The Integration of Simple Growth and Cycle Models

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INTRODUCTION

VARIOUS ceiling models of cycles or cyclical growth have appeared. In all except one, Kurhara's model, the rate of growth of the ceiling is exogenous. However, the saving and investing that takes place as income is at or below the ceiling implies that the ceiling grows. This paper investigates the conditions under which the rate of growth of ceiling income, as generated by the demand-determined division of income between investment and consumption, is sufficiently large that self-sustained growth can take place.

Existing econometric income models can be divided into two broad classes: short-run forecasting and long-run growth. The short-run forecasting models are basically extensions of the simple Keynesian aggregate demand-determining models. The long-run growth models assume that sufficient aggregate demand always exists and investigate the implications of various patterns of input changes for the growth of capacity.

In many ways, the most interesting analytical and forecasting range is neither the very short run nor the very long run. An intermediate horizon, of ten to fifteen years, is of great practical interest for economic policy, for this is the time span that encompasses the possibility of major or deep depression cycles. Although it is legitimate in constructing short-run forecasting models to ignore the impact of investment upon productive capacity and of finance upon the stock of financial instruments outstanding, over a ten- or fifteen-year period these small changes will cumulate and be of decisive importance in determining system behavior. On the other hand, the standard strategy in constructing long-run models is to assume that the impact of financial variables...
can wash out. Thus, both practical and theoretical possibilities open up when an intermediate horizon is adopted.

Recent work in the long waves in economic growth rates\(^2\) and on mild and deep depression cycles\(^6\) also indicate that a complete model of the income-determining process that can be iterated to generate a ten- to fifteen-year time series is of interest.

Both the short-run and the long-run models are one-sided, in that they are concerned with either aggregate demand or aggregate supply, and incomplete, in that they do not include, in any deep sense, monetary and financial phenomena. Friedman and Schwartz\(^4\) have imputed the observed pattern of cycles to the behavior of quite narrowly defined money; Tobin,\(^5\) implicitly, and Minsky,\(^6\) explicitly, have examined the implications of financial factors for the longer waves. Aside from the previously mentioned paper by Kurihara, scant attention has been paid to how the productive capacity ceiling is generated, or to the interaction of the production ceiling with demand determination. In this short paper we shall undertake only a part of the total analytical work, and we shall essentially ignore the monetary-financial feedbacks in the growth process. What will be undertaken is to integrate aggregate demand and supply determination in an income model.

Special attention will be paid to those conditions which must be satisfied if self-sustained growth is to take place. Our results show that self-sustained growth is not likely, except as an intermittent phenomenon, unless inflation succeeds in curtailing consumption, or technological progress, whether embodied or disembodied, raises the rate of growth of ceiling income. With a sufficiently rapid rate of growth of ceiling income, the ceiling constraints will not necessarily trigger a downturn. Thus, once again, we have to turn to the characteristics of the aggregate demand-determining relation to generate a downturn. In the conclusion it is suggested that if the coefficient of induced investment decreases as a result of financial changes, a downturn can take place because the rate of growth of ceiling income needed to maintain growth increases. That is, as a self-sustained growth process matures, it becomes necessary to run faster in order to stay in the same place.

**THE INGREDIENTS**

A simple income model that allows for both the behavior of aggregate demand and supply can be built out of well-known ingredients. To be precise, the model that will be discussed here consists of:

1. a demand generating relation which is the familiar Hansen-Samuelson\(^7\) accelerator-multiplier model,
2. a maximum supply (or productive capacity or ceiling income) generating relation derived from the Harrod-Domar\(^8\) growth models,
3. a minimum supply (or floor income) generating relation which is based on the assumption that there exists (a) a part of consumption (and perhaps investment) demand which is independent of current income, although not necessarily of past incomes or of the value of the capital stock, and (b) a maximum to the disinvestment that can take place per period, which is related to the size of the capital stock and hence to the maximum supply.
4. a reconciliation relation which states that actual income equals aggregate demand unless aggregate demand exceeds the maximum aggregate supply or falls below the minimum aggregate supply, in which case actual income will equal the appropriate aggregate supply.

Due to our present interest in self-sustained growth, the implications of assumption (3) will be ignored. This will enable us to simplify our demand-determining functions and write these as homogeneous relations. In another paper,\(^9\) I have examined how the nonhomogeneous portions of these equations affect both the interval of time for which self-sustained growth can take place, if the ceiling is not growing rapidly enough to sustain growth permanently, and the depth of the depression.

Self-sustained growth takes place when actual income and maximum supply income grow without the existence of any exogenous growth stimulating factor, i.e., an internally sustained state of steady growth. Within the framework of the Hansen-Samuelson plus Harrod-Domar integrated model under discussion, this means that the maximum aggregate supply is growing at a sufficiently high rate so that, with actual income equal to this maximum supply income, the demand induced by the achieved level and rate of change of income is sufficient to utilize fully the increasing productive capacity.

Standing by itself, the Hansen-Samuelson model states that income is determined by aggregate demand. In periods when demand is not constrained by aggregate supply, this Keynesian assumption is valid, especially if the nonhomogeneous part of the consumption function depends upon wealth which, of course, is a reflection of the economy's capital stock. With this interpretation of the Hansen-Samuelson model, the consumption function of this part of the integrated model is said to determine ex-ante consumption, and the accelerator based investment function is interpreted as determining ex-ante investment.

The second-order difference equation of the Hansen-Samuelson model is simple framework that yields the variety of time series necessary for cyclical analysis and also the possibility of a one-shot turning
point based upon initial conditions, which is vital for our analysis.

We assume that at any date the maximum available supply depends upon the existing capital stock. This capital stock changes by the amount of net investment. The rate of change of aggregate supply depends upon the net investment that occurs, and its productive efficiency. This obviously means that the saving coefficient of the Harrod-Domar part of the integrated model is an ex-post saving coefficient.

The productive efficiency of investment put into place relates the change in aggregate supply to the change in capital stock. As such it is an incremental output/capital coefficient. The way in which the Harrod-Domar growth model is typically written focuses attention on the reciprocal of the output/capital coefficient, the capital-output ratio. This way of writing the productive efficiency of investment makes it easy to assume that the productive efficiency of investment in the aggregate supply-determining relation is the reciprocal of the coefficient of induced investment in the demand-determining relation.

The coefficients of induced investment and of the productive efficiency of investment are two quite different things. The coefficient of induced investment—the accelerator coefficient in the relation that determines ex-ante investment—is in part based upon the productive efficiency of investment, but it is also related to the willingness of investors to take risks and the terms upon which investors can finance their endeavors. In spite of this recognition of the difference between the coefficients of induced investment and the productive efficiency of investment, we will initially assume that they are equal. This enables us to focus on the extent to which adjustments in consumption make it possible for self-sustained growth to occur.

The reconciliation relation, as used here, is a purely formal assertion that supply is, if necessary, an effective constraint. The really deep economics in any ceiling model focus on how supply is rationed. Whether consumption or investment demand, or both in varying degrees, are cut back is a result of market processes.

The model as set out here is not sufficiently complete to cover these phenomena. The function of financial markets is to ration investment funds. The available nominal supply of investment funds depends upon the functioning of the financial system. The ability of the financial system to constrain consumption depends upon the existing and desired portfolios of households. An integration of financial phenomena with the real demand- and supply-generating relations would be necessary to enable us to deal more precisely with the reconciliation relations.

We can look at the rationing process a bit more closely even without constructing a formal model of the financial system. In the diagram below,

\[ \log Y_0 \]

\[ \log Y_1 \]

\[ \log Y_2 \]

\[ \log Y_3 \]

\[ \log Y_4 \]

\[ \log Y_5 \]

\[ \log Y_6 \]

\[ \log Y_7 \]

\[ \log Y_8 \]

\[ \log Y_9 \]

\[ \log Y_{10} \]

\[ \log Y_{11} \]

\[ \log Y_{12} \]

\[ t \]

Log \( Y_e \) is the ceiling income and at each date \( t \), aggregate demand is greater than the ceiling, i.e., the ceiling is an effective determinant of income. Given that the demand for consumption goods is determined by income of the \( t-1 \text{st} \) period, consumers have the “cash in hand” to finance, at existing prices, the purchase of the consumption component of \( Y_d \). However, investors, independent of the separation between saving and investing units, have to finance investment in excess of planned saving.

This presumably requires changes that increase velocity or the money supply. To the extent that investment can be cut back to the difference between productive capacity and planned consumption without any rise in interest rates, the income-generating process need not be affected. However, if the rationing phenomenon results in a rise in interest rates (or its equivalent, a rise in the price of investment goods as against consumption goods), then the income-generating process will be affected. If we make the Keynesian assumption that consumption demand is independent of interest rates, but assume that investment demand, and hence the \( \beta \) coefficient, depends upon interest rates, then a rising set of interest rates will lower the \( \beta \) coefficient. A fall in \( \beta \) raises the minimum rate of growth of capacity that leads to demand's rising faster than capacity, when income is at the ceiling. Thus the reconciliation process can affect the effect of the ceiling by raising the rate of growth required to sustain growth.

The assumption that changes in the size of the capital stock are the sole determinants of the rate of growth of productive capacity is, of course, heroic. The alternative is to adopt a production function which allows for factor substitution and relate ceiling output growth to the growth of the labor force as well as capital equipment. However, once we assume that technological change occurs, the growth of capacity will not be dependent solely upon the growth of the capital stock. As is usual, the technological change coefficient becomes a catchall that allows not only for technical progress, but also for differential growth rates of the labor
force and capital and the improvement of the labor force due to education, public health, etc.

As a result, within a Harrod-Domar framework for the growth of capacity, we allow for both embodied and disembodied technical change. Embodied technical change works by way of the capital put into place and, in our formulation, will result in a rise in the productive efficiency of investment. Disembodied technical change results in a rise in productive capacity that is independent of the amount of investment put into place. Such “progress” is as inevitable and well-nigh as universal as the passage of time; and, like time, it covers a multitude of sins.

**THE FORMAL MODEL**

The formal model can be written as:

\[ Y^*_t = Y^*_{t-1} + \frac{I^*_{t-1}}{\beta} \]

(1)

\[ C^*_t = \alpha Y^*_t \]

(2)

\[ C^*_t = \lambda_1 C^*_t + \lambda_2 C^*_t \]

(3)

\[ I^*_t = \beta(Y^*_t - Y^*_{t-1}) \]

(4)

\[ I^*_t = (1 - \alpha)Y^*_t \]

(5)

\[ Y^*_t = \lambda_1 I^*_t + \lambda_2 I^*_t \]

(6)

\[ Y^*_t = C^*_t + I^*_t \]

(7)

\[ \lambda_1 = 1 \text{ if } Y^*_t = C^*_t + I^*_t \leq Y^*_t \]

\[ = 0 \text{ otherwise} \]

(9)

\[ \lambda_1 + \lambda_2 = 1. \]

(10)

\[ C, I, \text{ and } Y \text{ have their usual meanings, the superscript } a \text{ means actual, } d \text{ means demand and } s = \text{ supply, } \alpha = \text{ ex-ante marginal (}= \text{ average in these models) propensity to consume, } \beta = \text{ ex-ante coefficient of induced investment, } \bar{\alpha} \text{ is the ex-post marginal (}= \text{ average in these models) propensity to consume, and } 1/\beta = \text{ ex-post productive efficiency of investment (i.e., the marginal output per unit of investment coefficient). The switching coefficients } \lambda_1 \text{ and } \lambda_2 \text{ have no interpretation aside from their definition in equations (9) and (10). The subscripts } a \text{ on } Y, C, \text{ and } I \text{ refer to the dates.} \]

**INTEGRATION OF SIMPLE GROWTH AND CYCLE MODELS**

**BEHAVIOR OF AGGREGATE SUPPLY**

Equation (1) states that the change in aggregate supply depends upon the investment put into place. Equation (1) plus equation (6) yields us the familiar Harrod-Domar growth model where the rate of growth depends upon the saving and the investment coefficients. For we have

\[ Y^*_t = Y^*_{t-1} + \frac{(1 - \alpha)}{\beta} Y^*_t \]

(11)

\[ \nu = \frac{Y^*_t}{Y^*_{t-1}} = 1 + \frac{1 - \alpha}{\beta} \]

and

\[ Y^*_t/Y^*_t = Y^*_t/Y^*_t-1 \cdot Y^*_t-1/Y^*_t = (1 + \frac{1 - \alpha}{\beta})(1 + \frac{1 - \alpha}{\beta}) = \nu^2, \]

so that

\[ Y^*_t = Y^*^\nu \]

(12)

where \( \nu \) is the rate of growth of aggregate supply when income actually equals supply income. The above is the familiar result: that the rate of growth of income is a constant, given that the ex-post saving coefficient and the ex-post marginal output-capital ratio are constants. This result, of course, holds within our model when \( \lambda_2 = 1 \). If \( \lambda_2 = 1 \), then

\[ Y^*_t = Y^*_{t-1} + \frac{\beta(Y^*_t - Y^*_t-1)}{\beta} \]

\[ \nu^*_{t-1} = Y^*_t/Y^*_t-1 = 1 + \frac{\beta}{\beta} Y^*_t-1(Y^*_t-2 - Y^*_t-3) \]

(13)

\[ \nu^*_t = 1 + \frac{\beta Y^*_t-1(Y^*_t-2 - Y^*_t-3)}{\beta Y^*_t-1(Y^*_t-2 - Y^*_t-3)} \]

The rate of growth of the maximum available supply depends upon:

(1) the ratio of the coefficient of induced investment to the capital-output ratio,

(2) the ratio of actual income to the maximum aggregate supply, and

(3) the rate of change of actual income in the previous two periods. As (2) and (3) are variables, the rate of growth of maximum supply income is also a variable. Of course \( \nu^*_t \) can be less than 1, which means that the maximum supply income can decrease.

Note that as long as

\[ \beta(Y^*_t - Y^*_t-1) < (1 - \alpha)Y^*_t-1, \quad \nu^*_t < \nu, \]

\[ \beta(Y^*_t - Y^*_t-1) < (1 - \alpha)Y^*_t-1, \quad \nu^*_t < \nu, \]
there is no way that lost growth in productive capacity can be made up unless $v_{t-1} < v$ implies that subsequent $1/\beta$'s will be larger than they otherwise would have been.

### Behavior of Aggregate Demand

Equation (2) plus equation (5), together with a definition of income as $C^* + I^*$, yields the Hansen-Samuelson accelerator-multiplier model. As is well known, the characteristics of the time series which this model will generate depend upon the values of $\alpha$ and $\beta$. We assume that normally a certain minimum buoyancy of entrepreneurs and investors exists so that the coefficient of induced investment is sufficiently greater than 1 that in the solution equation

$$Y_t = A_1 \mu^*_t + A_2 \mu^*_0,$$

we have that $\mu_1 > \mu_2 > 1$. The values of $\mu_1$ and $\mu_2$ are

$$\mu_1 = \frac{\alpha + \beta + \sqrt{(\alpha + \beta)^2 - 4\beta}}{2},$$

$$\mu_2 = \frac{\alpha + \beta - \sqrt{(\alpha + \beta)^2 - 4\beta}}{2}.$$

The values of $A_1$ and $A_2$ are determined by the initial conditions.

Assuming that the two initial conditions are $Y_0$, $Y_1 > 0$ and that $Y_1 = \tau Y_0$, $\tau > 1$. We then have

$$Y_0 = A_1 + A_2,$$

$$\tau Y_0 = A_1 \mu_1 + A_2 \mu_2,$$

so that

$$A_1 = \frac{\tau - \mu_2}{\mu_1 - \mu_2} Y_0$$

and

$$A_2 = \frac{\mu_1 - \tau}{\mu_1 - \mu_2} Y_0.$$

If $\mu_1 > \tau \geq \mu_2$, $A_1 > 0$, $A_2 > 0$; however, if $\mu_1 > \mu_2 > \tau$, then $A_1 < 0$. As $A_1$ is the coefficient of the larger root, $A_1 < 0$ implies that in time $A_1 \mu^*_t + A_2 \mu^*_0 < 0$, so that the “explosion” of income will be in the direction opposite from the initial displacement. Even if the roots of the solution equation are real and greater than 1, the time series generated by the solution equation can generate one turning point. The cause of this turning point lies in the initial conditions. If the initial conditions do not supply a sufficient push to income, a turning point will result. The minimum push that will yield a monotonic explosive series is given by $\mu_2$, the smaller root of the solution equation.

### Behavior of the Integrated Model

We can now sketch how the integrated model operates. The essential question is what happens when demand income exceeds supply income. As the pattern of behavior of the model is independent of where we begin, we can in all generality assume that the two initial incomes, $Y_0$ and $Y_1$, are both less than the maximum supply income and that

$$Y_1/Y_0 = \tau > \mu_2$$

so that a particular solution of the income-generating function $Y'_t = A_1 \mu'_t + A_2 \mu'_0$ with $A_1, A_2 > 0$ and $\mu_1 > \mu_2 > 1$ will be in motion to generate future demands. As long as $Y'_t < Y^*_t$, actual income will be determined by this particular income-generating relation. However, as $A_1 > 0$, the rate of change of actual income will in time approach $\mu_1$, the larger of the two roots. But values of $\alpha, \beta$ which lead to $\mu_2$ in the neighborhood of achieved rates of growth, generate a $\mu_1$ that is far larger than observed rates of growth. Hence in time

$$Y'_t = A_1 \mu'_t + A_2 \mu'_0 > Y^*_t$$

will result. This means that actual income will be $Y^*_t$ and all of demand will not be realized.

Before examining how the reconciliation process is carried out when $Y'_t > Y^*_t$, and noting the implications of some reconciliations for the generation of self-sustained growth, it is best if we interpret the switch that occurs when $Y'_t > Y^*_t$. $Y^*_t$ is the result of a self-sustaining demand-generating process based upon the structural characteristics of the economy and some initial conditions. Such an income-generating process once set in motion will not generate actual incomes for all times in the future. The path of actual income will be affected by exogenous events and constraints as well as the structural elements and history embodied in the ruling demand-generating relation. These exogenous events and constraints are interpreted as determining new initial conditions for a particular demand-determining relation that will determine aggregate demand as long as no external event or constraint prevents this demand income from being realized. Hence whenever $Y^*_t \neq Y^*_t$, $Y^*_t$ and $Y^*_{t-1}$ are new initial conditions for a demand-determining relation. Within our framework this new demand-determining relation will determine actual incomes until the incomes so determined are
inconsistent with the supply constraints, for we are ignoring external shocks in this paper.

When \( Y^* \) is inconsistent with \( Y^* \), then actual values of \( C \) and/or \( I \) will differ from their demand or ex ante values. The problem now becomes to what extent the outback \( Y^* \) to \( Y^* \) takes the form of a reduction of consumption or of a reduction of investment. Equations (3) and (6), which tell us how income, when it is equal to aggregate supply, is divided between consumption and investment, do not describe how the reconciliation process affects consumption and investment.

When \( Y^*_n > Y^*_n \), then \( Y^*_n = Y^*_n \). This means that new initial conditions, \( Y^*_n \) and \( Y^*_n-1 \), determine \( A \) and \( A_2 \) in a specific demand-generating relation. If \( Y^*_n / Y^*_n-1 < \mu_2 \), then \( A_1 < 0 \) and a single turning point will be generated, whereas if \( Y^*_n / Y^*_n-1 \geq \mu_2 \), then \( A_1 \geq 0 \) and \( Y^*_n+1 / Y^*_n > Y^*_n+1 / Y^*_n \) will be generated so that \( Y^*_n+1 \) becomes the \( n + 1 \) period's actual income. In this case we know that \( Y^*_n+1 / Y^*_n = \mu_2 \) and steady growth will take place if \( \mu_2 > \mu_2 \) and a single turning point will be generated if \( \mu_2 > \mu_2 \). Steady growth is the result of setting off new demand-generating processes each period which in the next period generate demand that is equal or greater than supply, whereas the turning point with the accompanying fall of income below supply occurs if the demand-generating process leads to a smaller increase in demand than in supply.

Hence, whether steady growth or a cyclical downturn occurs when the available supply becomes a determinant of actual income depends upon the rate of growth of aggregate supply; this model is a ceiling model of cycles and growth. However, as aggregate supply is growing, it is the rate of growth of aggregate supply rather than the existence of some fixed ceiling to productive capacity that is the critical factor. As there is no doubt that the rate of growth of supply that can be sustained when the economy is at or close to full employment is lower than the rate of growth of income that does take place when the economy is recovering from a depression, a decrease in the rate of growth of actual income occurs when income approaches aggregate supply income. This decrease in the rate of growth of actual income is the critical constraint in this model.\(^{10}\)

THE POSSIBILITY OF SELF-SUSTAINED GROWTH

The rate of growth of aggregate supply is given by

\[
\bar{v} = 1 + \frac{1 - \bar{\alpha}}{\bar{\beta}}
\]

and the lower root of the solution equation is given by

\[
\mu_2 = \frac{\alpha + \beta - \sqrt{\alpha + \beta)^2 - 4}}{2}
\]

From these equations we get:

\[
\beta = \frac{\mu_2(\mu_2 - \alpha)}{\mu_2 - 1},
\]

\[
\bar{\beta} = \frac{1 - \bar{\alpha}}{\bar{v} - 1},
\]

which are straight lines in \( \alpha \), \( \beta \) and \( \bar{\alpha} \), \( \bar{\beta} \). (Given that \( \mu_2 > 1 \), the domain of \( \alpha \) and \( \beta \) is restricted.) If we assume \( \alpha = \bar{\alpha} \), \( \beta = \bar{\beta} \), \( 0 < \alpha < 1 \), and \( \beta > 0 \), then for any \( \alpha \), \( \beta \) pair \( \mu_2 > \bar{v} \); that is, the rate of growth of productive capacity will be below the minimum rate of growth of income that must take place if self-sustained growth is to occur.

This is illustrated in Figure 1. For example, at point \( A \), \( \alpha \approx .92 \) and \( \beta \approx 2.824 \) yield \( \mu_2 = 1.05 \) and \( \bar{v} = 1.03 \). Hence, if \( Y^*_n = Y^*_n \), \( Y^*_n-1 = Y^*_n-1 \), the demand-generating relation set into motion with \( Y^*_n \), \( Y^*_n-1 \) as initial conditions will have \( A_1 < 0 \) which implies that growth will not be self-sustained.

For self-sustained growth to occur, it is necessary for \( \alpha \) and \( \beta \) to be "greater" than \( \bar{\alpha} \) and \( \bar{\beta} \). For example, if \( \alpha \) and \( \beta \) are such that they lie along the line \( \mu_2 = 1.04 \), then \( \bar{\alpha} \) and \( \bar{\beta} \) must be such that they lie on or below the line \( v = 1.04 \) if self-sustained growth is to occur. In Figure 1, self-sustained growth would be attainable if the set of lines \( \bar{v} = 1.03 \), etc. could be shifted upward so that for every \( \bar{v} = \mu_2 \) the line for \( \bar{v} \) would lie above the line for \( \mu_2 \).

For this to occur, some combination of factors that tend to yield \( \bar{\alpha} < \alpha \) and \( \bar{\beta} < \beta \) must be operative.

EX-Ante AND Ex-Post Consumptions

The assumption that \( \alpha = \bar{\alpha} \), given that \( C^*_n = \alpha Y^*_n \) and \( C^*_n = \alpha Y^*_n-1 \), and that \( Y^*_n = \bar{v}Y^*_n-1 \), implies that \( C^*_n = C^*_n > C^*_n \). The rise in income between the \( n \)th and the \( n - 1 \)th period results in ex-post consumption being larger than ex-ante consumption. As supply income effectively determines income because \( Y^*_n > Y^*_n \) and \( C^*_n > C^*_n \), the entire burden of adjustment is upon investment.

Rather than assume that ex-post consumption exceeds ex-ante consumption, we can assume that ex-post consumption equals ex-ante consumption. If this occurs,

\[
Y^*_n = Y^*_n + \frac{Y^*_n-1 - \alpha Y^*_n-2}{\bar{\beta}}
\]
process still results in ex-post investment being lower than ex-ante investment. The rate of growth of supply that results is too low to maintain self-sustained growth.

**The Impact of Inflation**

In order to have \( p \geq \mu \), it is necessary that when \( Y' > Y' \) (recall that we are assuming that \( \beta = \bar{\beta} \), ex-post consumption be less than ex-ante

As is illustrated in Figure 2, the lines for equation (19) also lie below the lines for equation (17), so that for any given \( \alpha, \beta = \bar{\beta} \) pair \( \mu > p \). Even if ex-post consumption is restricted to ex-ante consumption, the adjustment
consumption. One way in which consumers can be forced to lower their consumption below the ex-ante level is for consumer prices to rise; this is, of course, particularly true if a large portion of consumers use all their income for consumption and have no means by which they can spend more than their income. Writing $p^*$ for $p_n/p_s-1$, we have

$$
\bar{\beta} = 1 + \frac{1 - \frac{\alpha}{\beta p^*}}{\beta}.
$$

Assuming $p^* > 1$, there exist values of $\alpha$ and $\beta$ which generate rates of growth of aggregate supply that are larger than the lower root of the demand-generating relation. This means that if inflation that decreases consumption below ex-ante consumption occurs, self-sustained growth can take place. (Even though consumption is lowered below ex-ante consumption by inflation, with a constant rate of increase in consumers' prices, real consumption will still be growing:

$$
\left( \frac{\alpha Y_{t-2}^*}{p^*} \right) / \frac{\alpha Y_{t-1}^*}{p^*} = \bar{\beta} > 1.
$$

In Figure 3, point A shows that if $\alpha \approx .875$, $\beta \approx 3.675$ then $\nu_2 = 1.05$ and $\bar{\beta} = 1.05$ with $p^* = 1.02$. That is, if ex-post consumption is approximately 98 per cent of ex-ante consumption so that $Y_{t-1}^* = .98 Y_{t-2}^*$ can be invested, real supply will grow at 5 per cent. Points B and C have similar interpretations.

It is doubtful that in the United States, as now organized, inflation is an efficient or an effective way of depressing consumption in order that investment be sufficient to generate a growth rate of income large enough to satisfy the conditions for self-sustained growth.

**THE EFFECT OF TECHNOLOGICAL CHANGE**

We can distinguish two types of technical change. Disembodied technical change, where productive capacity increases independently of investment, and embodied technical change, where investment is the carrier of technical change.
This results, when income is at the ceiling, in

\[ \frac{Y_n^*}{Y_{n-1}^*} = r + \frac{1 - \bar{\alpha}}{\bar{\beta}} \]

and, with less than capacity income, in

\[ \frac{Y_n^*}{Y_{n-1}^*} = r + \frac{\bar{\beta}}{\bar{\beta}} \left( \frac{Y_{n-1}^* - Y_{n-2}^*}{Y_{n-1}^*} \right) \]

The rate of growth of ceiling income, when income is at the ceiling, is

\[ \bar{r}_1 = r + \frac{1 - \bar{\alpha}}{\bar{\beta}} \]

which yields

\[ \bar{\beta} = \frac{1 - \bar{\alpha}}{\bar{r} - r} \]

As is shown in Figure 4, with a 2 per cent per year growth in productive capacity due to disembodied technical change, it is possible with \( \alpha = \bar{\alpha} \), \( \beta = \bar{\beta} \) for the rate of growth of ceiling income \( \bar{r}_1 \) to be greater than the critical value \( \mu_2 \) derived from the demand-generating relation.

**Embodied Technical Change**

We will assume that embodied technical change results in \( \bar{\beta} < \beta \); i.e., the productive efficiency of investment is higher than “expected” because of technological progress.

In Figure 2, note that if \( \alpha = .9 \) and ex-post consumption equals ex-ante consumption, \( \beta = 3.15 \) and \( \bar{\beta} = 2.85 \) will yield \( \mu_2 = \bar{r}_1 = 1.05 \), i.e., it takes but a small decline in \( \bar{\beta} \) below \( \beta \) to satisfy the conditions for self-sustained growth.

**CONCLUSIONS**

Given that technological change, whether embodied or disembodied, takes place, and that the effect of technological change is to increase the rate of growth of ceiling income beyond that which would result just from accumulations, it has been shown that the ceiling income can grow fast enough so that self-sustained growth is possible. Therefore, in a technically dynamic world, we have to look beyond productive capacity constraints to explain the observed pattern of cyclical growth.

The coefficient of induced investment \( \beta \) is not a technical production function characteristic as is \( \bar{\beta} \), but rather reflects investors' and entrepreneurs' attitudes toward risk. Thus, \( \beta \) would be a variable that depends, at least in part, on the menu of financial instruments available to asset owners and the liability structure of investing units.

In the demand-generating relation, the smaller root of the solution equation was the critical value for sustaining growth. However, the derivative of this coefficient, \( \mu_2 \), with respect to \( \bar{\beta} \), the coefficient of induced investment, is negative.\(^{11}\) Anything that tends to lower \( \beta \) will raise \( \mu_2 \) and thus increase the minimum rate of growth of ceiling income that will sustain growth.
Cumulative unbalanced changes in the menu of available financial instruments take place during a period of self-sustained growth. The unbalanced nature of financial developments should affect the relative interest rates at which the public and financial institutions are willing to hold their available stock of primary liabilities; this, together with the fact of increasing risk as the independence of the expected performance of financial assets decreases, will feed back upon the willingness to invest. Thus the rate of growth of ceiling income (including the effect of technical change) that is sufficient to sustain growth with one set of portfolios can become inadequate with another. A cyclical growth pattern can emerge due to cumulative changes that affect demand rather than from any necessary insufficiency of the rate of growth capacity.

It seems evident that the integration of ceiling models of growth with the financial flows that accompany growth is a fruitful research path in any attempt to develop econometric models that have the interesting intermediate time horizon.

Notes


4. Ibid.


10. The formal apparatus is set out in Minsky, *ibid.*


12. Tobin, op. cit.