Optimizing Glide-Flight Paths

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by
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Abstract

Flight is no rare event in today's society, and aviation is a global industry that significantly contributes to carbon emissions and global warming. Thus, my project theorizes how aviation might be better optimized at a fundamental level to improve aerodynamic efficiency and reduce carbon emissions. This is done by analyzing two systems of flight: gliding and powered flight. In pursuit of an understanding of a hybrid of these flight systems, I first look to qualitatively analyze the benefit of gliding over powered aviation. Powering an aircraft involves an engine that generates thrust, while gliding only involves three forces: lift, drag, and gravity. How can gliding be used to reduce the amount that an aircraft relies on generating thrust? How can we use gravity to our advantage? The following theoretical work hones in on these ideas, and expands to the intricacies of fluid dynamics around an airfoil, optimizing the general aircraft design that would be required from an aircraft with a hybrid glide/powered mode, and optimizing a flight path that would reimagine air travel in a fuel-efficient manner. The theoretical work introduces both questions and concerns about this hybrid flight plan, and ultimately attempts to motivate further work and thinking in the context of modeling aerodynamically-efficient flight plans and design ideas.

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Dedication

To my mother, father, sister, and brother.

Acknowledgments

I'd like to first thank Matthew Deady, who gave me opportunities to succeed throughout college, this project, and in life in general. When I doubted the math, when I doubted the physics, or when I (most often) doubted myself, Matt was always able to see things clearly and inspire me to continue working, be curious, and believe in myself. Words can hardly capture the magnitude of skills that I learned from Matt. Matt is "The Man in the Yellow Hat" from *Curious George*, for me and many others. I'd like to thank the physics department, in particular Hal Haggard, Paul Cadden-Zimansky, and Antonios Kontos, for teaching me and furthering my interest in all things physics.

1
Introduction

Aerodynamics is an aggregate study. The conditions of the atmosphere and design of an aircraft form a relationship that is not fully understood and optimized. This paper is designed to look at the benefits of incorporating more unpowered glides in a flight plan, to reduce fuel-energy consumption and improve general aerodynamic efficiency.

On its trip back to Earth, the space shuttle uses gravity to propel its way home, trading massive amounts of potential energy (mgh) for kinetic energy ($\frac{1}{2}mv^2$). Because it begins its descent from such a high altitude, the space shuttle has to slow itself down from extremely high speeds (starting at around $17,300mph$, it has to slow itself down to a mere $250mph$ at landing)[8]. It descends as a glider, with a lift to drag ratio around 1[8] so that it can reduce its speed over its return.

The space shuttle motivated my thinking that glide descents can be useful. While the space shuttle uses massive amounts of fuel energy in other ways, it uses no energy upon its return home. In identifying the relevant parameters for wing and body design for a fast moving glider at high altitudes, and by sufficiently describing and accounting for the temperature and density conditions of the atmosphere, we can model the important parameters of the situation.

Gliding is the most efficient way to fly without having to dump energy into generating thrust. Flying is not exclusively achieved by one thing, and flying is not exclusively powered flight.

Powered aircraft rely on the same laws of lift and drag that unpowered gliders rely on, and powered aircraft can learn from and still use glide technology. Some basic aspects of a body's ability to glide through a medium tend to be forgotten on the front of powered commercial aircraft, and my goal is to model the efficiency of glider technology on an aircraft. How efficient could a glider be for long distance descent? Could a powered airplane turn on a "Glider Mode" that would cease the amount of energy an engine has to do? This can be studied by describing and understanding the existing laws of fluid dynamics and aerodynamics, and then applying that understanding to a theoretical model of a hybrid glide/powered flight plan. These ideas require an understanding of an airfoil moving through a fluid, and how much that fluid is affected by that motion. Whatever can be figured out about that fluid and the environment it's in is equally relevant.

2
Ascent, Descent

2.1 The Cost of an Engine

This project commenced with a single theoretical question: can gliding be used to make aviation more energy-efficient in an industry that suffers from an immense carbon footprint? The Air Traffic Action Group, a not-for-profit association that exists as one global, industry-wide body, reports that aviation is responsible for about 2%[14] of human-induced carbon dioxide emissions. Air travel is costly for the environment, but it is used by industries across the board. The economic value of goods being traded by air is considerably large: aviation makes up 0.5%[14] of the volume of the world's trade shipments, but it is over 35%[14] in value. As a result, the aviation industry is in high demand for projects that improve aerodynamic efficiency, decrease energy consumption, and significantly reduce the carbon cost of flight. This paper discusses the optimistic and dead-end characteristics of a model glide/powered flight-path. The idea involves splitting a flight up into two sections: a powered ascent and a gliding descent. I also explore other theories that attempt to manage fuel consumption and improve aerodynamic efficiency.

2.1.1 The Forces of Flight

There are four forces that govern the flight of an aircraft: lift, drag, weight, and thrust. Lift pushes an aircraft up and opposes gravity, which weighs the aircraft down toward the center of the earth. A powered plane uses an engine that burns fuel to provide thrust and counteract drag. Drag is the force that resists the movement of an aircraft through the air. While a powered plane experiences all of these forces, a glider does not experience thrust, because it relies on gravity. Although the work in this paper covers a technical spectrum, it always circles back to these four force vectors.

2.1.2 Energy Management

Aviation involves the combination of four crucial types of energy: potential energy which is proportional to the altitude and mass of the aircraft, kinetic energy which is proportional to the airspeed^2, chemical energy or "fuel," and airmass energy (the thermal energy left behind an aircraft when it passes through and stirs air). For the purpose of this thesis, I focus mostly on the relationship between potential, kinetic, and chemical energy.

Energy can neither be created nor destroyed. Potential energy can be converted to kinetic energy, chemical energy can be burned to generate altitude and speed (potential and kinetic energy), and so on, but the total amount of energy in the system does not change.

There are some energy conversion processes, however, that are irreversible. Burning fuel and using up the chemical energy of a vehicle is a process that cannot go backwards. Similarly, there is no current way to recapture energy that is lost to drag. The goal of my study is to identify the parameters that will optimize an aircraft's use of fuel. I am using glide-technology to promote conversions from altitude/potential energy to airspeed/kinetic energy. Unlike burning fuel, gliding transforms energy stored in the altitude of the plane into airspeed that the plane can use. Thus, the aim of this thesis is to provide an argument for the use of glide technology on energy consumption to provide aerodynamic energy efficiency. In this project, I am interested in an optimal glide-flight path that reduces the amount of chemical energy needed during flight.

3
Optimizing Powered vs. Glider Aircraft

In order to frame the discussion on optimizing the design of gliders and planes, there are first some properties of fluid dynamics that need to be defined.

3.1 Fluid Properties

From the perspective of fluid mechanics, matter can be in one of two states: solid and fluid. From a technical standpoint, their distinction lies in their response to an applied shear, or tangential stress force.

From Bertin and Cummings: "A fluid is a substance that deforms continuously under the action of shearing forces."[3]

Without relative motion in the fluid, there are no shear or stress forces acting on the fluid particles, and the fluid particles all have the same velocity and direction. This state is called the *hydrostatic stress condition.*

A fluid can be a liquid or a gas. For aerodynamic calculations we consider air to be a hydrostatic fluid, because gas molecules around Earth have no definite volume and expand until it forms an atmosphere that is essentially hydrostatic.[3]

It is mathematically useful to treat air as a continuum. Employing this concept allows us to quantitatively describe the gross behavior of a fluid using observable, measurable, macroscopic properties. These properties and terms are listed and defined as the following:

1. Temperature

In qualitative terms, we tend to describe temperature by how hot an object feels to the touch. This is an okay every-day qualitative description, but in order to quantitatively describe atmospheric temperature we must use a better system. Thus, we define the "equality of temperature." That is, when two bodies have equality of temperature, there is no change in any observable property when they come into thermal contact. Additionally, two bodies that are equal in temperature to a third body must be equal to each other. We then can define an arbitrary scale of temperature in terms of a convenient property of a standard body. Most of this paper measures temperature in Kelvin, and sometimes converts to $^{\circ}C$ when appropriate.

2. Pressure

Particles in a fluid have random motion due to their thermal energy. When a surface is placed in a fluid, these individual molecules will continuously strike that surface. Newton's second law states that a force is exerted on the surface equal to the time rate of change of the momentum of rebounding molecules. Pressure is the magnitude of this force, per unit area of the surface. Standard atmospheric pressure at sea level (SAP), according to Bertin and Cummings, is defined as "the pressure that can support a column of mercury $760mm$ in length when the density of the mercury is $13.5951g/cm^3$ and the acceleration due to gravity is the standard value." This paper uses a SAP of $101,325Pa$, or $1.01325e5N/m^2$.

3. Density

The density of a fluid at a point in space is the mass of the fluid per unit volume surrounding the point. Given our assumption that the fluid is a continuum, for a thermally perfect gas, the density at a point is defined as $\rho = \frac{p}{RT}$.

4. Viscosity

The fluids of interest in this paper are Newtonian in nature. Therefore, the shearing stress in the fluid is proportional to the rate of shearing deformation. The constant of proportionality is called the coefficient of viscosity, μ.

$$shearstress = \mu \times (transversegradientofvelocity) \tag{3.1.1}$$

Thus, the higher the viscosity of a fluid, the higher the shear stress within the fluid.

5. Kinematic Viscosity

The relationship between the viscosity and air density ρ is often encountered by an aerodynamicist. This relationship is defined as the Kinematic Viscosity v: $v = \frac{\mu}{\rho}$. Recall that μ is the coefficient of viscosity. This equation has the dimensions $[\frac{L^2}{T}]$, where L is length and T is time. In summary, the higher the viscosity is proportional to, in layman terms, the "thickness" of the fluid.

3.2 Fluid Dynamics: Visualizing the Flow Field Around an Airfoil

The problem for anyone studying physics, as long as they have some knowledge in Partial Differential Equations and Vector Calculus, is less about quantitatively describing a flow field, and more about qualitatively summarizing that flow field.

Treating air as a continuum, Newton's laws are still correct through the flow-field, in that his second law physically accounts for forces internal to the fluid. Solving equations pertaining to lift requires constraining the body moving through the field (airfoil), and then solving sets of equations around the body for air pressure, and vector velocities. But flow fields over a large area are complicated, and have varying pressures and velocities from point to point. Thus, understanding this apparatus from a qualitative point of view requires thinking about air as broken up into granular pieces, or air parcels. By breaking air into these grains, one can imagine each parcel interacting with other parcels, and transferring momentum and energies to each other. This might seem intuitive, but if we did not consider the air as interacting with itself, and if we drew a picture of our airfoil moving through this fluid, the air would seem to fire like bullets in a straight line at the wing, which we know is not realistic and not true:

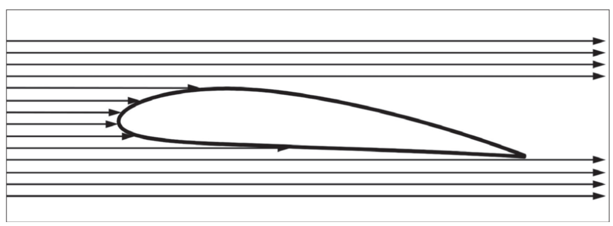

Figure 3.2.1. A wing with air flowing in a stream at it.[1]

What air actually does through an extended flow-field is more complicated, because velocities and pressures may change from point to point. Because we know that air particles interact with each other and are not simply stationary, a more realistic flow (with laminar flow) around a wing looks like this:

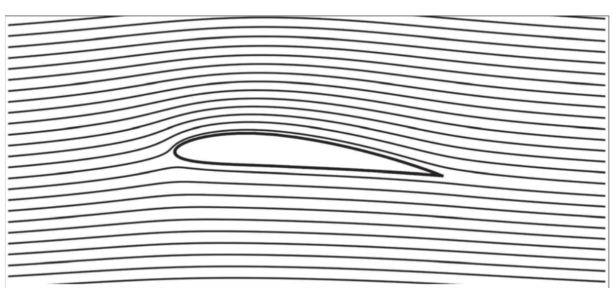

Figure 3.2.2. A wing in a more accurate air flow.[1]

Designing a flight path requires studying behavior of an aggregate system with many interacting parts, and describing the cause-and-effect between the forces and the motions. According to Bernoulli (under certain conditions), there is a pressure and velocity difference for the fluid above and below the airfoil. These differences have to do with the airfoil's motion through the fluid field and the fluid field's inertia. Bernoulli correctly argues that the air flowing over the wing has a high velocity and low pressure, and the air flowing below the wing has a low velocity and high pressure. This results in a net force upward from the high pressure on the bottom of the airfoil. Because this pressure difference relies on the speed at which the airfoil moves through the fluid, a body must generate a high enough velocity to create lift. These velocities vary from aircraft to aircraft.

When the airfoil moves through the flow field, air molecules create lift and push the airfoil up, which results in air molecules being pushed down due to Newton's second and third laws.

Consequently, a vehicle's angle of attack (the angle that the front end of an aircraft has from the flat, horizontal plane which is parallel to the surface of the Earth) creates lift. When a plane tilts upward (positive yaw), there is more surface area of the bottom of the airfoil for particles to interact with. Thus, lift is created for angles of attack.[1]

Because of Bernoulli's Principle, the airfoil changes velocity and pressure fields.

3.3 Factors That Effect Dimension and Design

Gliders are designed to optimize lift so that they can have high lift to drag ratios. This is greatly done by reducing several forms of drag that contribute to the total drag of an aircraft through the air.

3.3.1 Total Drag

At first, solving for drag seemed trivial. I did not consider how many terms would go into affecting the drag on an aircraft. The truth is, there are many aerodynamic effects that contribute to the

[1] For simplicity, this effect is neglected in the calculations in chapter 5.

resistance of a vehicle through the fluid. This was important to know, because drag is a significant term in calculating the energy being taken away from an airplane. The total drag is calculated by summing two forms of drag: parasite drag and lift-induced drag.

3.3.2 Parasite Drag

Parasite drag is simply the resistance that air has to anything moving through it. Parasite drag increases with the square of the speed, so if the speed of an aircraft is doubled, the parasite drag is quadrupled. There are three types of parasite drag: form drag, skin friction, and interference drag. Form drag comes from the turbulent wake of a surface moving through an airflow. See figure 3.3.1. A flat plate has much more form drag than a streamlined object.

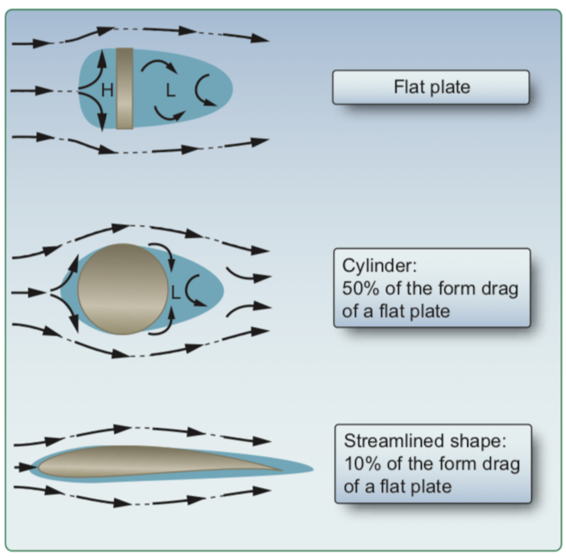

Figure 3.3.1. A streamlined shape is used to reduce form drag.[7]

Skin friction drag is related to the roughness of the glider?s surfaces. Even when wing surfaces may appear smooth, they may be quite rough when viewed under a microscope. This roughness enables a thin layer of air to cling to the surface and create small eddies, or areas of lower pressure that contribute to drag. Due to the roughness of the surface of the object, a boundary layer is created where the velocity of the fluid particles cancel. Because air is viscous and fluid particles have shear forces on each other, this boundary layer acts on the particles around it which are flying by with an upstream velocity U. Thus, there is an open area of research to reduce skin friction drag to reduce the effect of the boundary layer on the total drag.

The boundary layer takes two forms: 1. Laminar: the fluid particles slide smoothly over their neighbors. 2. Turbulent: dominated by areas of lower pressure and turbulent flow (less shear forces between particles).

Interference drag occurs when the varied currents of air passing over the glider interfere with each other.

3.3.3 Lift-Induced Drag

As an airfoil is driven through the air to develop the difference in air pressures that we call lift, induced drag is created. When the higher pressure air on the lower surface of the wing curves around the end of the wing and fills in the lower pressure area on the upper surface, the lift is lost, but the energy to produce the different pressures is still expended. The result of this process is drag, because it is wasted energy.[7]

3.3.4 Total Drag

In summary, the total drag is the sum of the parasite drag and the lift-induced drag.

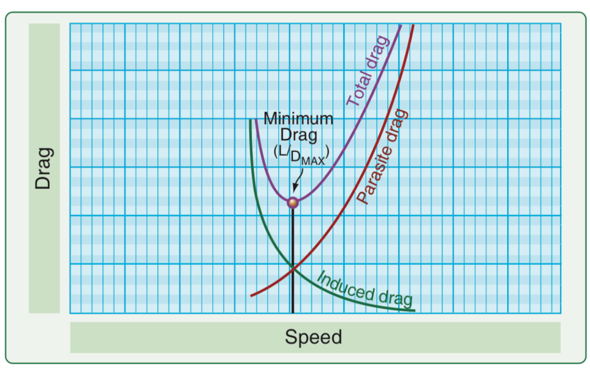

Figure 3.3.2. This is what a typical graph of total drag looks like for an aircraft.[7]

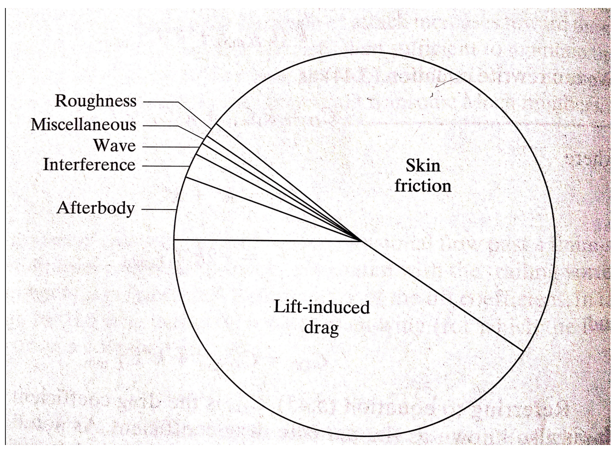

Figure 3.3.3. A pie graph of the drag terms that contribute to total drag, provided by Bertin and Cummings. For more on this, see [4]

4
The Physics Within the Model

4.1 Atmospheric Physics

As I said in preceding text, the analysis of a flight model involves a number of systems. Aerodynamics does not just involve wings, the body of the aircraft, and the fluid immediately surrounding the aircraft. A body moving through air, particularly when it descends from a high altitude,[1] experiences a variety of effects from all over the atmosphere. In particular, the air density and temperature changes as a function of altitude. These are integral to understanding the ascent and descent paths for a hybrid glide/powered flight plan. Therefore, when I began calculations for the path of an aircraft, I started by analyzing all of the atmospheric conditions that the aircraft would move through in its descent. Matt and I split up this work into two parts: first we analyze air pressure as a function of altitude in an isothermal system, and then we move to understanding the exponential atmosphere in the adiabatic case.

4.1.1 Atmospheric Calculations: Isothermal

Professor Deady and I first wanted to understand how pressure changes exponentially as a function of altitude. We first estimate the atmosphere to be an isothermal system, meaning we hold

[1] The space shuttle, which acts as a bulky glider on its descent to Earth's surface, experiences all of Earth's atmosphere on its trip. This is trivial, since it descends from space, but its trip from the top of the atmosphere is becoming more common in aviation. Consider Virgin Atlantic's space tourism, which exceeds 50 miles of altitude.[12]

the temperature constant throughout the calculation. These conditions can be understood with an isothermal pressure equation, which demonstrates that air pressure drops off exponentially as

$$P(h) = P_o e^{\frac{-mgh}{kT}}. \tag{4.1.1}$$

[3]

Professor Deady and I found this equation useful, even if it only showed us isothermal results. In this equation's exponent, mgh is the potential energy at height h and kT is the thermal energy at a temperature T. We set this temperature to stay at $273K$, or just around room temperature. We both knew intuitively that as h went to infinity, air pressure would drop down to 0, which is what is called “space.” What we did not know, was how quickly the atmosphere transitioned from being an atmosphere to being space, and how a vehicle could navigate the exponentially changing air pressures if it is trying to maintain lift.

Our given variables for this problem include the air-pressure measured at sea-level in Pascal (P_0), room temperature in Kelvin (K), Avogadro's number (N_A), the molar mass of N_2, the Universal Gas Constant (R) in $\frac{J}{mol*K}$, and the acceleration of gravity (g). If we consider our atmosphere as being mostly made up of N_2 (this assumption is often used by aerodynamicists in wind tunnels as the test gas substitute for air), then with a molar mass of $28g/mol$, we solve for the pressure at any given height:

$$P_0 = 101,325Pa$$

$$T = 273K$$

$$N_A = 6.02 * 10^{23} units/mol$$

$$N_2 = 28g/mol$$

$$R = 8.3145 \frac{J}{mol * K}$$

$$g = 9.81 m/s^2$$

$$k = \frac{R}{N_A} = 1.38 * 10^{-23} J/K.$$

Using these variables to solve for the exponent $\frac{mg}{kT}$, our equation for pressure becomes:

$$P(h) = P_0 * e^{-(0.121km^{-1})*h}. \tag{4.1.2}$$

The pressures obtained from this equation are not completely accurate, given that the temperature is held constant throughout, and the atmosphere is not entirely made up of N_2. But this model does show approximately how quickly air pressure drops off for a range of altitudes that airplanes will see. In order to visualize this, I created a graph of the isothermal pressure function that ranges from $0km$ altitude to $49.95km$ (with increments of $0.05km$):

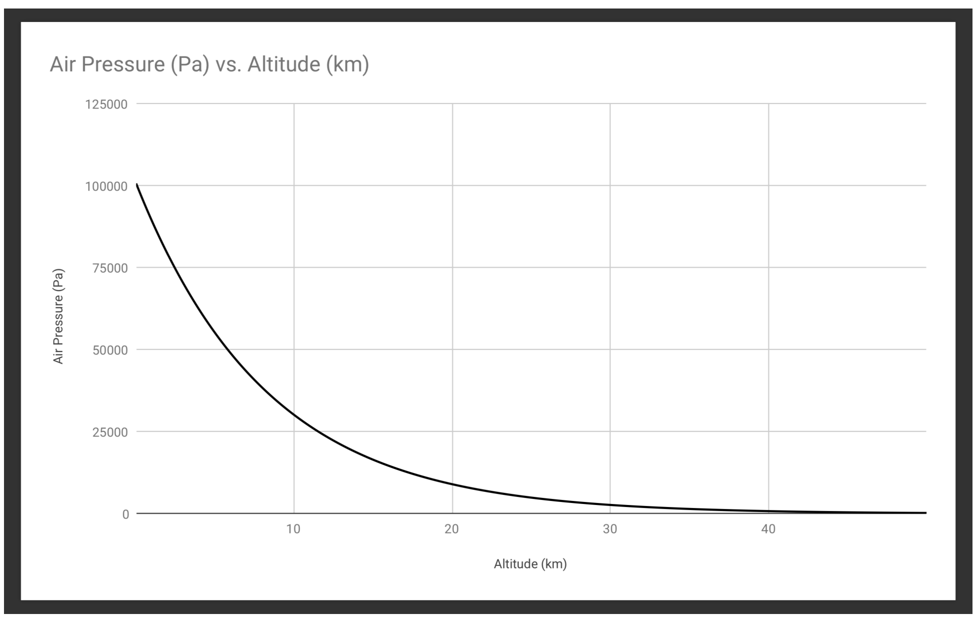

Figure 4.1.1. Air Pressure (Pa) as a function of Altitude (km) in isothermal conditions.

Air pressure drops off quickly as altitude increases. At sea-level the air pressure is $101,325Pa$, but at an altitude of $10km$, that pressure becomes $30,214Pa$. To give some context, $10km$ ($32,808ft$) is around the cruising altitude of most commercial airplanes ($33,000ft$ to $41,000ft$ for a Boeing 747). Thus, a commercial aircraft quickly experiences a range of air pressures during ascent, which explains why your ears pop so frequently during a flight's initial climb. The space shuttle, which stays in a low-Earth orbit (ranges from $304km$ to $528km$, or 190 to $330mi$), indeed begins its gliding descent in "space," outside of our atmosphere. Design choices for an aircraft that go to these high altitudes need to take into account the change in air density during the trip.

4.1.2 Atmospheric Calculations: Adiabatic

I now move away from an isothermal calculation to an adiabatic, in order to visualize the change in atmospheric temperature according to altitude.[2] Temperature changes as you move to different altitudes because of convection. When heat is applied to the bottom layer of a system, the hot, less dense air rises while the cool, more dense air sinks. This can also be referred to as "vertical mixing." See figure 4.1.2 below.

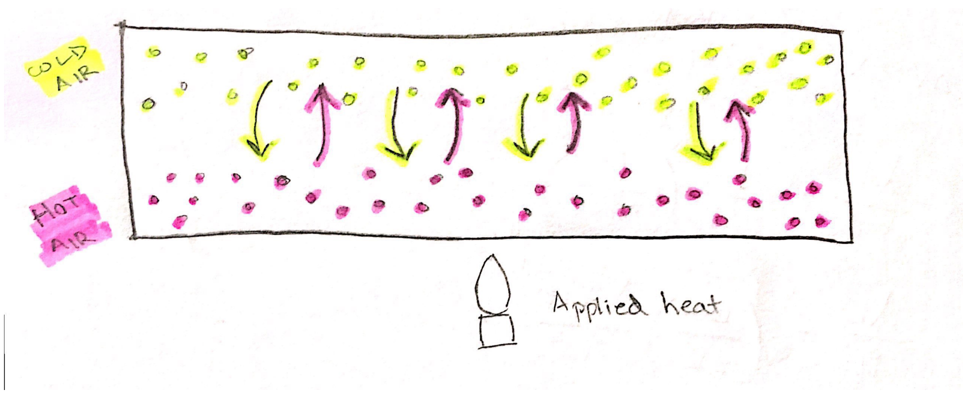

Figure 4.1.2. In order to visualize convection, I drew a small diagram.

Convection has a number of effects on aviation and our atmosphere. The sun shines on our planet and heats the ground. The air in the atmosphere is heated from below, and because of this, atmospheric temperature changes at different heights, and atmospheric turbulence also occurs. Atmospheric turbulence occurs when mixing air particles have small-scale, irregular motions that are observed in winds with varying speed and direction. This effect is more extreme at lower altitudes due to the increased flow disturbances around surface obstacles. To quantitatively understand this, I calculated the adiabatic lapse rate[3] for our atmosphere of N_2.

[2] An adiabatic system is a system in which there is no heat transfer. That is, heat does not enter or leave the system.
[3] Adiabatic Lapse Rate: The rate at which atmospheric temperature decreases with increasing altitude in conditions of thermal equilibrium.

Solving for a change in temperature over change in height involves splitting the height (which we can call z) up into infinitely small slabs of length dz. Using the fact that we already know the pressure as a function of height or z, we can also find $\frac{dP}{dz}$. Then, in order to find the change atmospheric temperature over a change in height, we can do

$$\frac{dT}{dz} = \frac{dT}{dP} * \frac{dP}{dz}. \tag{4.1.3}$$

For an ideal gas,[4]

$$PV = NkT. \tag{4.1.4}$$

To complete this calculation, we also need the thermodynamic correction

$$PV^{\gamma} = C, \tag{4.1.5}$$

where C is a constant, and γ is the ratio of the specific heat coefficient at constant pressure (C_p) and the specific heat coefficient at constant volume (C_v).

Then, solving for the volume V in equation 4.1.4, we obtain

$$V = \frac{NkT}{P}.$$

We can plug this volume into equation 4.1.5 to solve for C.

$$C = P * \frac{NkT}{P}^{\gamma}.$$

We can now split up our equation for C: on one hand, we solve for the temperature T, and on the other, we solve for $\frac{d}{dP}$. In these two new equations, we define a new constant

$$C' \equiv (Nk)^{-\gamma} C.$$

[4]Note that although I'm using NkT instead of nRT, there is no difference because this term eventually cancels anyway.

Then, dividing out both equations, the constant C' cancels, and we can solve for $\frac{dT}{dP}$. These steps are shown below.

T:

$$T^{\gamma} = P^{\gamma-1}(Nk)^{-\gamma}C$$

$$T^{\gamma} = P^{\gamma-1}C' \tag{4.1.6}$$

$\frac{d}{dP}$:

$$(\gamma)T^{\gamma-1}\frac{dT}{dP} = (\gamma-1)P^{\gamma-2}C' \tag{4.1.7}$$

Dividing equation 4.1.7 by equation 4.1.6:

$$\frac{(\gamma)T^{\gamma-1}\frac{dT}{dP} = (\gamma-1)P^{\gamma-2}C'}{T^{\gamma} = P^{\gamma-1}C'},$$

and we get

$$(\gamma)T^{-1}\frac{dT}{dP} = (\gamma-1)P^{-1}.$$

Finally,

$$\frac{dT}{dP} = \frac{\gamma-1}{\gamma}(\frac{T}{P}). \tag{4.1.8}$$

For a diatomic gas (assuming N^2 is a good model for air),[3]

$$\gamma = \frac{C_p}{C_v} = \frac{(\frac{1}{2})f + 1(2)}{(\frac{1}{2})f} = \frac{f+2}{f}.$$

Using this γ, where f is the number of degrees of freedom in molecular motion, we can plug in to equation 4.1.8 to find $\frac{dT}{dP}$ in terms of just f, T, and P. This simplified form lets us plug in and solve for the dry adiabatic lapse rate, $\frac{dT}{dz}$. Additionally, to find $\frac{dP}{dz}$, we take the derivative of equation 4.1.1 in terms of height or z to find $\frac{dP}{dz} = -(\frac{mg}{kT})P$. Putting it all together:

$$\frac{dT}{dz} = [(\frac{2}{5+2})(\frac{T}{P})][-(\frac{mg}{kT})P] = -(\frac{2}{7})(\frac{mg}{k}).$$

For our rising air parcel, assuming it is made up of mostly nitrogen, we can solve for the dry adiabatic lapse rate:

$$\frac{dT}{dz} = -(\frac{2}{7})[\frac{(0.0288kg/mol)(9.81m/s^2)}{8.315J/K*mol}] = -0.0097K/m = -9.7°C/km \quad (4.1.9)$$

This means that for every $1km$ increase in altitude, a body experiences a change in temperature of about $-10°C$. See figure 4.1.3.

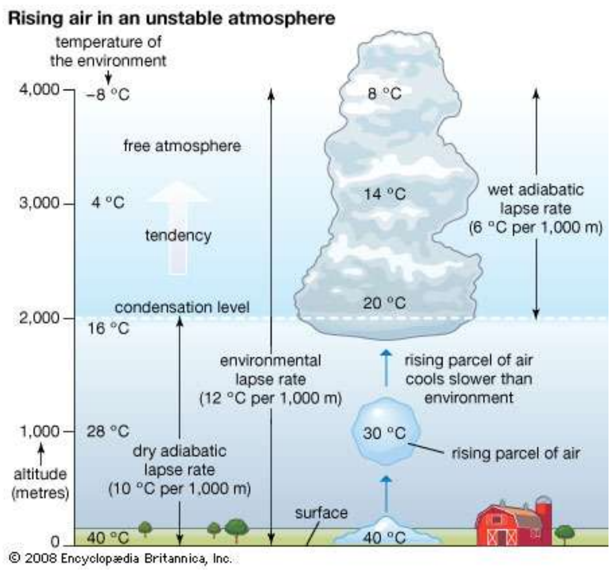

Figure 4.1.3. A picture of convection in the atmosphere, resulting in a dry adiabatic lapse rate until about $2km$, where the atmosphere then becomes the free atmosphere and we reach the condensation level.[13]

4.2 Reynolds Number

Obtaining theoretical solutions of the flow field around a vehicle is difficult. Because of this, experimental programs have been conducted to directly measure the parameters involved with the flow field. In order to determine under what conditions the experimental results obtained for one flow are applicable to another flow (which is confined by boundaries that are simply the geometry of the aircraft), we can derive the Reynolds number, which is a dimensionless measure of the ratio of inertial forces to viscous forces. Thus, the Reynolds number tells us that we can manipulate the inertial properties of an aircraft (such as the chord width of an airfoil) to change our movement through a fluid. It also tells us that the viscous properties of the fluid can change the nature of the flow (higher viscous force, lower Reynolds number; higher inertial term, higher Reynolds number). The objective of the parameters within the Reynolds number are as follows:[4]

1. To obtain information necessary to develop a flow model that could be used in numerical solutions.
2. To investigate the effect of various geometric parameters on the flow field.
3. To measure directly the aerodynamic characteristics of a complete vehicle.
4. To verify numerical predictions of aerodynamic characteristics for a particular configuration.

Synthesizing these objectives, the Reynolds number gives us a numerical approach to the particular geometric characteristics of a hybrid glide/powered vehicle. In addition, it helps us understand the flow field around the entire vehicle. From this, we can identify what we can change about the flight of an aircraft through air to optimize it for a particular air density and flow field.

Reynolds number (Re):

$$Re = \frac{Inertiaforce}{Viscousforce} = \frac{\rho v l}{\mu} = \frac{Ul}{\nu}, \tag{4.2.1}$$

[4]

where ρ is the density of the fluid, U is the velocity of the fluid (in the direction of the stream of the fluid), l is the chord width of the particular airfoil, μ is the dynamic viscosity of the fluid, and ν is the kinematic viscosity of the fluid. Increasing the inertial terms yields higher Reynolds numbers, which means more laminar flow in the flow field. That is, the lower Re is, the more sheet-like, smooth flowing the air is. At a higher Re, turbulence occurs as a results of differences in speed and direction of the fluid. For every aircraft, big or small, different Reynolds numbers drastically change the performance of that aircraft. Any aerospace program does not just study the scale factor of the wings and rigid-body of an aircraft. The Reynolds number (Re) dictates how that body will interact with the air flow around it.

Many external flows with which we are familiar with are associated with moderately sized objects. These objects tend to have a characteristic length on the order of $0.01m < l < 10m$. In addition, typical upstream velocities are on the order of $0.01m??/s < U < 100m??/s$, and the fluids involved are typically water or air. The resulting Reynolds number range for such flows is approximately $10 < Re < 10^9$.

Flows with $Re > 100$ are dominated by inertial effects, whereas flows with $Re < 1$ are dominated by viscous effects. Therefore, most familiar external flows are dominated by inertia.

Small Reynolds numbers mean that the viscous effects dominate, while large Reynolds numbers mean that the inertial terms dominate. Thus, small Reynolds numbers characterize situations where there is more turbulent flow around the object moving through the fluid. High Reynolds numbers typically describe situations with more laminar flow.

To visualize this, I've included a code that I have been working on in MATLAB, made available by [10]. Pay attention to the direction of the arrows in the pictures, which shows the direction of the velocity of a fluid particle at a particular point. In the case with a high Reynolds number, there is more variation in the direction of the particle flow over the entire grid.

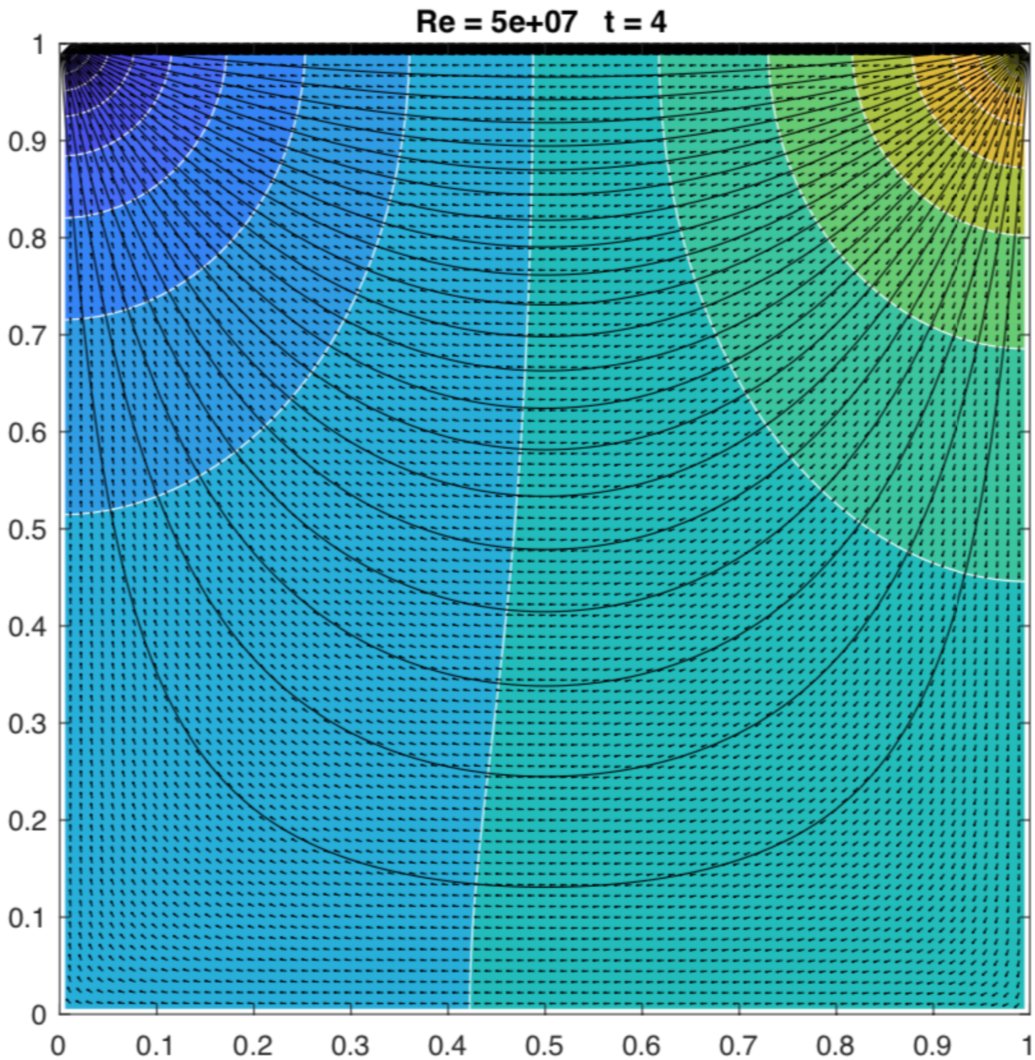

Figure 4.2.1. Consider a Navier-Stokes incompressible flow-field with a Reynolds number of $5.3560e7$. Notice that the velocity vectors, which are visualized by the small arrows, are steady throughout the flow field. That is, their direction does not vary as much as with a lower Reynolds number and turbulent flow. To read more about this solution to the Navier-Stokes equation, see [10].

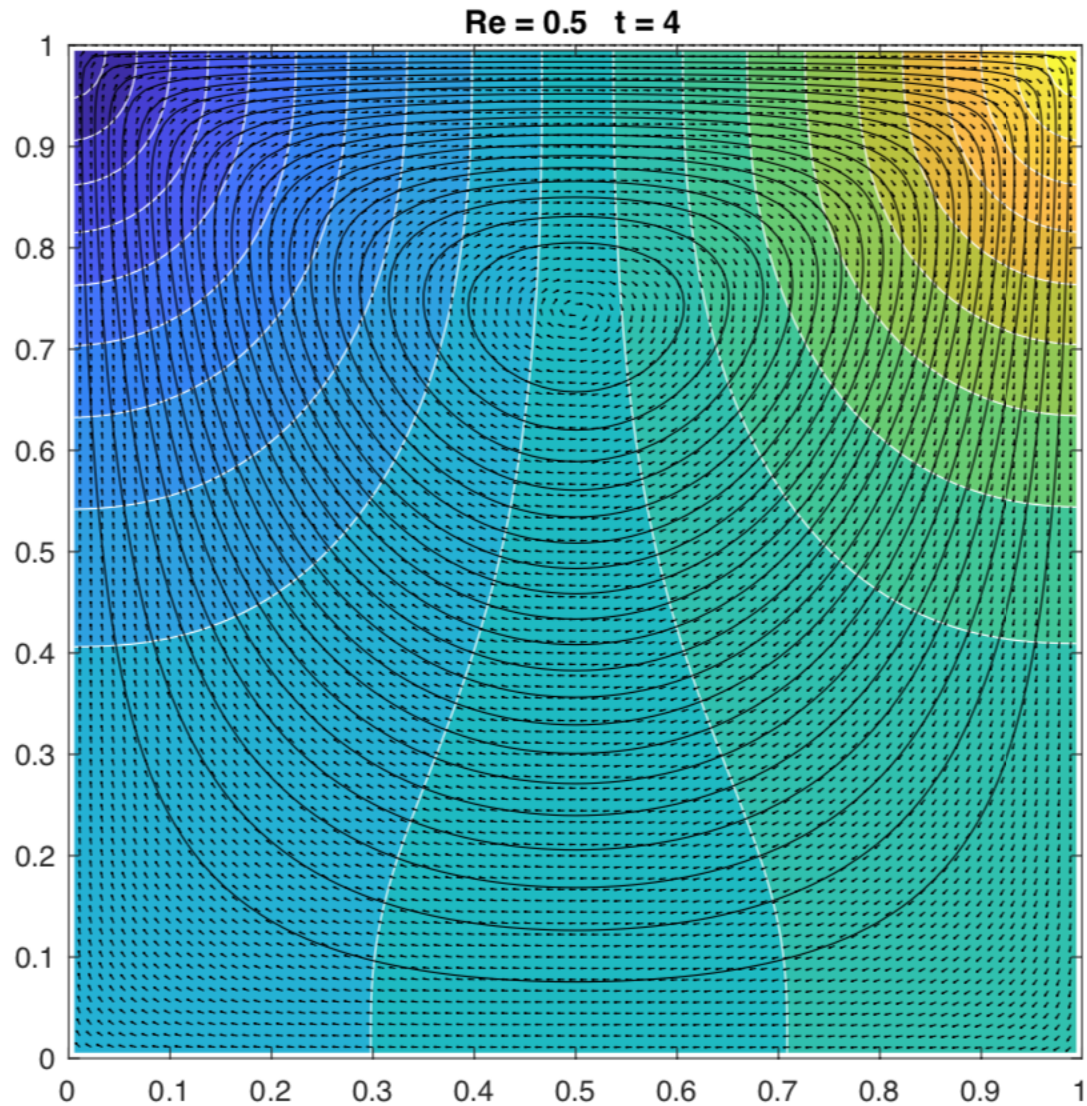

Figure 4.2.2. I used the same Navier-Stokes solution to produce this image, but significantly decreased the Reynolds number.[10]

4.3 Lift and Drag

In order to solve for the forces on a plane in a flight path model, one must find out some extremely difficult parameters about the aircraft. I was interested in being able to calculate the lift and drag forces on an object so that I could solve for the energy loss due to drag in the following chapter.

When a body moves through a fluid, its interaction between it and the fluid can be described in terms of the stresses. The wall shear stresses on the body, τ_w, is due to viscous effects and normal stresses due to the pressure p. Both τ_w and p vary in magnitude and direction along the surface of an airfoil. See figure 4.5.1.

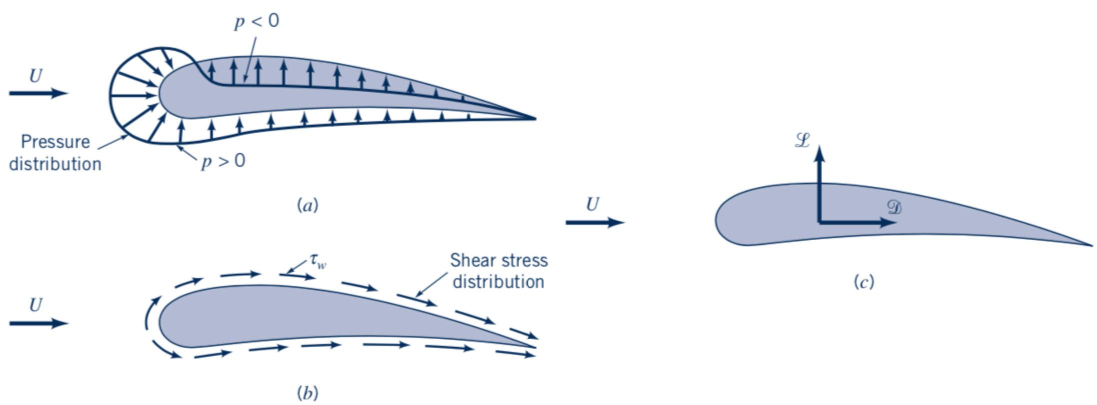

Figure 4.3.1. Provided by Munson, we can see the forces from the fluid surrounding a two dimensional slice of a wing.[3]

Solving for lift and drag includes summing the pressure and stress forces on the top and bottom surface areas of an object moving through a fluid. We obtain the lift and drag forces on an object by integrating the effect of the pressure and shear forces on the body surface. The pressure and shear forces on a small element of the surface of a body can be visualized like this:

The x and y components of the fluid force on the area element dA are, according to figure 4.5.1,

$$dF_x = (pdA)cos\theta + (\tau_w dA)sin\theta$$

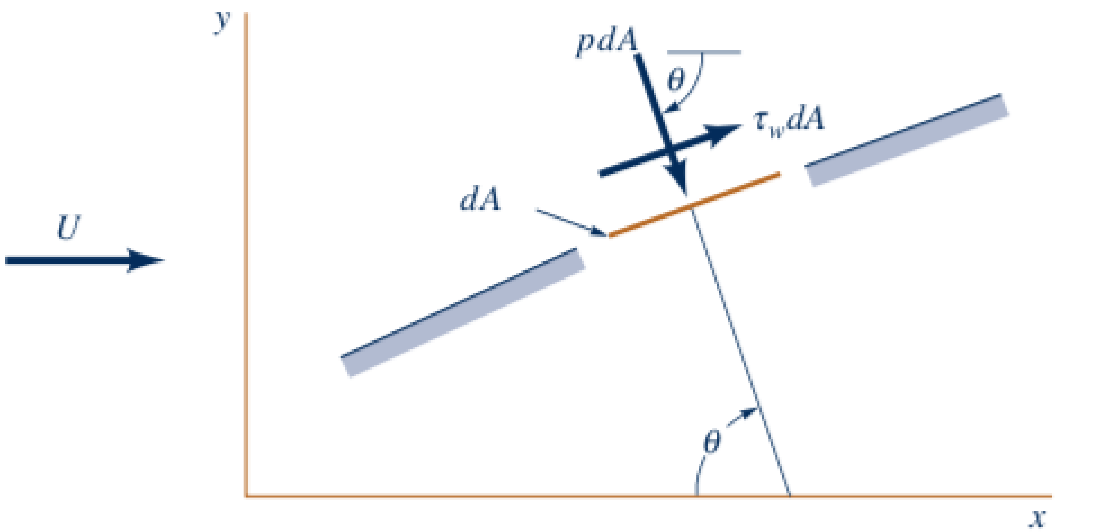

Figure 4.3.2. The pressure and shear force elements that affect the calculated lift and drag forces on an object.[3]

and

$$dF_y = -(pdA)sin\theta + (\tau_w dA)cos\theta.$$

The lift and drag forces are thus,

$$\mathscr{L} = \int dF_x = \int (pcos\theta)dA + \int (\tau_w sin\theta)dA \tag{4.3.1}$$

.

and

$$\mathscr{D} = \int dF_x = \int (pcos\theta)dA + \int (\tau_w sin\theta)dA. \tag{4.3.2}$$

Both the shear force and pressure force contributes to the lift and drag. In order to calculate lift and drag, you have to determine the the pressure and shear force distributions on the body. From an engineering standpoint, this is where one wing differentiates itself from another. Manipulating the shape of the wing changes the shear stress distribution and magnitude, as well as the pressure distribution and magnitude. Additionally, the angle of attack clearly affects the lift and drag of an airfoil and aircraft. Equations 4.5.1 and 4.5.2 are true for any body, but you still have to find the appropriate shear stress and pressure distributions on a body. Instead of this, I used a simplified method, which involves defining dimensionless lift and drag coefficients (C_L and C_D respectively). These are defined as:

$$C_L = \frac{\mathscr{L}}{\frac{1}{2}\rho * U^2 A} \tag{4.3.3}$$

$$C_D = \frac{\mathscr{D}}{\frac{1}{2}\rho * U^2 A}. \tag{4.3.4}$$

A is the characteristic area of the object, which I take in my calculations to be the frontal area (the projected area observed by someone looking toward the object from a direction normal to the upstream velocity, or U).

Calculating the characteristic area involves breaking the airfoil up like so:

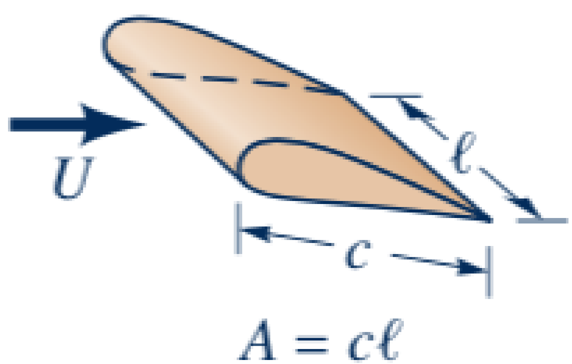

Figure 4.3.3. The frontal area of a finite, three-dimensional airfoil.[3]

I use this method of obtaining the area and solving for the drag coefficient C_D to find the drag force $\mathscr{D}$ and total energy of the model flight path.

Note a few properties of these formulas:

1. Drag increases proportional to the upstream velocity squared. This would be parasite drag, described in chapter 3. 2. The density of air ρ increases the drag force linearly. 3. The characteristic area of the object increases the drag force linearly. 4. The coefficient of drag increases linearly with the drag force (somewhat trivial).

5

The Model Flight Plan

In designing the appropriate flight model, I used the following aspect ratio and design specifics (figure 5.0.1), provided by the FAA glider handbook:

Matt and I made some assumptions about the flight of the aircraft to model a simple flight path. We identified the total energy as the parameter of interest, . The velocity of the aircraft is held constant so that the drag force is held constant. Additionally, we keep the density of the fluid constant. See the following hand-drawn diagram to understand the modeled flight path in the subsequent table of values.

I used the following parameters to model the energy-optimization flight path:

1. **Characteristic area**: $219.5ft^2 = 20.39m^2$[1]
2. **Aircraft mass**: $1040lb = 471.74kg$

3. **Reynolds Number**: $Re = 10^5$

[1] The characteristic area and aircraft mass come from figure 5.1.1.

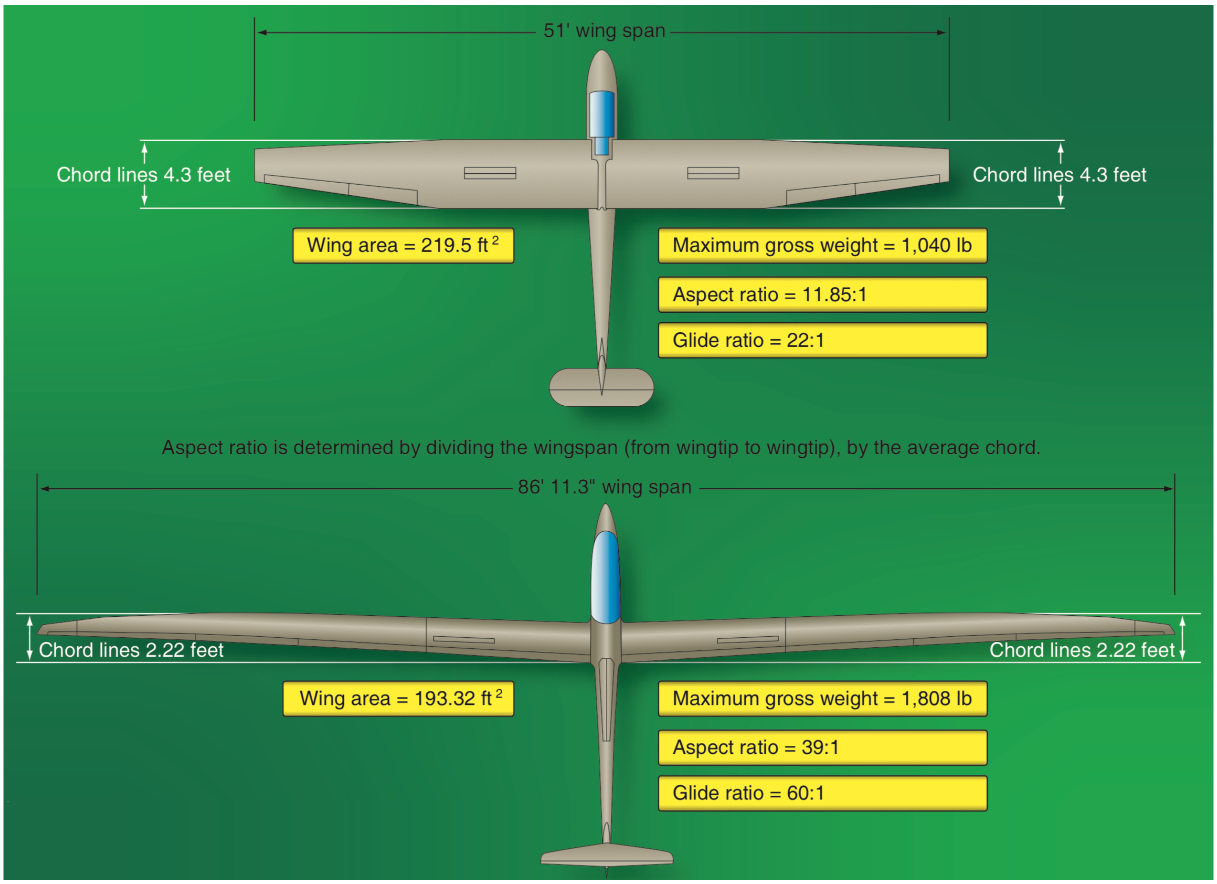

Figure 5.0.1. Gliders have a large aspect ratio to maintain lift at lower angles of attack.[7]

4. Angle of decline: $a = 3^{\circ}$

5. Velocity: $73 knots = 37.55 m/s$

6. Drag coefficient: $C_D = 0.12$

For data Sets 1 and 2: the temperature is held constant at room temperature, $273K$. For data sets 3 and 4, temperature and air density at different altitudes reflects the U.S. Standard Atmosphere measurements published by the United States in 1976.[5]. The aircraft velocity was taken from an FAA document online that cited the best glide speed for a PA 28 161 (see [11]).The drag force, using equation 4.3.4, depends on the characteristic area A, the upstream velocity U, the drag coefficient C_D, and the air density, which depends on the temperature and air pressure at a particular altitude.

The Drag coefficient and associated Reynolds number are set to values that ensure laminar flow in the path of the vehicle.

5.1 Data Sets For Energy Optimization

When we modeled this flight, we visualized it in a simple geometric flight path. We held the angle of descent, a, constant, so we could vary the angle of ascent b and observe how much powered energy would be needed for different paths along the descent line, G. The maximum height of the flight path h and the horizontal length component of the glide-path x were found using trigonometric identities and a system of equations, shown at the top my drawing. The total horizontal distance d of the flight path was held constant at $1000km$.

The length of the gliding descent is labeled G, while the length of the powered ascent is L. The relationship between these two lengths is of interest.

In data sets 1 and 2, the air pressure is calculated using the same air pressure formula derived in chapter 4 (equation 4.1.2). This air pressure is used to calculate air density, using the equation $\rho = \frac{p}{RT}$[3]. The value used for the gas constant R is the SI value: $R = 287.05N * m/kg * K$.

In data sets 3 and 4, I did not calculate the air density, nor did I hold the temperature at a constant room temperature. The values for air density (kg/m^3) and temperature (K) are taken from the Geometric U.S. Standard Atmosphere measurements published by the United States in 1976[5].

The kinetic energy KE of the plane is calculated by taking $\frac{1}{2}mV^2$ where m is the mass of the vehicle and V is the aircraft velocity. The potential energy PE is found by calculating mgh where m is the mass of the vehicle, g is the gravitational acceleration $9.81m/s^2$, and h is the maximum height that the aircraft reaches before gliding.

Drag is calculated using $\mathscr{D} = (C_D)\frac{1}{2}\rho * U^2A$, which comes from the equation for the drag coefficient found in chapter 4. Once I calculated the drag force $\mathscr{D}$, I could then multiply it by the length of the path of the plane to get a “Drag Energy.” This is used to solve for the Powered Energy and Glide Energy. The drag energy is split up into two parts: a powered drag energy and a glide drag energy. These values reflect the drag energy over the powered path and gliding path of the aircraft, respectively. Thus, the drag glide energy (DGE) is proportional to the length G

and the drag powered energy (DPE) is proportional to the length L. In addition, notice that the parameters affecting drag are C_D, ρ, U^2 (which is just the aircraft velocity V^2), and A.

What I call "Powered Energy" is calculated by adding the potential energy and the energy lost due to drag. The potential energy reflects the amount of energy needed to reach a height h. This value is considered to be equal to the amount of energy that the powered flight would need to produce to get to that height. The energy lost to drag is considered to be equal and opposite in sign to the energy needed for thrust to maintain the same velocity. Since velocity is held constant, the drag force is equal and opposite to the thrust force. Thus, $PoweredEnergy$ is the amount of energy that the powered plane requires from its engine to ascend.

What I call "Glide Energy" is calculated by subtracting the drag energy, calculated over the path of the glide flight G, from the kinetic energy KE. The glide energy is thus directly proportional to the amount of drag. I solved for this energy because the drag energy for the glide path comes directly out of the kinetic energy of the glider. I'm assuming that the glider can maintain its velocity by manipulating its shape and creating parasite drag (for example, using flaps to create more resistance to the flow). This energy reflects the amount of "free energy" that the aircraft has cashed in from its potential energy. This energy is used to counteract drag energy.

I'm more interested in energy exchanges than total energy. I care more about the transition from one type of energy to another. Powered Energy tells me the sum of how much energy is needed from an engine to get up to a height h and how much energy is needed to counteract drag. Glide energy tells me how much kinetic energy I have (I bought this kinetic energy by getting up to a height h) minus the amount of energy taken away by drag (on the glide flight path G). The final calculation I made takes the difference of the Glide Energy and Powered Energy to calculate "Energy Efficiency." This value reflects the optimization parameter: when the Energy Efficiency has the highest value, the plane has taken the optimal flight path to reduce the amount of powered energy required and maximize the amount of glide energy achieved. I plotted the Powered Energy vs. Angle b and the Energy Efficiency vs. Angle b for all data sets.

Glide: $h = (0.0524)x$ — $\tan(b) = \tan(3°)$

Climb: $h = D\tan b - x\tan b$

} system of equations, can solve for x and h in every case

Interested in: Total E (KE + PE), E needed for thrust (Negative the energy needed for drag) ⇒ Glide Energy & Powered Energy

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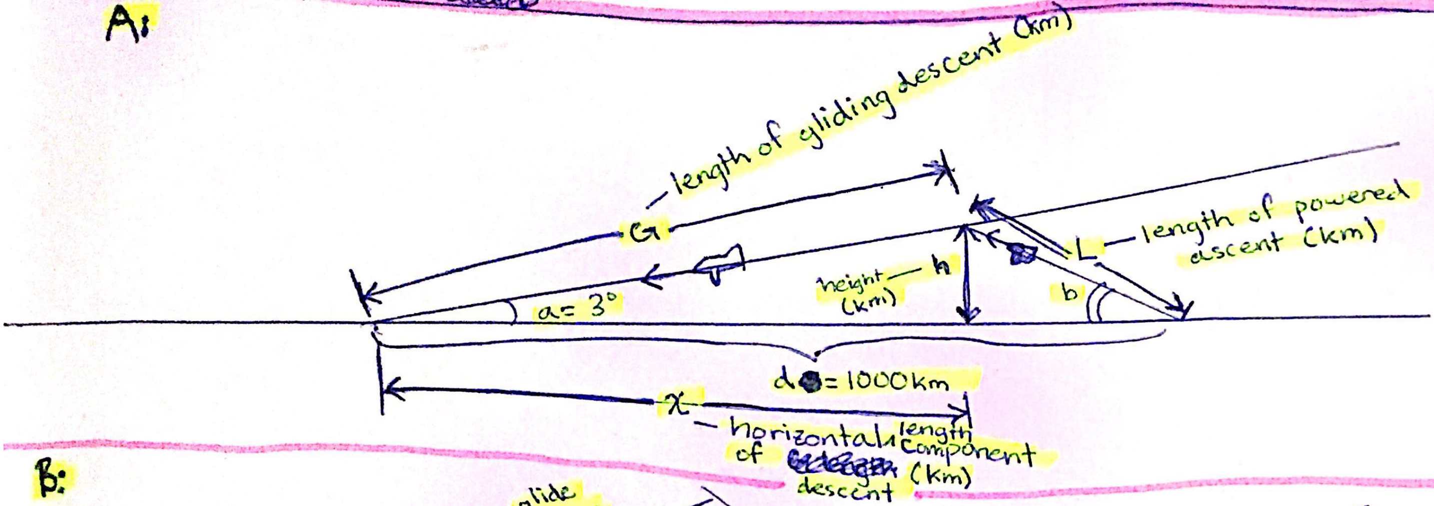
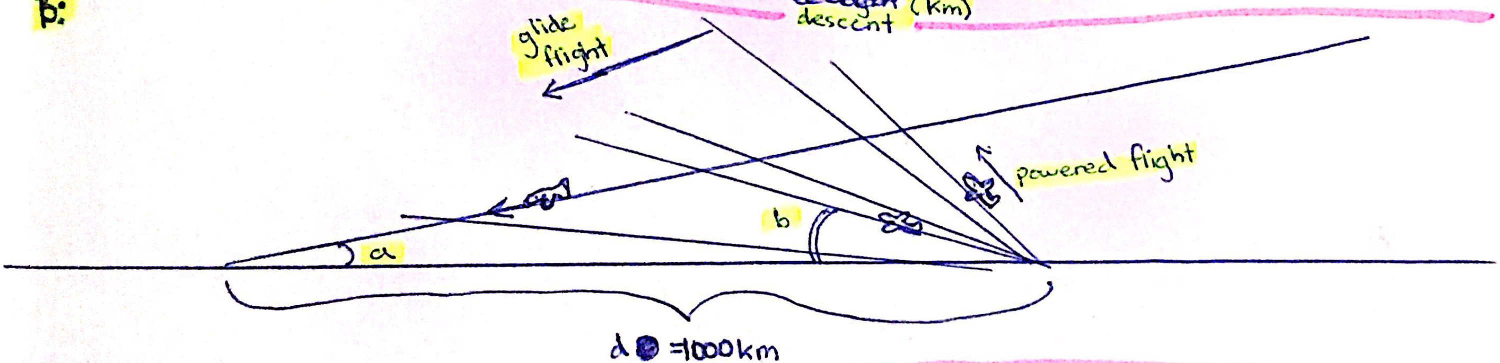
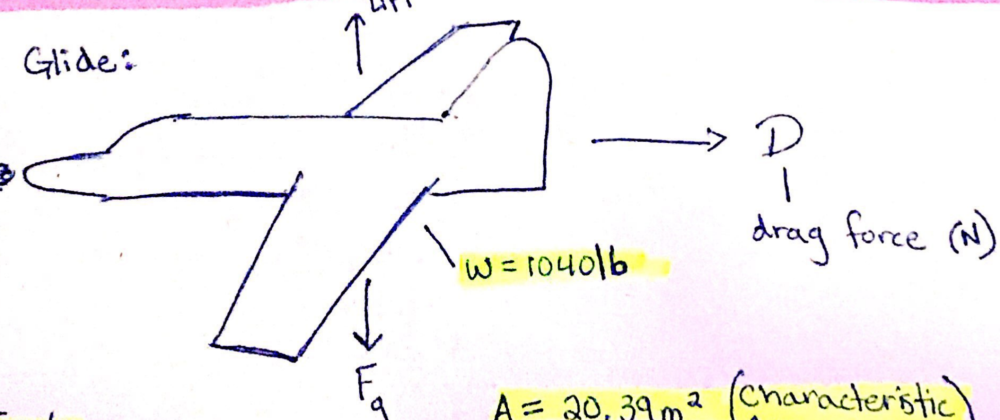
A:



B:



C:



$|\vec{v}| = |\vec{U}|$

aircraft velocity — upstream velocity (m/s)

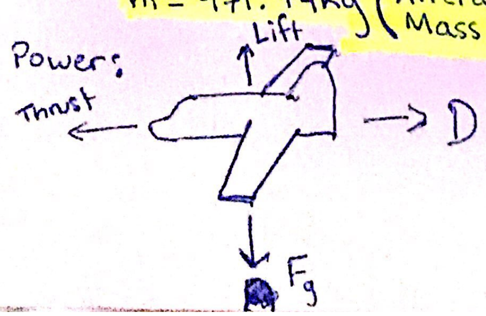
$v = 73$ knots $= 37.55$ m/s

Re is magnitude 10^5

$C_D = 0.12$ (laminar flow)

$A = 20.39\,m^2$ (Characteristic Area)

$m = 471.74$ kg (Aircraft Mass)



5.1.1 Data Sets 1 and 2

Data Set 1:

Angle a (°)	Aircraft Mass (kg)	Aircraft Velocity (m/s)	Pressure (Pa)	Temperature (K)	Air Density (kg/m^3)
3	471.74	37.55	397	273	0.00506
3	471.74	37.55	431	273	0.00550
3	471.74	37.55	476	273	0.00607
3	471.74	37.55	537	273	0.00686
3	471.74	37.55	626	273	0.00799
3	471.74	37.55	764	273	0.00975
3	471.74	37.55	1000	273	0.01276
3	471.74	37.55	1473	273	0.01880
3	471.74	37.55	2702	273	0.03448
3	471.74	37.55	8026	273	0.10242

Data Set 2:

Angle a (°)	Glider Mass (kg)	Aircraft Velocity (m/s)	Pressure (Pa)	Temperature (K)	Air Density (kg/m^3)
3	471.74	37.55	764	273	0.00975
3	471.74	37.55	810	273	0.01034
3	471.74	37.55	864	273	0.01103
3	471.74	37.55	927	273	0.01183
3	471.74	37.55	1000	273	0.01276
3	471.74	37.55	1087	273	0.01387
3	471.74	37.55	1191	273	0.01519
3	471.74	37.55	1317	273	0.01681
3	471.74	37.55	1473	273	0.01880
3	471.74	37.55	1669	273	0.02130
3	471.74	37.55	1921	273	0.02452
3	471.74	37.55	2253	273	0.02875
3	471.74	37.55	2702	273	0.03448
3	471.74	37.55	3332	273	0.04252
3	471.74	37.55	4254	273	0.05429
3	471.74	37.55	5678	273	0.07245
3	471.74	37.55	8026	273	0.10242

h (km)	d (km)	x (km)	d*tan(b)	tan(a)	tan(b)	Angle b (°)	L (km)	G (km)
45.81	1000	874.15	363.97	0.0524	0.3640	20	133.93	875.22
45.12	1000	861.13	324.92	0.0524	0.3249	18	146.02	862.18
44.30	1000	845.49	286.75	0.0524	0.2867	16	160.73	846.53
43.30	1000	826.33	249.33	0.0524	0.2493	14	178.98	827.34
42.04	1000	802.23	212.56	0.0524	0.2126	12	202.19	803.21
40.40	1000	770.91	176.33	0.0524	0.1763	10	232.63	771.85
38.17	1000	728.41	140.54	0.0524	0.1405	8	274.25	729.31
34.97	1000	667.31	105.10	0.0524	0.1051	6	334.52	668.13
29.95	1000	571.64	69.93	0.0524	0.0699	4	429.41	572.34
20.96	1000	399.91	34.92	0.0524	0.0349	2	600.45	400.40

h (km)	d (km)	x (km)	d*tan(b)	tan(a)	tan(b)	Angle b (°)	L (km)	G (km)
40.40	1000	770.91	176.33	0.0524	0.1763	10	232.63	771.85
39.90	1000	761.54	167.34	0.0524	0.1673	9.5	241.78	762.47
39.37	1000	751.40	158.38	0.0524	0.1584	9	251.69	752.32
38.80	1000	740.40	149.45	0.0524	0.1495	8.5	262.48	741.31
38.17	1000	728.41	140.54	0.0524	0.1405	8	274.25	729.31
37.48	1000	715.30	131.65	0.0524	0.1317	7.5	287.16	716.17
36.73	1000	700.89	122.78	0.0524	0.1228	7	301.36	701.74
35.89	1000	684.97	113.94	0.0524	0.1139	6.5	317.06	685.81
34.97	1000	667.31	105.10	0.0524	0.1051	6	334.52	668.13
33.93	1000	647.59	96.29	0.0524	0.0963	5.5	354.04	648.38
32.77	1000	625.42	87.49	0.0524	0.0875	5	376.01	626.18
31.46	1000	600.31	78.70	0.0524	0.0787	4.5	400.93	601.04
29.95	1000	571.64	69.93	0.0524	0.0699	4	429.41	572.34
28.22	1000	538.58	61.16	0.0524	0.0612	3.5	462.28	539.24
26.20	1000	500.04	52.41	0.0524	0.0524	3	500.65	500.65
23.82	1000	454.51	43.66	0.0524	0.0437	2.5	546.01	455.07
20.96	1000	399.91	34.92	0.0524	0.0349	2	600.45	400.40

KE (J)	PE (J)	Drag (N)	Drag Powered Energy (J)	Drag Glide Energy (J)	DGE - DPE (J)
332,577	211,977	8.74	1170	7646	6476
332,577	208,819	9.49	1385	8181	6795
332,577	205,028	10.48	1684	8869	7185
332,577	200,382	11.83	2117	9788	7670
332,577	194,537	13.78	2787	11071	8284
332,577	186,941	16.81	3911	12976	9065
332,577	176,637	22.01	6036	16052	10016
332,577	161,820	32.42	10847	21664	10817
332,577	138,620	59.47	25538	34039	8501
332,577	96,977	176.68	106087	70742	-35344

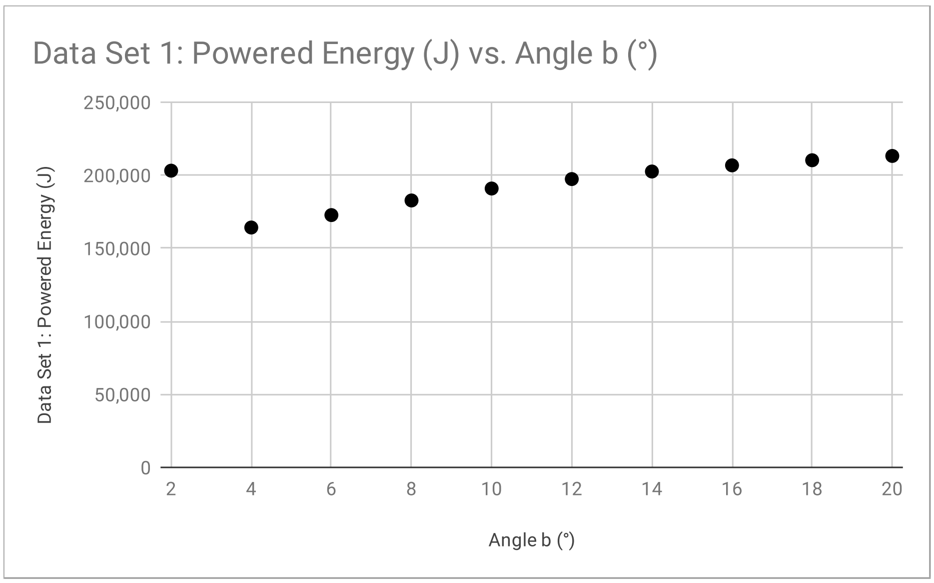
KE (J)	PE (J)	Drag (N)	Drag Powered Energy (J)	Drag Glide Energy (J)	DGE - DPE (J)
332,577	186941	16.81	3911	12976	9065
332,577	184670	17.84	4313	13603	9290
332,577	182212	19.02	4788	14313	9524
332,577	179544	20.40	5354	15122	9768
332,577	176637	22.01	6036	16052	10016
332,577	173456	23.92	6868	17130	10262
332,577	169962	26.21	7898	18391	10493
332,577	166103	28.99	9191	19881	10690
332,577	161820	32.42	10847	21664	10817
332,577	157037	36.74	13009	23824	10815
332,577	151660	42.29	15902	26481	10580
332,577	145572	49.59	19881	29804	9923
332,577	138620	59.47	25538	34039	8501
332,577	130603	73.34	33904	39549	5644
332,577	121257	93.64	46883	46883	0
332,577	110217	124.98	68239	56874	-11365
332,577	96977	176.68	106087	70742	-35344

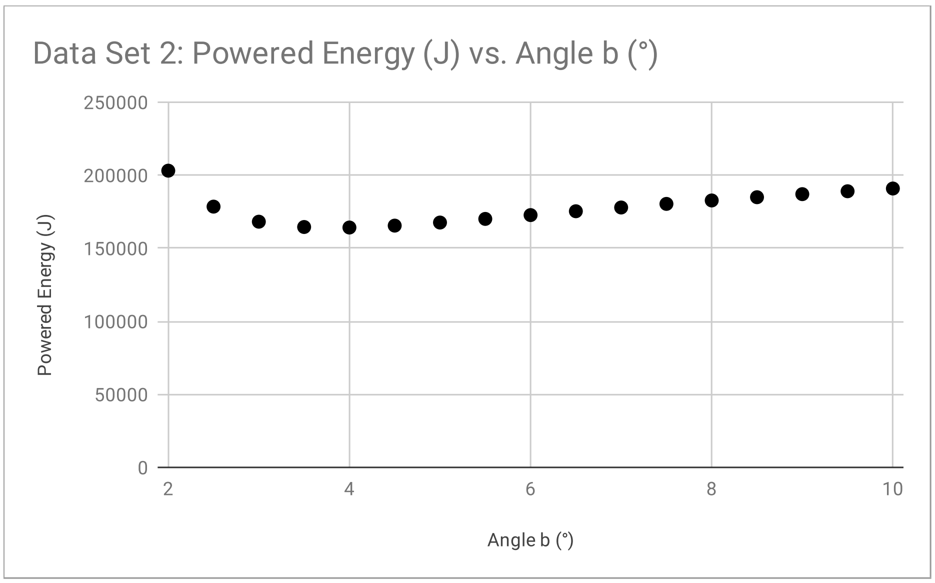
Powered Energy (J)	(calories)	(Btu)
213,147	50870	202
210,204	50168	199
206,712	49335	196
202,499	48329	192
197,324	47094	187
190,852	45549	181
182,673	43597	173
172,666	41209	164
164,158	39178	155
203,064	48464	192

Powered Energy (J)	(calories)	(Btu)
190852	45549	181
188983	45103	179
187000	44630	177
184898	44128	175
182673	43597	173
180325	43037	171
177859	42449	168
175294	41836	166
172666	41209	164
170046	40584	161
167562	39991	159
165453	39488	157
164158	39178	155
164508	39262	156
168140	40129	159
178457	42591	169
203064	48464	192

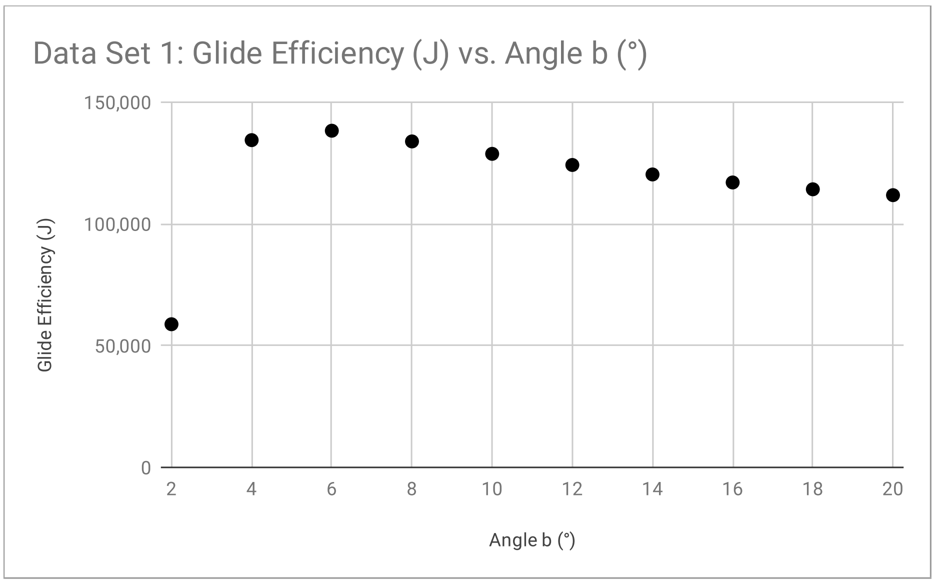
Glide Energy (J)	(calories)	(Btu)	Glide Efficiency (J)
324,931	77549	308	111,784
324,396	77422	307	114,192
323,708	77257	307	116,996
322,789	77038	306	120,290
321,506	76732	304	124,182
319,601	76277	303	128,749
316,525	75543	300	133,852
310,913	74204	294	138,247
298,538	71250	283	134,381
261,835	62490	248	58,771

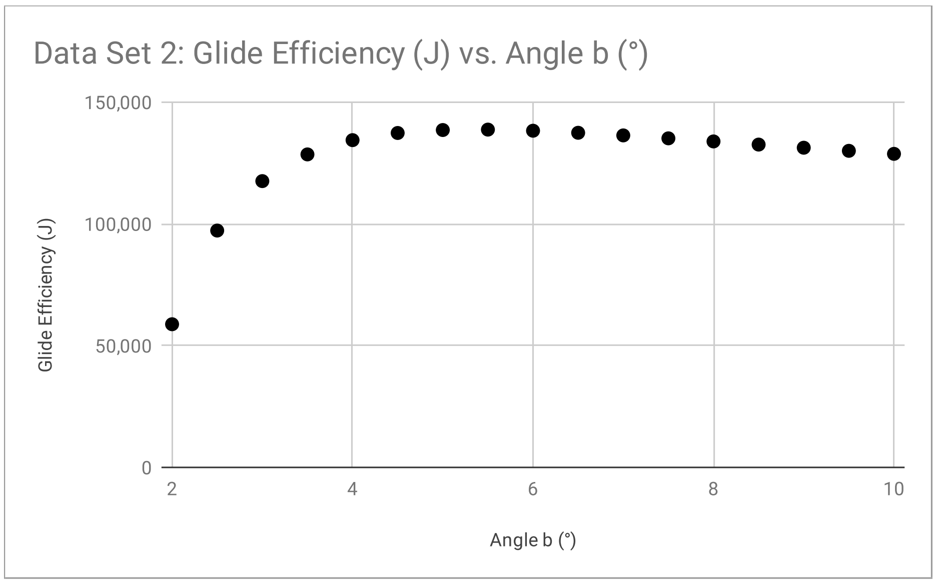
Glide Energy (J)	(calories)	(Btu)	Glide Efficiency (J)
319,601	76277	303	128,749
318,974	76127	302	129,991
318,264	75958	301	131,264
317,455	75765	301	132,557
316,525	75543	300	133,852
315,447	75286	299	135,122
314,186	74985	298	136,327
312,696	74629	296	137,401
310,913	74204	294	138,247
308,753	73688	292	138,707
306,096	73054	290	138,534
302,773	72261	287	137,320
298,538	71250	283	134,381
293,028	69935	278	128,521
285,694	68185	271	117,554
275,703	65800	261	97,246
261,835	62490	248	58,771


Data Set 1: Powered Energy (J) vs. Angle b (°)
250,000
200,000
150,000
100,000
50,000
0
Data Set 1: Powered Energy (J)
2
4
6
8
10
12
14
16
18
20
Angle b (°)



Data Set 2: Powered Energy (J) vs. Angle b (°)
250000
200000
150000
100000
50000
0
Powered Energy (J)
2
4
6
8
10
Angle b (°)


Data Set 1: Glide Efficiency (J) vs. Angle b (°)
Glide Efficiency (J)
150,000
100,000
50,000
0
2
4
6
8
10
12
14
16
18
20
Angle b (°)


Data Set 2: Glide Efficiency (J) vs. Angle b (°)
Glide Efficiency (J)
150,000
100,000
50,000
0
2
4
6
8
10
Angle b (°)

5.1.2 Data Sets 3 and 4

Data sets 3 and 4 include values for air density (kg/m^3) and temperature (K) taken from the Geometric U.S. Standard Atmosphere measurements published by the United States in 1976[5].

Data Set 3:

Angle a (°)	Aircraft Mass (kg)	Aircraft Velocity (m/s)	Temperature (K)	Air Density (kg/m^3)
3	471.74	37.55	266.373	0.001762
3	471.74	37.55	264.164	0.001913
3	471.74	37.55	261.403	0.002193
3	471.74	37.55	258.641	0.002527
3	471.74	37.55	255.878	0.002995
3	471.74	37.55	251.456	0.003770
3	471.74	37.55	244.818	0.005209
3	471.74	37.55	236.513	0.008463
3	471.74	37.55	226.509	0.018410
3	471.74	37.55	217.581	0.075715

Data Set 4:

Angle a (°)	Glider Mass (kg)	Aircraft Velocity (m/s)	Temperature (K)	Air Density (kg/m^3)
3	471.74	37.55	270.65	0.0038
3	471.74	37.55	227.5	0.0040
3	471.74	37.55	249.769	0.0044
3	471.74	37.55	262.147	0.0048
3	471.74	37.55	266.277	0.0052
3	471.74	37.55	270.409	0.0059
3	471.74	37.55	270.65	0.0066
3	471.74	37.55	255.878	0.0073
3	471.74	37.55	236.036	0.0085
3	471.74	37.55	260.771	0.0099
3	471.74	37.55	266.277	0.0102
3	471.74	37.55	269.031	0.0149
3	471.74	37.55	270.65	0.0184
3	471.74	37.55	266.925	0.0243
3	471.74	37.55	192.79	0.0337
3	471.74	37.55	258.019	0.0484
3	471.74	37.55	264.9	0.0757

h (km)	d (km)	x (km)	d*tan(b)	tan(a)	tan(b)	Angle b (°)	L (km)
45.81	1000	874.15	363.97	0.0524	0.3640	20	134
45.12	1000	861.13	324.92	0.0524	0.3249	18	146
44.30	1000	845.49	286.75	0.0524	0.2867	16	161
43.30	1000	826.33	249.33	0.0524	0.2493	14	179
42.04	1000	802.23	212.56	0.0524	0.2126	12	202
40.40	1000	770.91	176.33	0.0524	0.1763	10	233
38.17	1000	728.41	140.54	0.0524	0.1405	8	274
34.97	1000	667.31	105.10	0.0524	0.1051	6	335
29.95	1000	571.64	69.93	0.0524	0.0699	4	429
20.96	1000	399.91	34.92	0.0524	0.0349	2	600

h (km)	d (km)	x (km)	d*tan(b)	tan(a)	tan(b)	Angle b (°)	L (km)
40.40	1000	770.91	176.33	0.0524	0.1763	10	233
39.90	1000	761.54	167.34	0.0524	0.1673	9.5	242
39.37	1000	751.40	158.38	0.0524	0.1584	9	252
38.80	1000	740.40	149.45	0.0524	0.1495	8.5	262
38.17	1000	728.41	140.54	0.0524	0.1405	8	274
37.48	1000	715.30	131.65	0.0524	0.1317	7.5	287
36.73	1000	700.89	122.78	0.0524	0.1228	7	301
35.89	1000	684.97	113.94	0.0524	0.1139	6.5	317
34.97	1000	667.31	105.10	0.0524	0.1051	6	335
33.93	1000	647.59	96.29	0.0524	0.0963	5.5	354
32.77	1000	625.42	87.49	0.0524	0.0875	5	376
31.46	1000	600.31	78.70	0.0524	0.0787	4.5	401
29.95	1000	571.64	69.93	0.0524	0.0699	4	429
28.22	1000	538.58	61.16	0.0524	0.0612	3.5	462
26.20	1000	500.04	52.41	0.0524	0.0524	3	501
23.82	1000	454.51	43.66	0.0524	0.0437	2.5	546
20.96	1000	399.91	34.92	0.0524	0.0349	2	600

G (km)
875
862
847
827
803
772
729
668
572
400

G (km)
772
762
752
741
729
716
702
686
668
648
626
601
572
539
501
455
400

KE (J)	PE (J)	Drag (N)	Drag Powered Energy (J)	Drag Glide Energy (J)	DGE - DPE (J)
332,577	211,977	3.04	407	2660	2253
332,577	208,819	3.30	482	2845	2363
332,577	205,028	3.78	608	3202	2594
332,577	200,382	4.36	780	3606	2826
332,577	194,537	5.17	1044	4149	3105
332,577	186,941	6.50	1513	5019	3506
332,577	176,637	8.99	2464	6553	4089
332,577	161,820	14.60	4884	9754	4870
332,577	138,620	31.76	13637	18176	4539
332,577	96,977	130.61	78424	52296	-26128

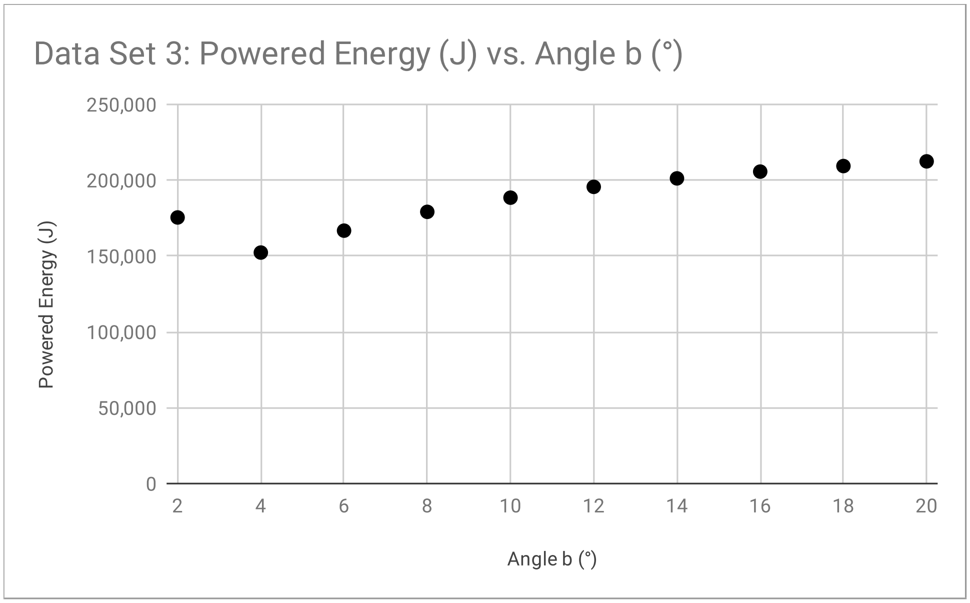
KE (J)	PE (J)	Drag (N)	Drag Powered Energy (J)	Drag Glide Energy (J)	DGE - DPE (J)
332,577	186940.92	6.50	1513	5019	3506
332,577	184669.54	6.89	1666	5255	3589
332,577	182211.99	7.52	1894	5661	3767
332,577	179544.01	8.22	2158	6094	3936
332,577	176636.87	8.99	2464	6553	4089
332,577	173456.42	10.13	2908	7253	4345
332,577	169961.65	11.43	3444	8019	4575
332,577	166102.91	12.52	3970	8586	4617
332,577	161819.55	14.60	4884	9754	4870
332,577	157036.61	17.06	6038	11059	5020
332,577	151660.42	17.60	6617	11020	4403
332,577	145572.32	25.62	10274	15402	5128
332,577	138619.74	31.76	13637	18176	4539
332,577	130603.16	41.93	19386	22613	3227
332,577	121256.56	58.17	29125	29125	0
332,577	110217.17	83.57	45631	38031	-7600
332,577	96977.11	130.61	78424	52296	-26128

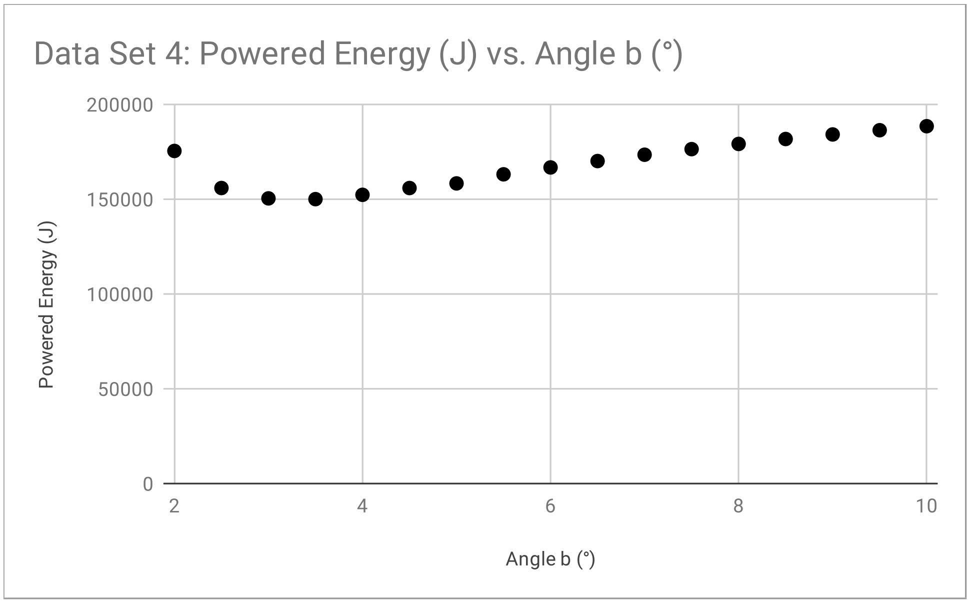
Powered Energy (J)	(in calories)	(in Btu)
212,384	50688	201
209,301	49952	198
205,636	49078	195
201,162	48010	191
195,582	46678	185
188,454	44977	178
179,101	42745	170
166,703	39786	158
152,256	36338	144
175,401	41862	166

Powered Energy (J)	(in calories)	(in Btu)
188454	44977	178
186336	44472	176
184106	43939	174
181702	43366	172
179101	42745	170
176365	42092	167
173405	41386	164
170073	40590	161
166703	39786	158
163075	38920	154
158278	37775	150
155846	37195	148
152256	36338	144
149989	35797	142
150381	35891	142
155848	37195	148
175401	41862	166

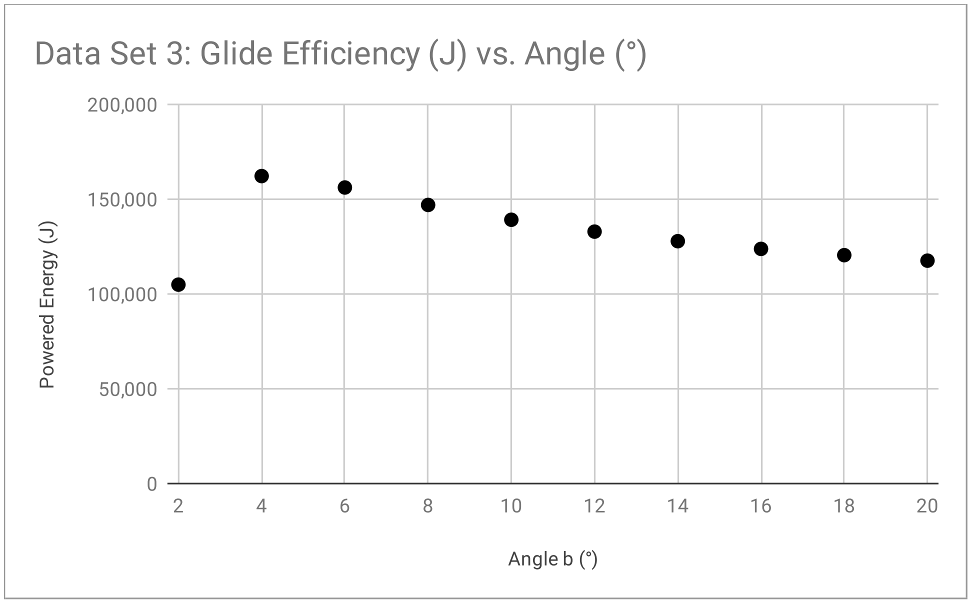
Glide Energy (J)	(in calories)	(in Btu)	Glide Efficiency (J)
329,918	78739	312	117,534
329,732	78695	312	120,432
329,375	78610	312	123,739
328,971	78513	312	127,809
328,428	78384	311	132,846
327,558	78176	310	139,104
326,024	77810	309	146,923
322,823	77046	306	156,120
314,401	75036	298	162,145
280,281	66893	265	104,880

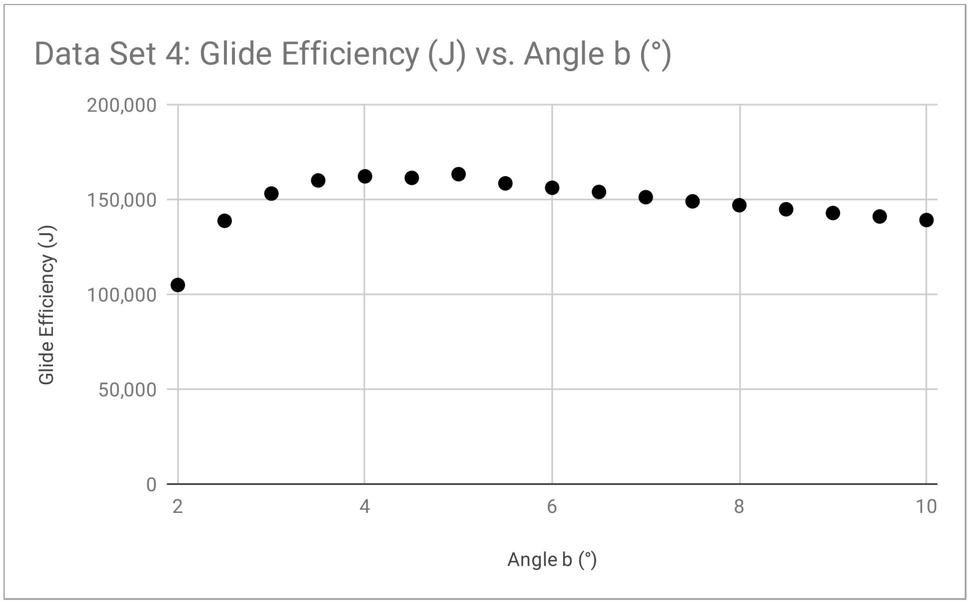
Glide Energy (J)	(in calories)	(in Btu)	Glide Efficiency (J)
327,558	78176	310	139,104
327,322	78120	310	140,986
326,916	78023	310	142,810
326,484	77920	309	144,782
326,024	77810	309	146,923
325,324	77643	308	148,960
324,558	77460	307	151,153
323,991	77325	307	153,919
322,823	77046	306	156,120
321,519	76735	305	158,444
321,557	76744	305	163,280
317,176	75698	300	161,330
314,401	75036	298	162,145
309,964	73977	294	159,976
303,453	72423	287	153,071
294,546	70297	279	138,698
280,281	66893	265	104,880


Data Set 3: Powered Energy (J) vs. Angle b (°)
250,000
200,000
150,000
100,000
50,000
0
Powered Energy (J)
2
4
6
8
10
12
14
16
18
20
Angle b (°)



Data Set 4: Powered Energy (J) vs. Angle b (°)
200000
150000
100000
50000
0
Powered Energy (J)
2
4
6
8
10
Angle b (°)


Data Set 3: Glide Efficiency (J) vs. Angle (°)
200,000
150,000
100,000
50,000
0
Powered Energy (J)
2
4
6
8
10
12
14
16
18
20
Angle b (°)


Data Set 4: Glide Efficiency (J) vs. Angle b (°)
200,000
150,000
100,000
50,000
0
Glide Efficiency (J)
2
4
6
8
10
Angle b (°)

5.2 Optimal Path

The Energy Efficiency plots show that there is a maximum value reached for lower values of b, ranging from $4°$ to $6°$. The Powered Energy plots show that at that maximum Energy Efficiency value, there is also a minimum amount of powered energy required from the aircraft. Notice that for tiny angles of b, the powered energy gets very big in all cases. At this angle, the drag powered energy overcomes the drag glide energy, which gives a negative value for $DGE - DPE$.

The parameters affecting drag are C_D, ρ, U^2 (which is just the aircraft velocity V^2), and A. The only parameter that varies in this equation is ρ, which varies due to air density.[2] Powered Energy scales with height and drag. Glide Energy scales with drag. The optimal flight path combines the geometry of the flight path which affects the powered energy, and the amount of drag (related to the air densities at different altitudes) which affects the Glide Energy. Because the weight and velocity of the aircraft is assumed constant, the kinetic energy term in the Glide Energy is unchanging.

The Powered Energy relies on reducing h, and also reducing L. Because we assume that the thrust force that the powered plane provides is equal to the drag force that we solve for, we want our powered flight path to have as short a distance as possible.

Thus, the optimal flight path relies on reducing h and L as much as possible, while also reducing the amount of drag over the entire trip. The drag depends on the altitude that the aircraft reaches. The first set of data shows that the Energy Efficiency still has an optimal value when the temperature is held constant. This confirmed for me that my pressure and air density calculations were relatively accurate, even if they did not account for different layers of the atmosphere where you see big differences in air pressure and density. The second sets of data, 3 and 4, provide an optimal Energy Efficiency for more accurate air pressure/density numerical values.

[2]Note that the drag forces calculated tend to be low. This is due to a low coefficient of drag (also to the fact that the plane has small dimensions), which was held constant. When that coefficient is made bigger, the drag forces calculated scale linearly.

6

Conclusions

Given the assumptions and parameters detailed in chapter 5, there seems to be a way to reduce the amount of powered energy required of an aircraft, and maximize the amount of glide energy achieved by an aircraft. The Energy Efficiency plots reach a maximum value for lower angles of b, between about $4°$ to $6°$. In summary, I am optimistic that there is a way to use glide technology in the flight plan of a powered vehicle to reduce the amount of chemical energy needed from that vehicle. I believe that this is done by optimizing the angle of powered ascent so that the aircraft reaches lower values of h and L, and by generally reducing the types of drag contributing to the model.

My goal with this plan was to reduce the amount of energy required from an engine by designing a flight path that uses gliding to convert potential energy into kinetic energy. In the future, there are more parameters that can be studied to make this model better from a theoretical standpoint. For instance, we held the velocity constant throughout the trip, meaning there must be something on the aircraft that increases drag during the glide descent to maintain that velocity. In future work, it would be interesting to see how much speed can be gained from an increasing kinetic energy on the glide path. This would mean letting gravity do its work on the glide path. If one could pick up speed on the glide path, and then immediately go back into the powered mode ascent/glide descent (and again pick up speed on the glide path), there might be an interesting way to reduce the amount of energy needed from burning fuel while also picking up speed from the gliding portion of the flight. It would thus be of interest to design a flight plan that involves utilizing this hybrid glide/powered flight path more than once during flight, perhaps to save on time, and efficiently increase aircraft velocity.

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