3-1984

Price Value Deviations

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$-A']e''^{-1}$

$-A']')e''^{-1}$

y

-1

$p = wl + rpA + rpK$
$\varphi = w\lambda + rpK[I-A]^{-1}$

$\lambda = \lambda A + l$ $\quad \lambda' = A'\lambda' + l'$

$[I-A']\lambda' = l' \rightarrow \lambda' = [I-A']^{-1}l' = ([I-A]')^{-1}l'$

Std ~~Price~~-Value Deviations Program

3/12/84

$l = [\cdots]$ $\quad l' = [\vdots]$

1. Input l, A, K [(R) if $H = A$]
   (i) Calculate $I-A$, $[I-A]^{-1}$, $\lambda = l[I-A]^{-1}$, $H = K[I-A]^{-1}$,
   (ii) Calculate R = dominant eigenvalue of H

2. Specify
   A. p^* at given w^*, $0 \le w^* \le 1$ → Input w^*
   B. p^* for all w^* $\quad w^* = 0, .1, .2, \ldots, 1.0$
   C. wage profit curves ← one industry / all industries

   If A, do (1) - once. If B, loop for all w^*.

   ~~(i) Calculate R~~

   (i) $r = R(1-w^*)$
   (ii) Calculate $w^*\lambda$, $[I-rH]$, $[I-rH]^{-1}$
   (ii) and $p^* = w^*\lambda \cdot [I-rH]^{-1}$

   λK

   (iii) Calculate $\dfrac{p^*_j - \lambda_j}{\lambda_j}$ $\quad$ (IV) Calculate $\lambda K \langle l_j \rangle^{-1} = \dfrac{C}{\Delta}$ and $\lambda H \langle l_j \rangle^{-1} = \dfrac{C}{\Delta}$
   & coefficients of variation (Subroutine)

3. If C (wage profit curves)
   (i) Calculate $\dfrac{w}{p_j} = \dfrac{1w^*}{p^*_j}$, ~~and~~ for each w^*

   (ii) Plot w/p_j versus r for each j.
   Superimpose straight line between intercepts

4 - Printing Subroutine

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad A' = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1-a_{11} & -a_{12} & -a_{13} \\ -a_{21} & 1-a_{22} & \\ -a_{31} & & 1-a_{33} \end{bmatrix} \qquad (I - A') = \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} \\ -a_{12} & 1-a_{22} & \\ -a_{13} & & 1-a_{33} \end{bmatrix}$$

$$(I - A)' = \begin{bmatrix} 1-a_{11} & & \\ -a_{12} & 1-a_{22} & \\ -a_{13} & & 1-a_{33} \end{bmatrix} = [I - A']$$

Numerical Example I (Matrix B is not economically correct; none the less, it too has hypothesized properties for iteration) N − 1

Iteration holding PER prices constant

$$A^T = \begin{bmatrix} .5 & .333 & 0 \\ 0 & .5 & .5 \\ .333 & 0 & .333 \end{bmatrix} \qquad l^T = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} .5 & 0 & .333 \\ .333 & .5 & 0 \\ 0 & .5 & .333 \end{bmatrix}$$

$|A| = .5[(.5)(.333) - (0)(.5)] \ldots$

$l = [1, 1, 1]$

$$w = \begin{bmatrix} .05 \\ 0 \\ .10 \end{bmatrix} \qquad B^T = (A + wl)^T = \begin{bmatrix} .55 & .38333 & .05 \\ 0 & .50 & .50 \\ .4333 & .10 & .4333 \end{bmatrix}$$

This is a mistake: as written, it is $A^T + wl$, not $A^T + (wl)^T$

$$|I - A^T| = .1111 \checkmark \qquad (I - A)^{-1} = \begin{pmatrix} 3 & 3/2 & 3/2 \\ 2 & 3 & 1 \\ 3/2 & 9/4 & 9/4 \end{pmatrix} \checkmark$$

$$(I - A) = \begin{pmatrix} \frac{1}{2} & 0 & -1/3 \\ -1/3 & 1/2 & 0 \\ 0 & -1/2 & 2/3 \end{pmatrix}$$

$$p^{(0)T} = \begin{bmatrix} 6.5 \\ 6.75 \\ 4.75 \end{bmatrix} \checkmark \qquad \lambda_1 = .98567941$$

(store)

$$x_1 = \begin{bmatrix} .582053 \\ .58518718 \\ .564597 \end{bmatrix} \begin{matrix} 04 \\ 05 \\ 06 \end{matrix}$$

$$y^{(0)} = p^{(0)T} = \begin{bmatrix} 6.5 \\ 6.75 \\ 4.75 \end{bmatrix}; \Sigma = 18 \qquad p^{(0)T} = \begin{bmatrix} 6.5 \\ 6.75 \\ 4.75 \end{bmatrix}$$

$$\lambda_1^{1} = \frac{\Sigma y^{(1)}}{\Sigma y^{(0)}} = .9833 \qquad y^{(1)} = B^T y^{(0)} = \begin{bmatrix} 6.40 \\ 5.75 \\ 5.55 \end{bmatrix}; \Sigma = 17.7 \qquad p^{(1)T} = \frac{1}{\lambda_0}\left[y^{(1)}\right] = \begin{bmatrix} 6.49635 \\ 5.836568 \\ 5.633357 \end{bmatrix}$$

$$\lambda_1^{(2)} = .983239 \text{ (R07)} \qquad y^{(2)} = \begin{bmatrix} 6.00 \\ 5.65 \\ 5.75333 \end{bmatrix}; \Sigma = 17.403 \qquad p^{(2)T} = \left(\frac{1}{\lambda_1}\right)^2 \left[y^{(2)}\right] = \begin{bmatrix} 6.18202 \\ 5.821406 \\ 5.927874 \end{bmatrix}$$

$$\lambda_1^{(3)} = .983426 \text{ (R08)} \qquad y^{(3)} = \begin{bmatrix} 5.7544 \\ 5.70166 \\ 5.65883 \end{bmatrix}; \Sigma = 17.11489 \qquad p^{(3)T} = \left(\frac{1}{\lambda_1}\right)^3 \left[y^{(3)}\right] = \begin{bmatrix} 6.018236 \\ 5.963851 \\ 5.918284 \end{bmatrix}$$

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$\lambda_1^{(1)} = .98333$
R69

$$y^{(4)} = \begin{bmatrix} 5.63351 \\ 5.68025 \\ 5.51591 \end{bmatrix}; \Sigma = 16.829$$

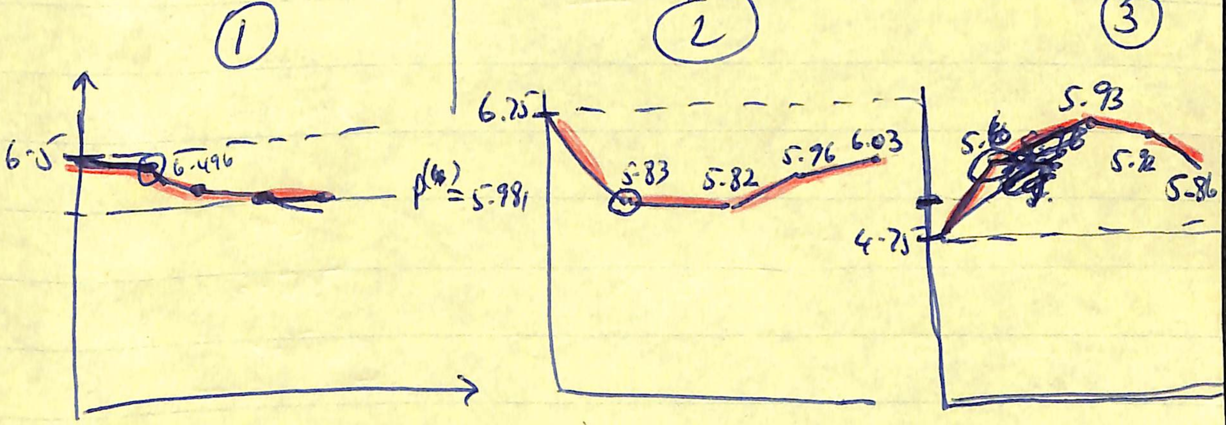
$$p^{(4)} = \left(\frac{1}{\lambda_1}\right)^4 \begin{bmatrix} y^{(4)} \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 5.98051 \\ 6.030126 \\ 5.855663 \end{bmatrix} \end{array} \begin{array}{c} 01 \\ 02 \\ 03 \end{array}$$

07 $\frac{01}{02}$

08 $\frac{04}{05}$

09 $\frac{02}{03}$

10 $\frac{05}{06}$



$p^{(0)}$	$p^{(1)}$	$p^{(2)}$	$p^{(3)}$	$p^{(4)}$
6.5	6.496 (−)	6.1820 (−)	6.0182 (−)	5.9805 (−)
6.75	5.8366 (−)	5.8214 (−)	5.9631 (+)	6.0301 (+)
4.75	5.6335 (+)	5.9278 (+)	5.9183 (−)	5.8557 (−)

Numerical Example II (Steedman, *Marx After Sraffa*, pp38-47)

$$A = \begin{bmatrix} .5 & .333 & 1.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad l = [1, .333, 1] \qquad w = \begin{bmatrix} 0 \\ 0 \\ .0625 \end{bmatrix} ✓$$

$$(I-A) = \begin{bmatrix} .5 & -.3333 & -1.5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{w}\hat{l} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .0625 & .020833 & .0625 \end{bmatrix} \qquad B = A + wl = \begin{bmatrix} .5 & .333 & 1.5 \\ 0 & 0 & 0 \\ .0625 & .020833 & .0625 \end{bmatrix}$$

$$(I-A)^{-1} = \begin{bmatrix} 2 & 2/3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \lambda = l[I-A]^{-1} = (1, \tfrac{1}{3}, 1)\begin{bmatrix} 2 & 2/3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [2, 1, 4]$$

$$A^T = \begin{bmatrix} .5 & 0 & 0 \\ .333 & 0 & 0 \\ 1.5 & 0 & 0 \end{bmatrix} \qquad l^T = \begin{bmatrix} 1 \\ .333 \\ 1 \end{bmatrix} \qquad B^T = \begin{bmatrix} .5 & 0 & .0625 \\ .333 & 0 & .020833 \\ 1.5 & 0 & .0625 \end{bmatrix}$$

$$H = A(I-A)^{-1} = \begin{bmatrix} \cancel{1/2} & \tfrac{1}{3} & 3/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2/3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2/3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|I - A^T| = .5 \ ✓$$

$$p^{(0)T} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} ✓ \text{ (p.40)}$$

$$\cancel{\lambda_1 = \tfrac{1}{1+r} = .6576526 \ (r = .52056) \ (p.46)} \qquad \cancel{X_1 = \begin{bmatrix} .3322353 \\ .4338940 \\ .8374463 \end{bmatrix}}$$

$$\lambda_1 = \frac{1}{1+r} = .6576526$$

$(r = 0.52056)$

Steedman $r = .5208$

$$\hat{X}_1 = \begin{bmatrix} .3604333 \\ .2[illegible]14878 \\ .9084937 \end{bmatrix}$$

Check: Steedman normalizes by $P_{gold} = 1 \Rightarrow x_2 = 1$

$\therefore \frac{x_1}{x_2} = 1.7043$ ✓ $\frac{x_3}{x_2} = 4.2961$ ✓ (p-46)

Note that this gives very different initial prices than Marx's own procedure

$$\frac{1}{\lambda_1} = 1.520559639$$

$$y^{(0)} = p(0)^T = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \quad \Sigma = 7 \quad R_{01} \qquad p(0)^T = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\frac{\Sigma y^{(1)}}{\Sigma y^{(0)}} = .75 = \lambda_1^{(1)} \quad y^{(1)} = \begin{bmatrix} 1.25 \\ 0.75 \\ 3.25 \end{bmatrix} \quad \Sigma = 5.25 \quad R_{02} \qquad p(1)^T = \left(\frac{1}{\lambda_1}\right) y^{(1)} = \begin{bmatrix} 1.9007 \\ 1.14042 \\ 4.941819 \end{bmatrix}$$

$$\frac{\Sigma y^{(2)}}{\Sigma y^{(1)}} = .64583 = \lambda_1^{(2)} \quad y^{(2)} = \begin{bmatrix} .828125 \\ .4843739 \\ 2.078125 \end{bmatrix} = 3.39068 \quad R_{03} \qquad p(2)^T = \left(\frac{1}{\lambda_1}\right)^2 y^{(2)} = \begin{bmatrix} 1.914709 \\ 1.1992168 \\ 4.80483617 \end{bmatrix}$$

$$\lambda_1^{(3)} = .65927 \quad y^{(3)} = \begin{bmatrix} .543945 \\ .3193352 \\ 1.3720703 \end{bmatrix} = 2.23530 \quad R_{04} \qquad p(3)^T = \left(\frac{1}{\lambda_1}\right)^3 y^{(3)} = \begin{bmatrix} 1.9123411 \\ 1.1226831 \\ 4.8237716 \end{bmatrix}$$

$$\lambda_1^{(3)} = .65730 \quad y^{(4)} = \begin{bmatrix} .3577271 \\ .20989943 \\ .90167236 \end{bmatrix} = 1.46930 \quad (R_{05}) \qquad p(4)^T = \left(\frac{1}{\lambda_1}\right)^4 y^{(4)} = \begin{bmatrix} 1.91234249 \\ 1.12208329 \\ 4.82017261 \end{bmatrix} ✓$$

$$\lambda_1^{(4)} = .65759 \quad y^{(5)} = \begin{bmatrix} .23521805 \\ .13802688 \\ .592945099 \end{bmatrix} = .96619 \quad R_{06} \qquad p(5)^T =$$

$$\lambda_1^* = .657653$$

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