find the difference between the SCR's in those two images. I repeat this calculation for each order of the Hadamard matrices I am testing, and for multiple sections of each image. Figure **??** in Part two shows some images of speckle results.

In the end there was no significant change in the speckle contrast ratio between images that were taken with the SLM on and off. This is likely due to the fact that the scale of the Hadamard cells on the SLM were not the appropriate size for the wavelength of our light and the geometry of our set up.

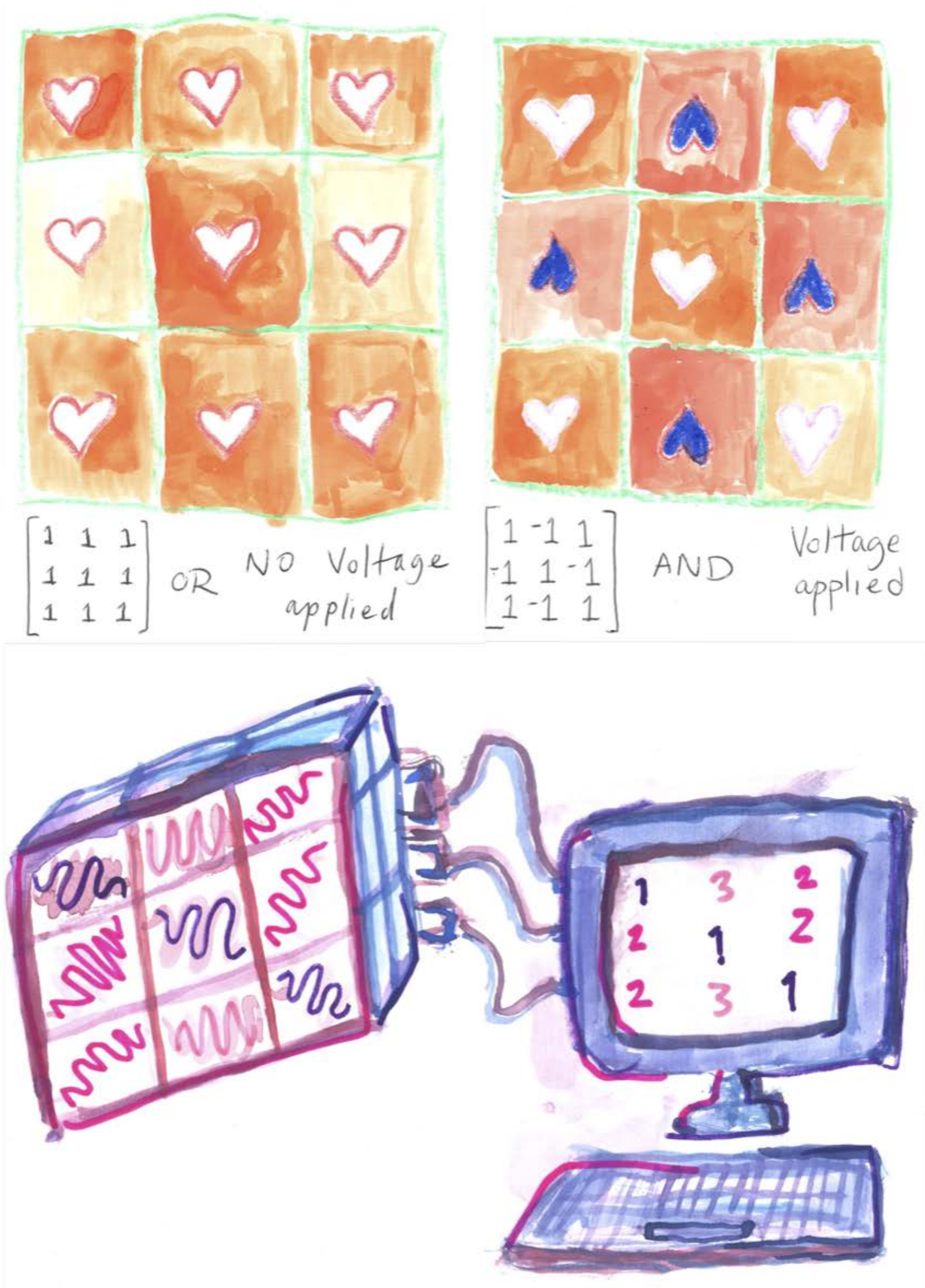



Figure 5.1.1: The SLM is programmed with an array of numbers that can change the orientation of the molecules in each pixel, imparting a phase shift onto the reflected light.

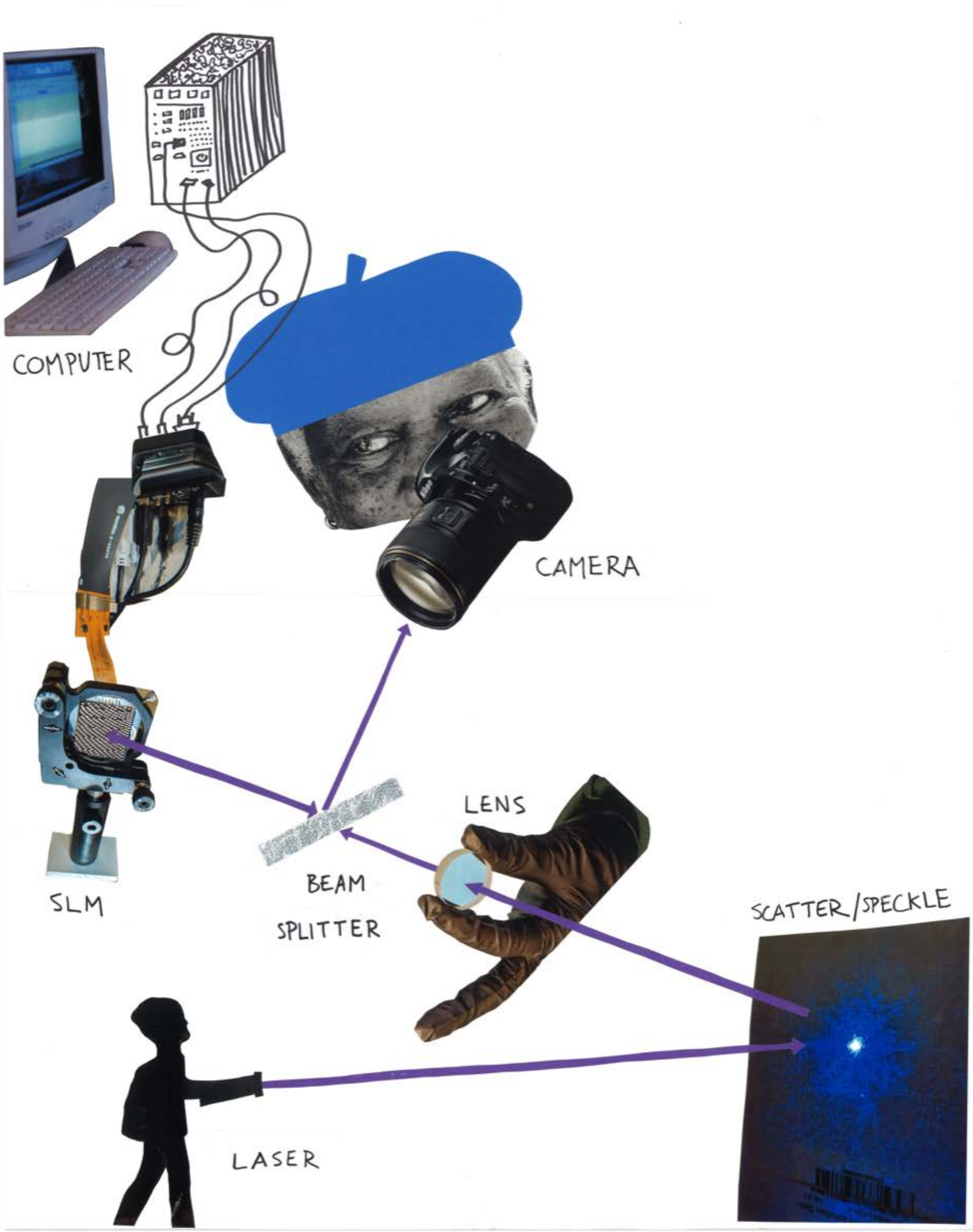



Figure 5.2.1: The despeckling setup.

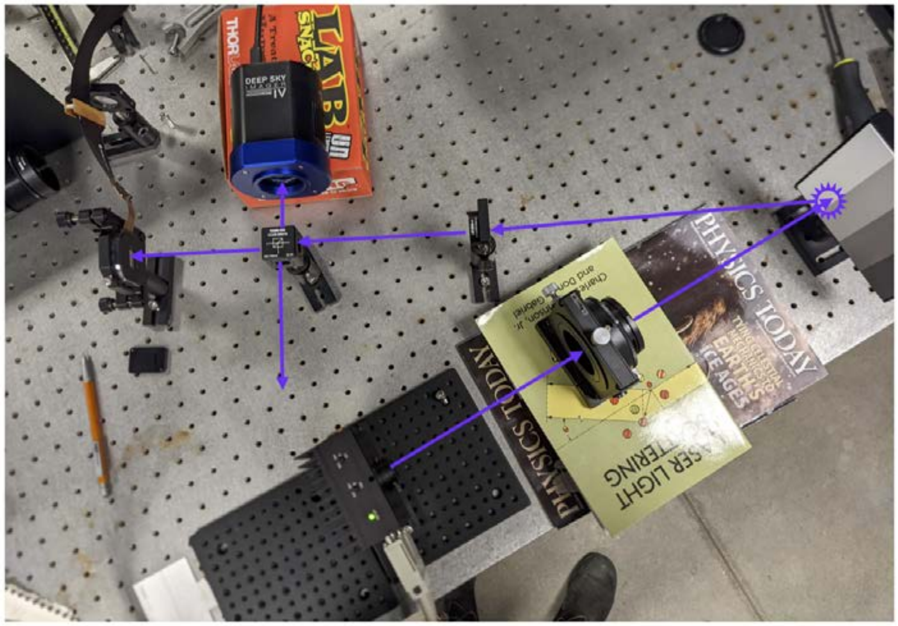



Figure 5.2.2: What the despeckling setup actually looks like.

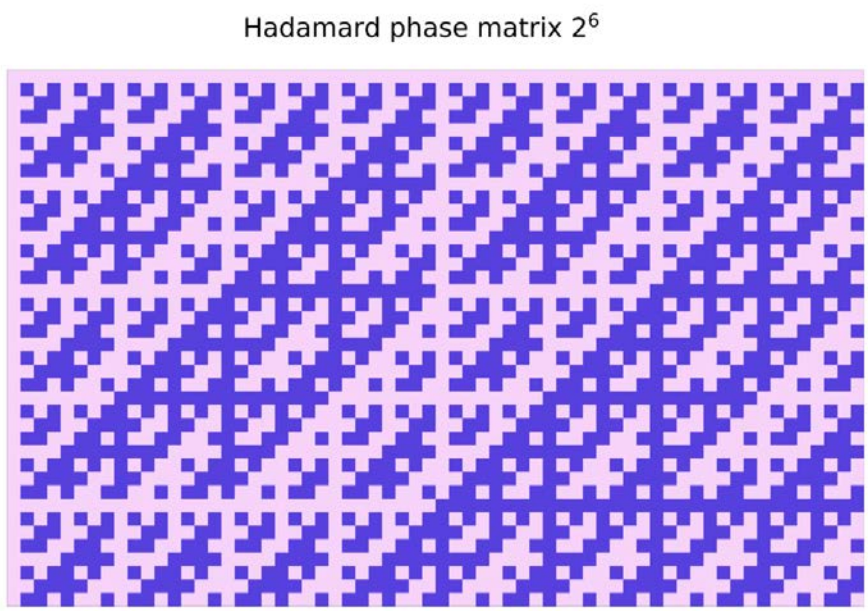



Figure 5.3.1: An example of a Hadamard matrix, of "order 6."

# 6
# Conclusions

Though the scientific results may sound anticlimactic, this experiment has clarified questions, and made visible new paths to go down in pursuit of despeckled light.

The results of this experiment were not conclusive. It is not clear from the preliminary results that there was consistent speckle reduction in any of the cases tested. And it is also not clear whether I was able to explain this in a way that non normal people can understand. This is a fairly typical outcome of a quest for me. Learn a little over a long period of time, feel intimidated and putz around for a long period of time, and then, right at the last minute, have a breakdown, cry, receive a visit from a fairy. Realize that the shadows were the ghosts of science men, snap into action, slap something together, and frown and shrug at myself because the tasks I was trying to accomplish turned out to be straightforward.

I will continue to work on this project, and hopefully the insights I have gained will help expedite next steps. This is not the first time I have been avoidant of research due to fear. Each time I think the next time will be amazing. I'm quite optimistic in this way. Things never improve as much as I would like them to. But this is how experiments go, they are circuitous and repetitive and if something is not working you just have to try every way you can think of to get something to work, and you may spend a long time trying to remember the value of $4 \times 4$. Maybe next time!

# Part II
## Warning: Abrupt Tonal Shift

In the following chapters I will use jargon and math to talk about the same things again.

# 7
# Background

## 7.1 LIGO

The Laser Interferometer Gravitational Wave Observatory (LIGO) is a global research collaboration that detects gravitational waves in an effort to improve maps of the distant universe and our understanding of astrophysical phenomena.

In 2015, 100 years after Einstein's prediction of the existence of gravitational waves, LIGO's two detectors in Livingston, Louisiana and Hanford, Washington made the first measurement of a gravitational wave. Since the initial detections 3 new detectors have been built: Virgo in Italy, GEO600 in Germany, and KAGRA in Japan [4].

## 7.2 Detectors

More than one detector is necessary to localize a gravitational wave in the sky and to confirm that the signal is indeed a gravitational wave and not spurious noise, but by further increasing the number of detectors in the network, sky localization becomes more accurate, as does the certainty of measurement. LIGO's detectors are Fabry-Perot Michelson interferometers, with arms stretching 4 km underground in high vacuum. As a gravitational wave passes through the detectors, the recombined beam experiences a phase shift that is interpreted as the differential arm length (DARM) [4]. The perturbation of spacetime that results from a gravitational wave is on the scale of $10^{-19}$ m. Their detection an extremely high precision measurement.

## 7.3 Noise

The raw readout of the detectors, exemplified in Figure **??**, is dominated by noise. Some noise sources are accounted for in post processing of the data, by calculating estimations of their contributions and subsequently subtracting them to reveal the characteristic signal of the gravitational waves. The frequency band in which the waves can be detected is determined by the masses of the bodies producing the gravitational wave. The sensitivity of the detectors in different frequencies is limited by the "noise budget," shown in Figure **??**. The colored lines represent different noise sources, with the vertical axis relating to their impact on the "strain," or the output signal of the detectors.

Noise is recorded and accounted for within the detectors by constant monitoring of the equipment. Accelerometers and seismometers are placed throughout the detectors. Some are connected to feedback systems that control aspects of the physical environment, such as temperature, pressure, power, and others are used to detect geological and anthropogenic noise, most of which frequencies between 0.03 Hz to 3 Hz [4]. Every part of the process of gravitational wave detection is limited by noise. In this work, I focus on noise caused by optical surfaces within the detectors, specifically the mirrors used in the Fabry-Perot cavity.

### *7.3.1 Mirrors*

LIGO mirrors are designed to highly reflective at normal incidence for 1064 nm light. They are coated with alternating layers of silicon dioxide and titanium dioxide doped tantalum pentoxide, creating zones of deconstructive interference between layers. Coatings are deposited via ion beam sputtering to ensure they remain in an amorphous state [12]. Mirrors are inspected by two independent observers in three stages, using three illumination devices that vary in intensity depending on the section of the mirror being inspected.

These mirrors must be pristine. For a sense of the scale: within the central 120 mm diameter, the maximum number of defects larger than 4 $\mu$ m is 10, and outside the central 120mm diameter 100 total defects are allowed. The mirrors themselves have a diameter of 350 mm [18]. Surface roughness on the mirrors due to variation in orientation of molecules, and deposition processes, is on the scale of 1 Å [15]. Though these mirrors are nearly perfect, it is inevitable that defects in the coatings will appear, either in the deposition process, or as a result of thermal heating of the optics while they are in the detectors. Defects in the coatings along with the surface roughness scatter light away from the cavity, adding noise to the output signal.

### *7.3.2 Scattering noise*

This work is motivated by efforts to reduce scattering noise, which impacts frequencies between 10 and 120 Hz. Gravitational waves in these frequencies are expected to be produced by very massive binary black hole mergers. Scattered light is either lost to the environment, reducing the power of the laser, or re-coupled into the beam, disrupting the coherence inside the cavity, and the accuracy of the strain [19]. At very low frequencies, the magnitude of the noise can reach $10^{-18}m/\sqrt{s}$. The amplitude of scattering noise related to the amount of scatter that recombines and the frequencies that it affects has to do with the relative motion of the test masses and the reflective chamber walls [4].

Total scattering from mirrors is measured by scanning and illuminating mirrors, and integrating the total reflectance. An example of the Total Integrating Sphere (TIS) measurement is shown in Figure 7.3.1 [2]. In order to mitigate scattering noise, we must understand not just how much light is scattered, but how it scatters, and the nature of the scattering defects.

### *7.3.3 Scattering experiments*

As mentioned above, scattering is the result of surface roughness, point defects that result from deposition process, or grain boundaries in crystals that can form as the mirrors are illuminated by the high power laser. New coating recipes and deposition processes are being explored in

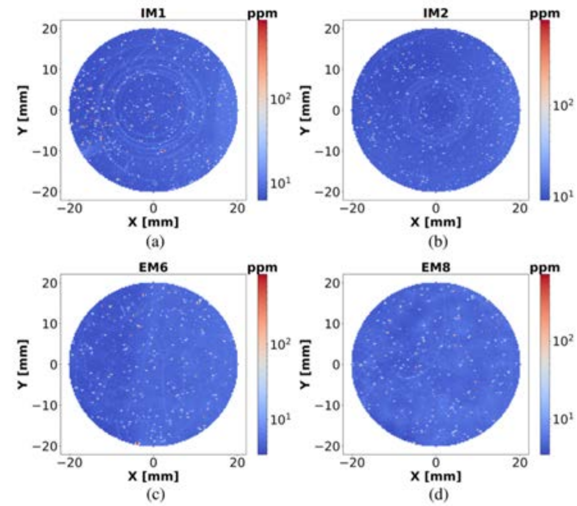



Figure 7.3.1: The Total integrated scatter is measured by illuminating every point on a mirror surface and collecting the total light scattered in all directions from that point. It creates a scattering map of the mirrors [2].

efforts to limit the population of defects in coatings and improve resilience of the mirrors after installation into the detectors. These coatings need to be tested to determine whether they are an improvement on the present coatings, and to help direct continuing research.

The main technique used to study scattering is imaging and analysis. Test mirrors are illuminated and imaged from many angles, to determine the bi-directional distribution function (BRDF) that characterizes the topography of the mirror surface. An example of the structure of a scattering experiment is shown in Figure [?scatteringexp]. One obstacle that arises in scattering imaging is uncertainty caused by speckle [7].

## 7.4 Speckle

Speckle is the interference effect that results from the scattering of coherent light. “Objective” speckle is the unadulterated pattern visible before an imaging system has processed the light. Because any imaging system will have a unique geometry and diffracting properties, the resultant speckle pattern will inherit additional features that are dependent on the way it is being imaged. Since these patterns will look different to different “viewers” (cameras), they are referred to as

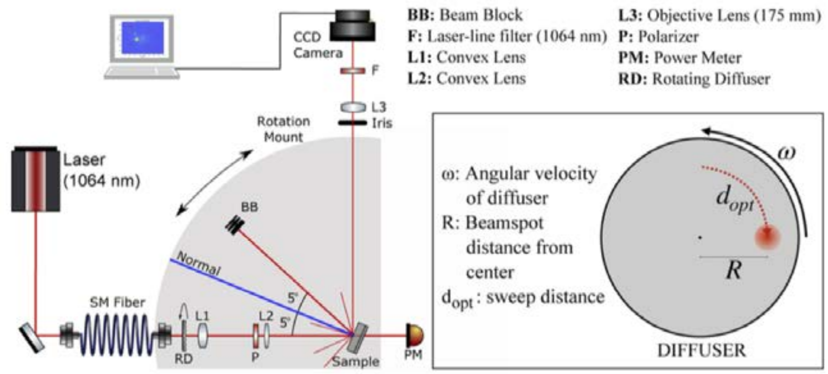



**Fig. 1.** Left: Basic layout of the setup. Light from the laser is guided through a fiber, a diffuser (RD), polarizer (P) and two lenses (L1,L2) onto the sample. Everything shown over the gray quadrant is placed on a motorized rotating mount and can rotate clockwise or counter-clockwise as viewed on the page. Scattered light from the sample surface passes through a 12 mm iris, an objective lens (L3) and a 1064 nm narrow-band filter, producing a focused image onto a CCD sensor. Right: Diffuser diagram depicting the relevant quantities in the speckle-reduction measurements. The red circle represents the beam position relative to the center of the diffuser. The offset is important, as for a given angular velocity $\omega$ and image exposure time, the beam sweeps a longer distance on the diffuser ($d_{opt}$).

Figure 7.3.2: A scattering setup. The mirror and laser source are placed on a rotating stage, and imaged from angles around the normal [7].

"subjective" [11]. Aptly named, speckle appears in images as speckles. Examples of objective and subjective speckle are shown in Figure **??**.

Speckle is a common source of noise in many fields of research, spanning biomedical applications—improving the image quality of ultrasounds, to the enhancement of personal electronic devices—developing pico-projection systems in cell phones [3] [8]Because speckle appears everywhere, a variety of techniques have been used in efforts to reduce it.

## 7.5 Speckle Contrast

Speckle is characterized by the speckle contrast ratio (SCR), which is defined as the ratio of the standard deviation to mean speckle intensity. It takes on values between zero and one, with one being maximum speckle contrast, and zero describing a completely speckle free image [6] [9].

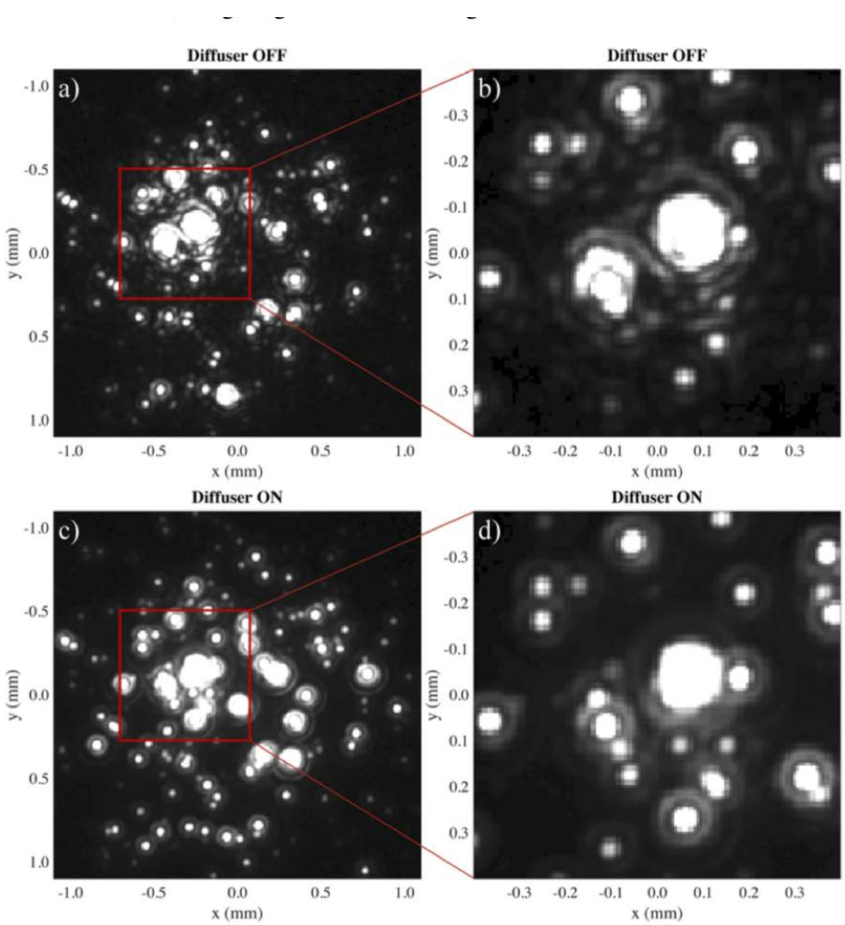



**Fig. 6.** Images at 20° viewing angle from the normal of sample A, a super-polished silica substrate coated with a single 374 nm $GeO_2$ layer. a) The diffuser is not rotating, and the speckle is apparent. Many individual spots get distorted due to destructive interference from nearby points. b) Close-up of (a) in a region with high-defect density. c) The diffuser is rotating and speckle is minimal. All individual points appear as round spots even when overlapping with others. d) Close-up of (c), as in (b).

Figure 7.4.1: Speckle in a scattering experiment, distorting the intensity of the image [7].

# 8
# Despeckling

In order to mitigate speckle, the coherence of the scattered light must be disrupted. The basic strategy is to average uncorrelated speckle patterns within the exposure time of the camera. This is most effective when each speckle pattern being averaged together are of the same mean intensity. If M speckle patterns are averaged within one unit of exposure time, the speckle contrast is reduced from 1 to what. If the averaged speckle has some correlation, there will be no change in the speckle contrast [9].

One technique used to mitigate speckle is the employment of a rotating diffuser. While the camera shutter is open, the beam passes through the rotating diffuser, which imparts a unique phase pattern to each wavefront that passes through. These are averaged at the camera sensor and speckle contrast is lowered. This technique is limited by the long exposure time required [7]. For situations in which short exposure times are necessary, the diffuser would have to spin at an impossibly fast rate, and in other systems that are constrained in space, the physical manipulation required to move a diffuser can be a limiting barrier. In contexts where these two factors are limitations, other studies have demonstrated speckle reduction through the use of a spatial light modulator, assigned with phase maps structured with Hadamard matrices [8] [11].

## 8.1 Spatial Light Modulator

A spatial light modulator is a mirror that is divided into pixels, each of which is connected to an electrode. When a voltage is applied, the optical axis within that section of the mirror is shifted, changing the index of refraction of the material and the phase of a reflecting beam. Any desired wavefront can be produced by programming the appropriate phase map into the SLM [11]. Hadamard matrices are often used as maps.

## 8.2 Hadamard Matrices

Hadamard matrices are composed of 1's and -1's, and are unique for their mutually orthogonal rows. H(n) is a Hadamard matrix of order n, with $2^n$ number of rows and columns.

Matrices below n = 16 are singular. Higher order Hadamard matrices, there are multiple inequivalent marices can be formed to satisfy the Hadamard condition. H(n) = 16 has 5 inequivalent matrices. That is also where the numpy function breaks, which (spoilers) could also be why I have null results. For orders 32, 36, and 40 there are millions of inequivalent matrices [6].

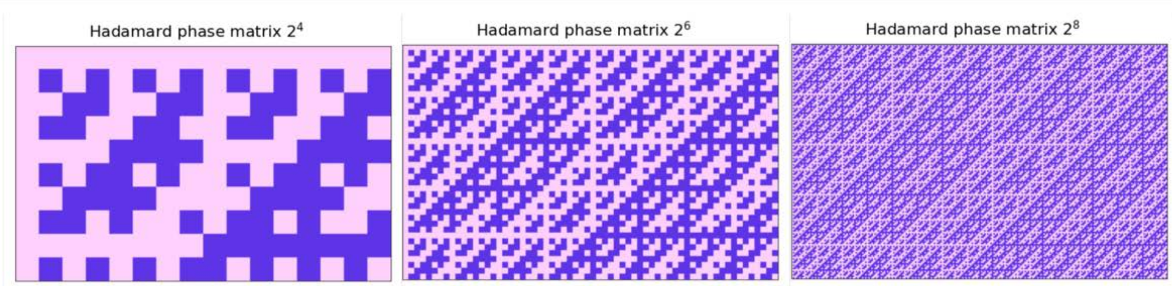



Figure 8.2.1:

The approach is to choose a Hadamard matrix of an order such that the detector resolution spot is covered by as many Hadamard cells as possible without enclosing more than one pattern. The cells are the smallest chunks of phase. The phase pattern is the set of cells that overlay one detector resolution spot. It is the variance of these cells in time that disrupts the interference of the coherent light, and reduces the speckle [9] [14].

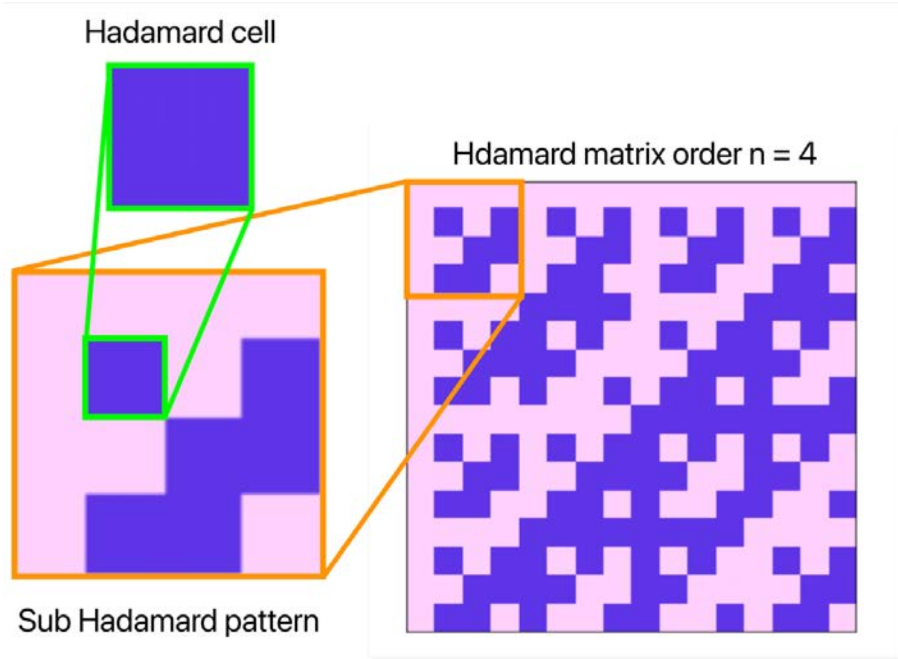



Figure 8.2.2:

The diffraction limitation for an imaging system is defined by

$$d = \frac{\lambda}{2n \sin\theta} = \frac{\lambda}{2NA}$$

The speckle size can be compared with the diffraction limited detector spot be comparing the numerical value of $d$, with the measured speckle size on the camera.

For a Hadamard matrix of order n, n incoherent sub-matrices can be generated, and the averaging of these in one resolution spot over the exposure time will result in SCR reduction by a factor of $n^{-}$ [6].

## 8.3 Simulations

I began by simulating light propagation in an attempt to understand the kind of subjective speckle I might expect to see in my experiment. Modeling the wavefront of the laser as an array of spherical point emitters, I first created a function that propagates one spherical point emitter through space.

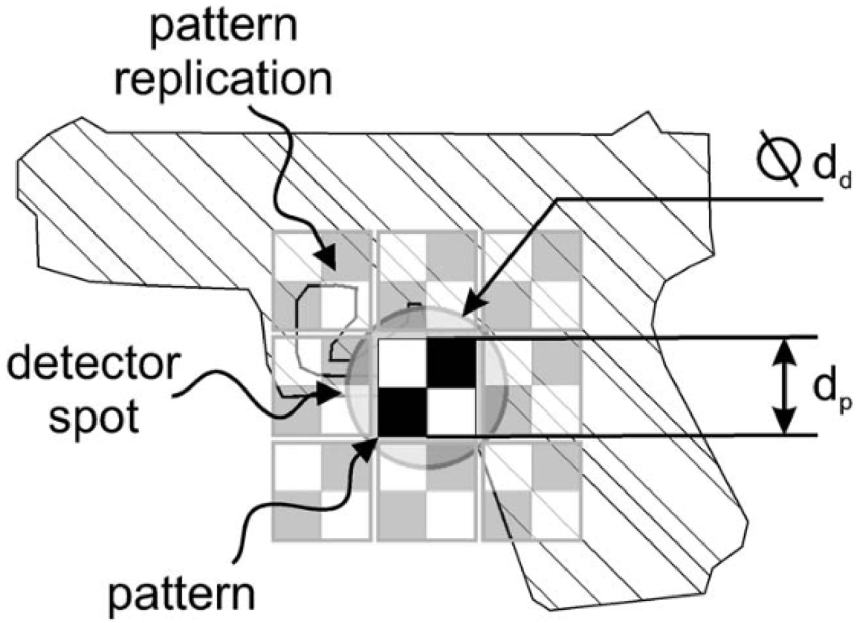



Figure 8.2.3: The detector resolution spot should fit as many Hadamard cells inside as possible without a whole Hadamard pattern fitting inside [14].

$$Ae^{ikx+\phi} \tag{8.3.1}$$

where k is the wave number: $\frac{2\pi}{\lambda}$, $\lambda$ is the wavelength and and $\phi$ is the phase of the EM wave.

A single point source propagated into the far field:

That was then passed through a circular aperture, modeled as a binary mask, taking values of zero and one, which, when applied to an incoming plane wave, leave an approximately flat plane wave front, if placed at a long enough distance from the point emitter.

I then created a function that propagates a whole field forward in space, so that from the location of the aperture, the array of pixels that pass through can each be considered a spherical point emitter, as in the Huygens model of a wavefront. Propagating an entire wavefront of pixels takes a lot of time and computing power, and it becomes difficult to image at the necessary/appropriate scale, so I did give up on this avenue of study. But first I made a lensing function.

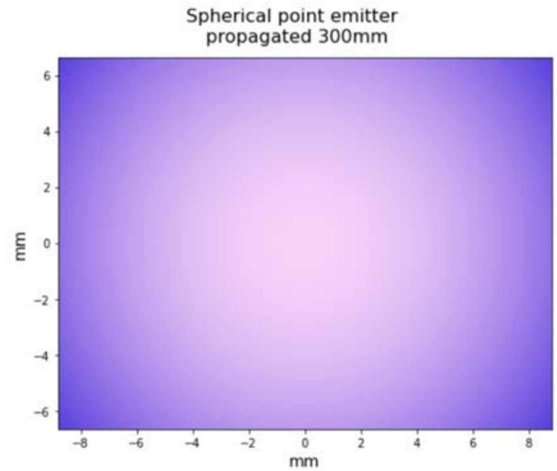



Figure 8.3.1: Propagation

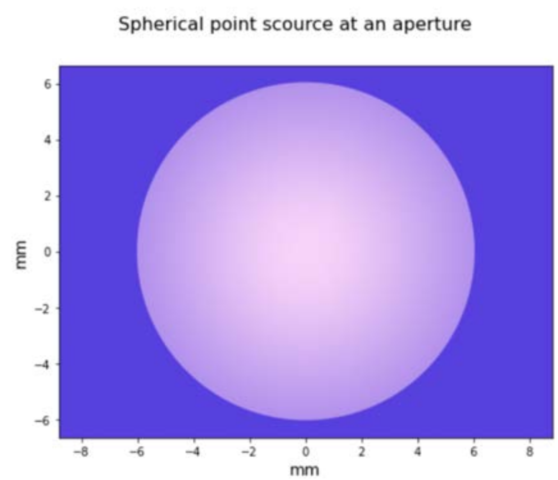



Figure 8.3.2: point source passing through aperture.

A lens can be modeled as a phase mask applied to a wavefront. In the thin lens and the paraxial approximation the phase transformation is

$$\Delta(x,y) = \Delta_0 - r_1(1 - \sqrt{1 - \frac{x^2+y^2}{r_1^2}} + r_2(1 - \sqrt{1 - \frac{x^2+y^2}{r_2^2}} \tag{8.3.2}$$

where $\Delta(x,y)$ is the change in phase as the sections of the wavefront pass through the lens,

$\Delta_0$ is the phase upon entering the lens,

and $r_1$ is the radius of curvature of the front or entering face,

and $r_2$ is the radius of curvature of the exit/back interface of the lens.

I have not yet managed to get this function to work, and decided to move on to the experimental work without simulating the expected subjective speckle.

In order to begin the experimental work, it was necessary to compute Hadamard matrices with which to program the SLM. Using the numpy function, I was only able to generate orders up to n = 15. The function is limited to smaller orders, I expect because at n = 16 there begin to exist multiple inequivalent matrices.

# 9
# Experiment

The 400nm laser illuminates a calibration screen, which is used for its lambertian scattering properties. A lens with 200mm focal length is set up 300mm away from the screen, at approximately a 30 degree angle.

The lens ($f = 200mm$) collects the scattered light, and sends it toward a beam splitter, which casts away half, and directs the rest to the Santec SLM-200, at 90 degree incidence. The beam receives the phase transition from the SLM and returns to the beam splitter, where it is redirected to the Meade Deep Sky Imager IV, placed at the imaging plane of the lens. The SLM is $9.6x15.36mm$ with $8x8\mu m$ pixel dimensions. The camera is $13.3x17.6mm$ with $3.8\mu m$ length sides.

Hadamard matrices of orders 4,6,10,12,13, and 14 were applied to the SLM. For each order, two images were taken with 0.5s exposure time. The first image was taken with a nonmoving phase pattern and a second with alternating Hadamard matrices, switching at an interval rate of 0. 05s. Since we are using low orders of Hadamard matrices, there are not inequivalent or uncorrelated matrices of the same order to be combined. In an attempt to decohere the matrices the second matrix is the same, but with 1's and -1's of the hadamard switched. It is not clear that these phase maps will result sufficiently different speckle patterns.

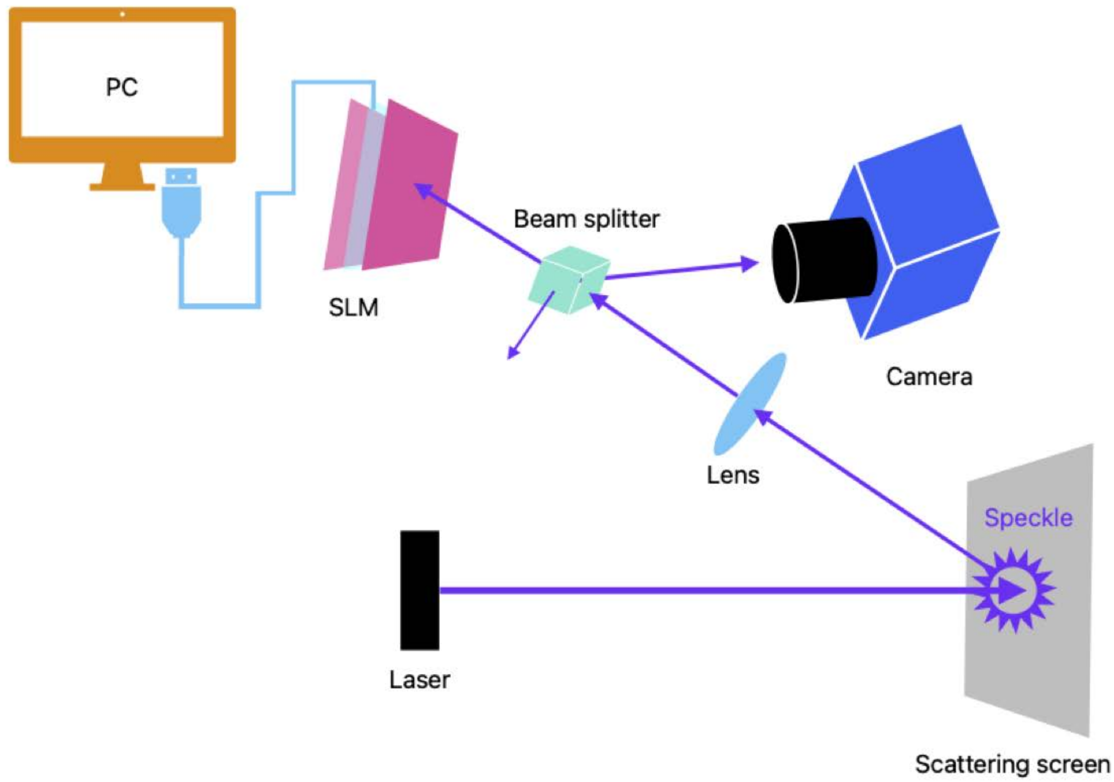



Figure 9.0.1: Schematic of experimental design.

Subsections of each image were selected in regions of approximately constant intensity, and SCR's compared between stationary and alternating cases. Theoretically speckle contrast can be improves by a factor of $\frac{1}{\sqrt{n}}$.

## 9.1 Calculation of expected

The diffraction limitation of a system is given by the equation

$$d = \frac{\lambda}{2n \sin\theta_i}$$

where $\lambda$ is the wavelength of light, $n$ is the index of refraction of the propagation medium, $\theta_i$ is the half angle that the lens makes to the converging spot on the detector, and $d$ is the minimum resolvable distance. Since

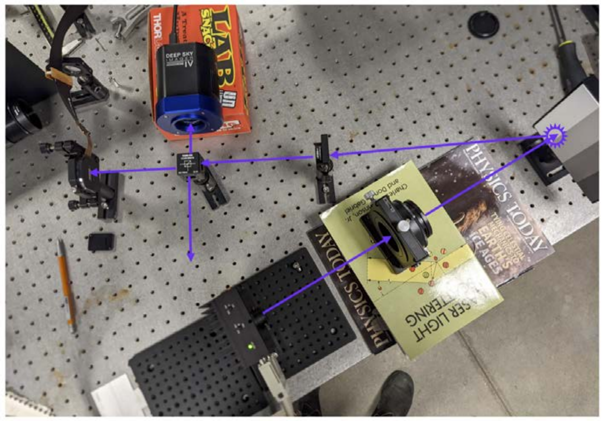


Figure 9.0.2: laser passing through aperture, hitting scattering screen, passing through lens and beam splitter, reflecting off of SLM and back through the beam splitter to the camera.

$$\sin\theta_O = 2\sin\theta_i$$

and in the small $\theta$ regime

$$\sin\theta_O = \frac{r}{O}$$

and

$$\sin\theta_i = \frac{r}{i}$$

where $O$ is the distance from the lens to the object, $i$ is the distance of the lens to the image plane, and $r$ is the radius of the lens, the smallest spot a camera can see when placed in the image plane is determined by

$$d = \frac{\lambda}{n\sin\theta_O}$$

In our system the diffraction limited resolution was calculated to be $d = 9.456\mu m$. And the measurement of speckle size (3 pixels per speckle, $3.79\mu m$ per pixel, was $11.37\mu m$.

The Hadamard cell sizes were also computed, after the measurements were taken. The respective cell sizes for orders 4, 6, and 10: $992\mu m$, $800\mu m$, and $80\mu m$. Since speckle reduction is only expected when the diffraction limited spot $d$ is larger than the cell size and smaller than the pattern size (which is always 4 times the cell size), we should not expect to see anything significant in our measurements, since our detector spot is smaller than the cell size in each of these cases. In higher orders the cell size reduces to one pixel.

# 10
# Outcomes

## 10.1 Evaluation: calculation of speckle contrast

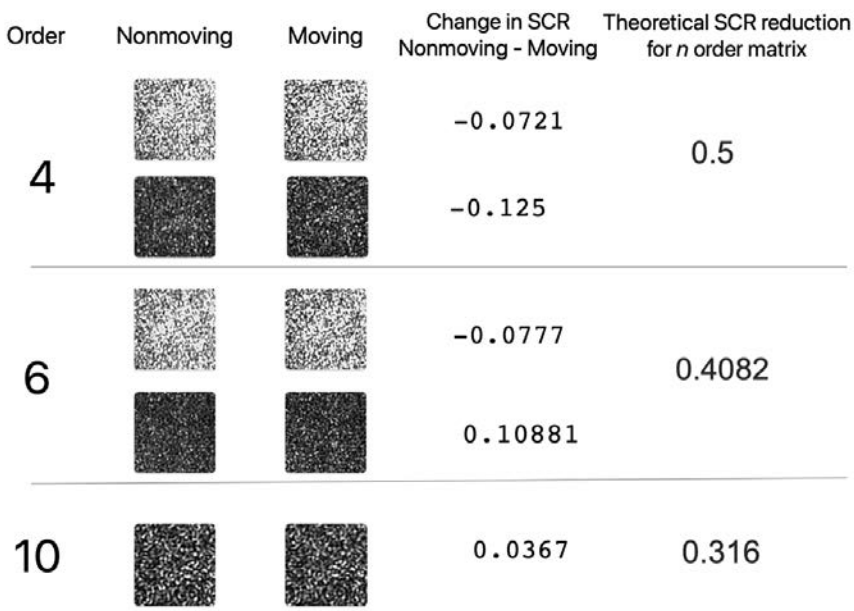



Figure 10.1.1: A preliminary sampling does not show a trend in SCR reduction across orders of Hadamard matrices. The values by which the SCR deviate are not significant compared with the predicted modification of SCR in corresponding orders.

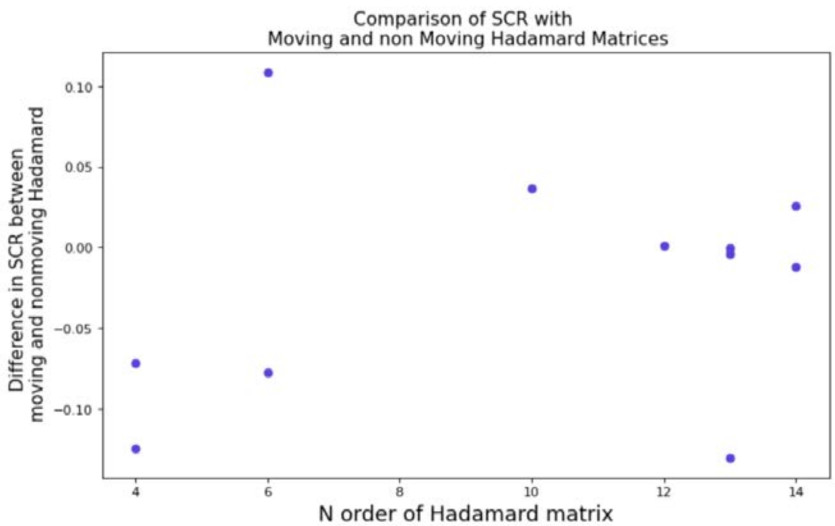
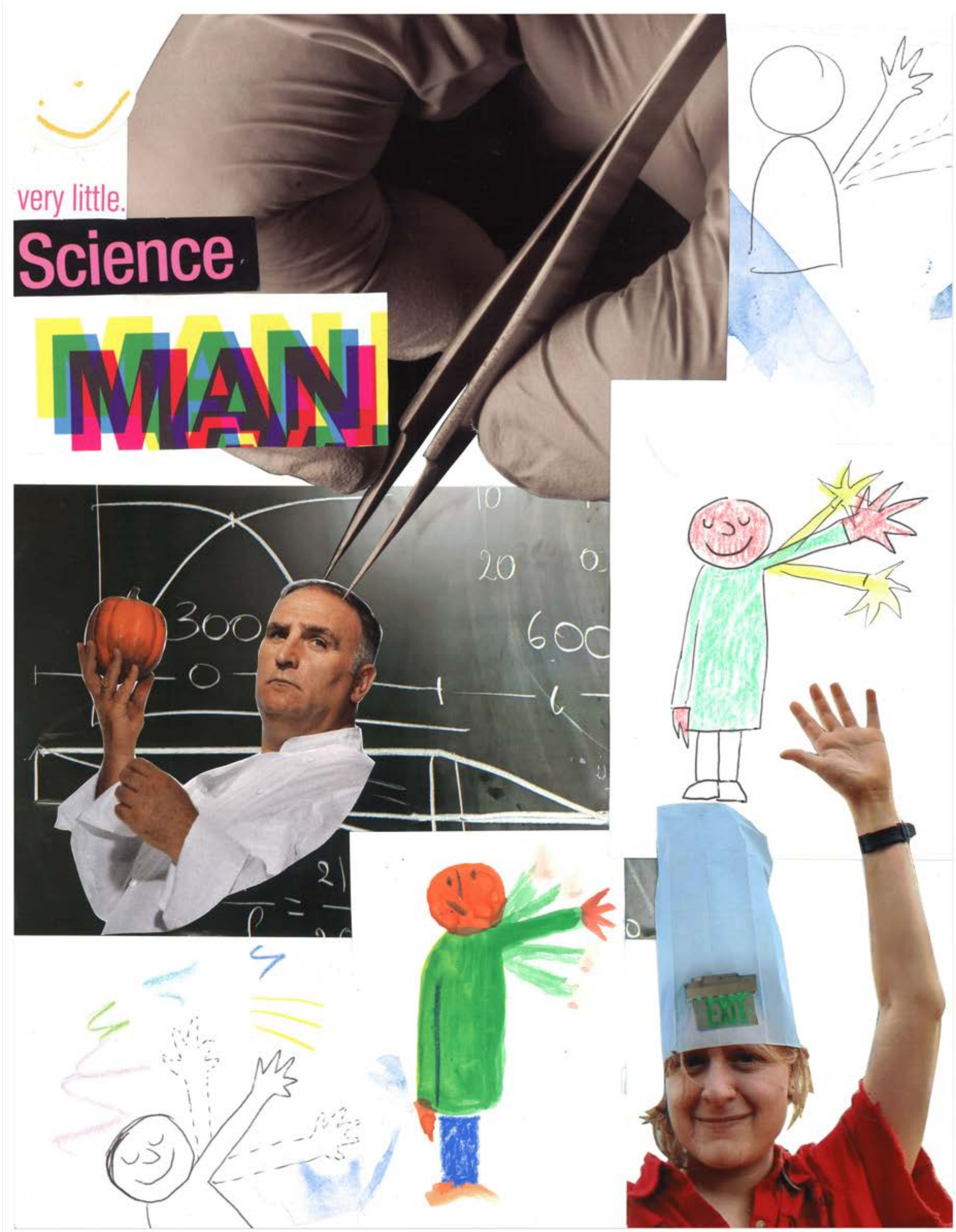



Figure 10.1.2: A graphical representation in the difference in SCR from stationary to moving hadamard matrices of different orders.

# 11
# Conclusions

The results are not particularly interesting, but the process answered some questions and paves paths forward. Many questions remain: Were the squares taken really of uniform intensity? Did I make a mistake in labeling them? This preliminary experiment was not designed to take into account any expectation of speckle size or diffraction limitation of the imaging system. The size of necessary Hadamard cells and pattern was not computed prior to this phase of the experiment. What does cause the changes in SCR in the different samples? How are sub Hadamard matrices constructed to be incoherent? Is there an optimal rate of alternation in different exposure times?


very little.
Science.
MAN

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