2-1978

LTV II

Anwar Shaikh PhD

LTV II

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(1) The article I mentioned is by Wilfred Parys, in the American Eco. Review Vol 72, No 5, Dec 1982, pp 1208–1212: "The Deviation of Prices from ~~Labor~~ Values"

(2) The attached notes (11/18/83, "Directions of Price-Value Deviations") lead to the following relation between "Sraffian" (completely transformed prices of production) and "Marxian" (vertically integrated first-order transformations), in terms of the standard commodity

From Equation F ~~Let~~ Sraffian Prices of Production $\equiv$ $\boxed{p' = \lambda' + r\left[p'H - \frac{1}{R}\lambda'\right]}$

From Equation L, L'#, Marxian P.O.P. $\equiv$ $\boxed{p^+ = \lambda' + r\left(\lambda'H - \frac{1}{R}\lambda'\right)}$

(1) We can immediately relate the two, since

$$d'(r) \equiv p' - \lambda' = r\left(p'H - \frac{1}{R}\lambda'\right)$$
$$d^+(r) \equiv p^+ - \lambda' = r\left(\lambda'H - \frac{1}{R}\lambda'\right)$$

so that

(a) $$\boxed{d'(r) = d^+(r) + r\left(p'H - \lambda'H\right)} = d^+ + r\,d'H$$

The first term represents the "Marxian" price of prod-value deviations, based on vertically integrated value compositions, & the second represents the "feedback" effect.

Adel:

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(ii) If we utilize equations I and N, we can write ⓐ as

$$d'_\gamma(r) = p'_\gamma(r) - \lambda'_\gamma = r[p' H^{(\gamma)} - \tfrac{1}{R}\lambda'] + r[p' H^{(\sigma)} - \lambda' H^{(\sigma)}]$$

$$p'_\gamma - \lambda_\gamma = r\left[\frac{C^T_\gamma}{X_\gamma} - \frac{1}{R}\lambda'_\gamma\right] + r\left[\frac{K^T_\gamma}{X_\gamma} - \frac{C^T_\gamma}{X_\gamma}\right]$$

$$= r\lambda_\gamma\left\{\left[\frac{C^T_\gamma}{\lambda_\gamma X_\gamma} - \frac{1}{R}\right] + \left(\frac{K^T_\gamma(r)}{\Lambda_\gamma} - \frac{C^T_\gamma}{\Lambda_\gamma}\right)\right\}$$

since $\frac{1}{R} = q_s = \left(\frac{C^T}{\Lambda}\right)_{std}$, and letting $\frac{K^T_\gamma(r)}{C^T_\gamma} \equiv \alpha_\gamma(r)$

ⓑ $$\boxed{\therefore \left(\frac{p'_\gamma - \lambda'_\gamma}{\lambda'_\gamma}\right) = \left(\frac{r}{R}\right)\left[\frac{q_\gamma - q_s}{q_s}\right] + \left(\frac{r}{R}\right)\left(\frac{q_\gamma}{q_s}\right)\left[\alpha_\gamma\left(\frac{r}{R}\right) - 1\right]}$$

$$\frac{p'_\gamma - \lambda'_\gamma}{\lambda'_0} = \frac{r}{R}[\alpha(r) - 1]$$

$$\frac{p'_\gamma - \lambda'_0}{\lambda'_0} = \frac{r}{R}\left[\frac{K^T_\gamma}{C^T_\gamma} - 1\right]$$

where

(% deviation of Sraffian POP from values) = $\left(\frac{r}{R}\right)$ {% deviation of Marxian POP from values} + $\frac{q_\gamma}{q_s}$ (% deviation of price of the vertically-integrated capital stock [$K^T_\gamma(r)$] from its value equivalent [C^T_γ])

I have always considered the first term [the "Marxian" component $\frac{r}{R}\left(\frac{q_\gamma - q_s}{q_s}\right)$] as the principal component, because the second term being the % deviation of the price of the γth vertically integrated capital stock from its own value, I figure it as being several "layers" back. But this of course assumes that vertical integration is effective

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(iii) Of all the terms in (b), the only nonlinear one is $\alpha(v) - 1 \equiv \frac{K_g^T(v) - C_g^T}{C_g^T}$.

But since this is, itself a price-value deviation, it can itself be decomposed into a linear component and a "residual" which is non-linear, etc., (See equations L, L' ~~in~~ attached notes 11/15/83, p.7)...

[Note that at $v=0$, $\alpha(0) = 1$, since $K_g^T(0) = C_g^T$]

(iv) Note that for this non-linear term to override the Marxian p.o.p. effect and hence ~~switch~~ reverse the direction of deviation, it is necessary that the price-value ratio $= 0$ at some $0 < x < 1$

Proof of Steady Convergence of iterates for $\lambda y = yB$

12/30/79

1. Raw iterates : $y^{(n)} = y^{(n-1)}B$ s.t. $\dfrac{y^{(n)}_j}{y^{(n-1)}_j} \to \lambda_1$ as $n \to \infty$

2. Scaled iterates: $p^{(n)} = \left(\frac{1}{\lambda_1}\right)^n y^{(n)} \longrightarrow \dfrac{p^{(n)}_j}{p^{(n-1)}_j} = \left[\dfrac{y^{(n)}_j}{y^{(n-1)}_j}\right]\dfrac{1}{\lambda_1}$

Since $0<\lambda_1<1$, it follows that behavior of scaled iterate ratios is same as that of raw iterates : If the latter are monotonically decreasing, so too are former.

3. Raw iterates are given by

$$(I) \quad y^{(n)} = y^{(n-1)}B = y^0 B^{n}$$

~~Since~~ $y^0 >> \phi$, and B is semi-positive, it follows that

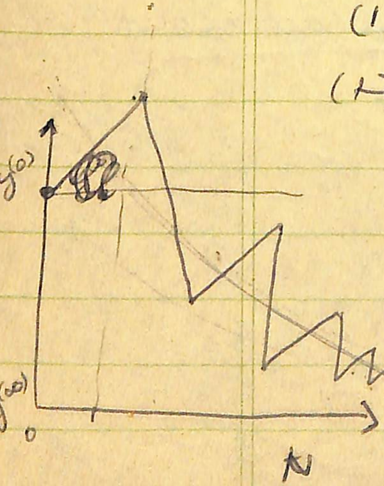
$$(II) \quad y^{(n)} >> \phi$$

Lastly, since ~~[illegible]~~ $\lambda_1 < 1$, we know that

$$(III) \quad y^{(\infty)} = \lim_{n\to\infty} y^{(n)} = \phi$$

4. It follows from the above that each iterate $y^{(n)}_j$ is positive and ~~[illegible]~~ each converges to zero. (see back)

(1) Argument is correct assuming step by step converge., which is proved No. Sequence pictured to be possible?



~~(2) Since initial ~~iterate~~ element is positive ($y^{(0)}_j > 0$), converge implies that the first iterate must be smaller in absolute value, which taken with ~~and~~ positivity of iterates implies that it must positive and smaller than $y^{(0)}_j$: ~~Convergence also implies that $|y^{(1)}_j| > 0$~~~~

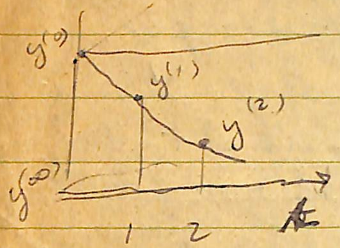
~~which taken with $y^{(1)}_j > 0$ implies~~

~~$0 < y^{(1)}_j < y^{(0)}_j$~~

(ii) Thus the first iterate converges and <u>does not overshoot</u> $y^{(\infty)}_j = 0$. <u>This rules out oscillation</u> (which depends on alternate overshooting and undershooting).

~~Similar or~~ This result depends on convergence holding at <u>each</u> <u>step</u>, <u>for each element</u>

$$\|y^{(n)} - y^{(\infty)}\| < \|y^{(n-1)} - y^{(\infty)}\|$$



Since $y^{(\infty)} = 0 \iff$

$$\|y^{(n)}\| < \|y^{n-1}\|$$

But $y^{(n)} = y^{(n-1)} B$

$$\|y^{(n)}\| \leq \|y^{(n-1)}\|\|B\| < \|y^{(n-1)}\| \qquad \text{since } \|B\| < 1$$

$$\therefore \|y^n\| < \|y^{n-1}\|$$

5. In sum:

<u>If converge is step by step</u>, and <u>if all iterates are positive</u>, <u>it follows that</u> NO <u>oscillations can occur</u>

Price – Deviations

2/1/78

I For any prices at all,

1. $$P_\gamma = W_\gamma^T + \pi_\gamma^T$$

2. We may define $\Lambda_\gamma \equiv \frac{W_\gamma^T}{w_0}$ = ~~skill~~ wage-weighted total labor-time

(i) $$P_\gamma = w_0 \Lambda_\gamma + \pi_\gamma^T = w \Lambda_\gamma \left[1 + \frac{\pi_\gamma^T}{W_\gamma^T} \right]$$

(ii) This can be expressed as

$$\frac{P_i}{P_\gamma} = \frac{\Lambda_i}{\Lambda_\gamma} \frac{\left[1 + \frac{\pi_i^T}{W_i^T} \right]}{\left[1 + \frac{\pi_\gamma^T}{W_\gamma^T} \right]}$$

II Now consider specific sets of prices. Assuming that money wages are given to capitalists, we could define prices

1. Prices Proportional to Value

$$P_\gamma^0 = w_0 \Lambda_\gamma \left[1 + \frac{\pi^T}{W^T} \right] \longleftrightarrow \frac{\pi_\gamma^T}{W_\gamma^T} = \frac{\pi^T}{W^T}$$

[Note that it is sufficient ~~that~~ prices to be proportional to values that vertically integrated profit-wage ratios be all equal. Howevwer, this will generally imply equal direct profit-wage shares]

2. Equilibrium Prices of Production

$$P_\gamma^* = w_0 \Lambda_\gamma + r K_\gamma^{*T} \longleftrightarrow \frac{\pi_\gamma^T}{W_\gamma^T} = \frac{r}{w_0} \left(\frac{K^{*T}}{\Lambda^T} \right)$$

[Here, K^{*T} = vertically integrated capital stock, measured in terms of prices of production. In other words, both inputs and outputs are in terms of the same prices (of production)]

3. Current Prices of Production [First iteration]

Given existing costs & wages, we could define prices which would yield the existing average rate of profit $\bar{r}$ in this industry. Here feedback effects are ignored.

(I) $P'_{\gamma} \equiv W_{\gamma} + M_{\gamma} + \bar{r} K_{\gamma} = w_0 L_{\gamma} + M_{\gamma} + \bar{r} K_{\gamma}$

where $L_{\gamma} = \frac{W_{\gamma}}{w_0}$ = wage-weighted direct labor requirement

M_{γ} = intermediate input costs, in market prices.

K_{γ} = capital advanced, in market prices.

(II) Given that market prices can be written as

$$P_{\gamma} = W_{\gamma} + W_{\gamma} + r_{\gamma} K_{\gamma} \quad ,$$

it follows that

$$\frac{P'_{\gamma} - P_{\gamma}}{P_{\gamma}} = (r_{\gamma} - \bar{r}) \frac{K_{\gamma}}{P_{\gamma}} = \left(\frac{r_{\gamma} - \bar{r}}{r_{\gamma}}\right) \frac{\Pi_{\gamma}}{P_{\gamma}} \cdot$$

(III) $$\boxed{\left(\frac{P'_{\gamma} - P_{\gamma}}{P_{\gamma}}\right) = \left(\frac{r_{\gamma} - \bar{r}}{r_{\gamma}}\right) m_{\gamma}}$$

where $\bar{r}$ = avg. rate of profit

r_{γ} = industry rate

m_{γ} = profit margin on sales

(IV) This tells us that profit-rate differentials can be very large while still yielding only small price deviations ("deviations of mkt prices from current prices of prod.")

On average, for U.S., $m_{\gamma} \cong 10\%$

Hence even an average profit-rate differential of 50% will imply an average price deviation of $(.5)(.1) = .05 = 5\%$

(V) Oligopoly theories attempt to explain the profit-rate differentials. I attempt to assess their impact on price deviations

Price Deviations 2/1/78

* Thus it is perfectly possible to observe large average deviations in profit rates ~~will not~~ which do not in any way contradict approximate equality of ~~market~~ prices and current prices of production.

III Price of Production — Direct Price Deviations.

1. $P_\gamma^* \equiv W_\gamma^T + r K_\gamma^{*T}$; $P_\gamma^\circ = W_\gamma^T + \left(\frac{\Pi^T}{W^T}\right) W_\gamma^T$

(i) We can rewrite $P_\gamma^\circ = W_\gamma^T + \rho_\gamma^T K_\gamma^{\circ T}$

where $K_\gamma^{\circ T}$ = Total cap. requirement in terms of direct prices

ρ_γ^T = vertically integrated profit rate when direct prices prevail = vertically integrated value rate of profit

(ii) Similarly, we can write $K_\gamma^{*T} = K_\gamma^{\circ T} + \Delta K_\gamma^{*T}$ so that

$$P_\gamma^* = W_\gamma T + r K_\gamma^{\circ T} + r \Delta K_\gamma^T$$

Thus

$$P_\gamma^* - P_\gamma^\circ = (r - \rho_\gamma^T) K_\gamma^{\circ T} + r \Delta K_\gamma^{*T} = \frac{(r-\rho_\gamma^T)}{\rho_\gamma^T} \rho_\gamma^T K_\gamma^{\circ T} + r \Delta K_\gamma^{*T}$$

$$\frac{P_\gamma^* - P_\gamma^\circ}{P_\gamma^\circ} = \left(\frac{r-\rho_\gamma^T}{\rho_\gamma^T}\right)\left(\frac{\Pi_\gamma^{\circ T}}{P_\gamma^\circ}\right) + r \frac{\Delta K_\gamma^{*T}}{P_\gamma^\circ}$$

But note: since $\frac{\Pi_\gamma^{\circ T}}{W_\gamma^T} = \frac{s}{v} \rightarrow \Pi_\gamma^{\circ T} = \frac{s}{v} W_\gamma^T$. But $P_\gamma^\circ = W_\gamma^T (1 + s/v)$

$\therefore \Pi_\gamma^{\circ T}/P_\gamma^\circ = \frac{s/v}{1+s/v} = \frac{s}{s+v} = \sigma = s/c$

INTEGRATED WICKSELL EFFECT

$$\text{(iii)}\quad \boxed{\left(\frac{p_\gamma^* - p_\gamma^\circ}{p_\gamma^\circ}\right) = \underbrace{\left(\frac{r - \rho_\gamma^T}{\rho_\gamma^T}\right)}_{\text{effect of changes in profit rates}} \cdot \sigma \;+\; \underbrace{r\,\frac{\Delta K_\gamma^{*T}}{p_\gamma}}_{\text{effect of changes in magnitude of capital advanced}}}$$

where $\sigma = \frac{S}{S+V} \cong \frac{\Pi}{Y}$

$$\Delta K_\gamma^{*T} \equiv K_\gamma^{*T} - K_\gamma^{\circ T}$$

2.

(i) It is here that direction of changes is crucial. If transformation raises the vertically integrated profit rate (i.e. $r > \rho_\gamma^T$) and lowers the money value of vertically integrated capital stock ($\Delta K_\gamma^{*T} < 0$). Then the two effects would tend to cancel each other out. Otherwise they would reinforce each other.

(ii) Note that $r = \rho_{PER}^T$, so the question becomes one of whether or not price-value deviations can be ordered in terms of the Marxian "average" industry (PER). If so, then $\rho_\gamma^T < \rho_{PER}^T = r \rightarrow \Pi_\gamma^{*T} > \Pi_\gamma^{\circ T}$

then $\frac{\Delta p_\gamma}{p_\gamma^\circ} > 0$, $r - \rho_\gamma^T < 0 \longrightarrow \Delta K_\gamma^{*T} > 0$,

since $\sigma > 0$, $r > 0$.

Thus if price-value deviations can be ordered in terms of deviations of $\left(\frac{C}{V}\right)_\gamma^T$ from $\left(\frac{C}{V}\right)_{PER}^T$, then these deviations will be less than σ times the profit-rate differentials.

3.

Secondly, profit-rate differentials are likely to be much smaller for vertically-integrated profit rates, than for direct ones.

Price - Deviations

2/1/78

(1) So if $6 \cong \frac{\pi}{4} \cong .30$, then even if direct profit rate differences average (say) .50, the vertically integrated ones are likely to average (say) .20, so that price of prod.-value deviations would be <u>less than</u> <u>.06 = 6%</u>.

Mapping: direct profit rate differentials $\longrightarrow$ vertically integrated profit rate differentials $\longrightarrow$ δ - vertically integ. $\longrightarrow$ price value deviat.

50% $\longrightarrow$ 30% $\longrightarrow$ 6% $\longrightarrow$ 4% (?)

4. Generally

$$\boxed{\begin{aligned} p^\circ &= (p^\circ A + wl)\langle 1+\rho_i\rangle = p^\circ A + wl(1+\tfrac{s}{v}) \\ p^* &= (p^* A + wl)(1+r) = (p^\circ A + wl)(1+r) + (\Delta p)A(1+r) \end{aligned}}$$

Assuming same money wage in both

$$p^* - p^\circ = \Delta p = (p^\circ A + wl)(1+r) + \Delta p A(1+r) - \underbrace{(p^\circ A + wl)\langle 1+\rho_i\rangle}_{p^\circ}$$

$$\Delta p[I-(1+r)A] = (p^\circ A + wl)\langle r - \rho_i\rangle$$

$$\Delta p\langle p_i^\circ\rangle^{-1}\langle p_i\rangle[I-(1+r)A]\langle p_i\rangle^{-1} = \left[1\langle p_i\rangle A\langle p_i^\circ\rangle^{-1} + wl\langle p_i^\circ\rangle^{-1}\right]\langle r-\rho_i\rangle$$

Define $\hat{\delta} \equiv \left[\frac{p_i^* - p_i^\circ}{p_i^\circ}\right] = \Delta\hat{p}\langle p_i\rangle^{-1}$, $r = \rho_{PER} = \rho^*$

Note that $p_i^\circ = w(1+s/v)\lambda_i$, so that

$$\frac{wl_i}{p_i^\circ} = \frac{l_i}{\lambda_i}\frac{1}{1+s/v} = \frac{v_i}{d_i}$$

Define $B = [b_{ij}] = \left[\frac{\lambda_i a_{ij}}{\lambda_j}\right] = \left[\frac{c_{ij}}{\lambda_j}\right]$

Define $\hat{k} = [k_i] = 1B + wl\langle p_i\rangle^{-1} = \left[\frac{c_j + v_j}{\lambda_j}\right] = \left[\frac{1}{1+\rho_j}\right]$

$$\hat{\delta}[I-(1+r)B] = \hat{k}\langle(1+\rho^*) - (1+\rho_j)\rangle = \hat{k}\left\langle\frac{1}{k^*} - \frac{1}{k_j}\right\rangle = \hat{k}\left\langle\frac{k_j - k^*}{k^* k_j}\right\rangle$$

$$= [k_1, k_2, k_3]\begin{pmatrix}\frac{k_1-k^*}{k^*}\frac{1}{k_1} & 0 & 0 \\ 0 & \frac{k_2-k^*}{k^*}\frac{1}{k_2} & 0 \\ 0 & 0 & \ldots\end{pmatrix} = \left[\frac{k_1-k^*}{k^*}, \frac{k_2-k^*}{k^*}, \frac{k_3-k^*}{k^*}\right]$$

$= \frac{V}{N}$

Define $\gamma_j = \dfrac{k_j - k^*}{k^*}$, where $k_j \equiv \dfrac{c_j + v_j}{d_j}$ = direct "capital-output" ratios

$$\boxed{\hat{\delta} = \hat{\gamma}\,[I - (1+r)B]^{-1}}$$

Normalized by
$p^* b = w = p^o b = w^o$

[Note: keeping money wages constant is equivalent to normalizing s.t. $p^* b = p^o b = w^o$]

[Note: $Q B Q^{-1} = \Delta = \langle \frac{1}{1+R_k} \rangle \rightarrow \hat{\delta} = \hat{\gamma}\, Q\, [I - (1+r)\Delta]^{-1} Q^{-1}$,

$q_k = \Delta^*_{(k)}$; $\hat{\delta} = \hat{\beta} \langle \frac{R_k - r}{1+R_k} \rangle Q^{-1} = \left[\beta_1 \frac{R_1 - r}{1+R_1}, \beta_2 \frac{R_2 - r}{1+R_2} \right] Q^{-1}$

where $\hat{\beta} \equiv \hat{\gamma}\hat{Q} = [\hat{\gamma}\hat{q}_1, \hat{\gamma}\hat{q}_2]$, $Q = [\hat{\Lambda}^+_{(1)}, \hat{\Lambda}^+_{(2)}]$

Total values of std commodity

Note that $\gamma_j = \dfrac{k_j}{k^*} - 1$

where $k^* = \sum \hat{k}_j \Delta^*_j$, Δ^*_j = value of PBR industry

But $\Delta^*_j \neq \Delta^+_j$, where Δ^+_j = value of std industry

A-Shaikh

Direction of Price-Value Deviations 2/15/84

1. We know that $\frac{p_j}{w} \uparrow$ as $r \uparrow$ [from $\hat{p}/w = \hat{\lambda}(I - rH)^{-1}$]

Therefore, $w/p_j \downarrow$ as $r \uparrow$, for all j

2. (i) $w/p_j = \frac{w^*}{p_j^*}$, where w^*, p_j^* are standard wage rate and unit j^{th} price, respectively

(ii) $\boxed{w^* = 1 - \frac{r}{R}}$

$$w/p_j = \frac{w^*}{p_j^*} = \frac{(w^*/\lambda_j)}{(p_j^*/\lambda_j)}$$

$\therefore$ p_j^*/λ_j = Std. price-value ratio = $\frac{w^*/\lambda_j}{w/p_j}$

(iii) $\boxed{p_j^*/\lambda_j = \frac{\frac{1}{\lambda_j}\left(1 - \frac{r}{R}\right)}{(w/p_j)}}$ = standard price-value ratio

Direction of Price-Value Deviations 2/15/84

3. The preceding expression for the γ^{th} standardized price-value ratio can be given a simple geometric interpretation

$$\left(\frac{p_\gamma^*}{\lambda_\gamma}\right) = \frac{\frac{1}{\lambda_\gamma}\left(1-\frac{r}{R}\right)}{w/p_\gamma} = \frac{f_\gamma(r)}{w/p_\gamma(r)}$$

(i) The term $f_\gamma(r) = \frac{1}{\lambda_\gamma}(1-r/R)$ is a straight line

whose intercepts are: $f_\gamma(0) = \frac{1}{\lambda_\gamma}$ at $r=0$

$f_\gamma(R) = 0$ at $r=R$

(ii) The ~~term~~ $w/p_\gamma(r)$ is the wage-profit rate tradeoff, and we know that it has intercepts

$$w/p_\gamma(0) = \frac{1}{\lambda_\gamma} \quad \left[\text{since } \frac{\hat{p}}{w} = \hat{\lambda}[I-rH]^{-1} = \hat{\lambda} \text{ at } r=0\right]$$

$$w/p_\gamma(R) = 0$$

(iii) ~~Thus~~ Both curves have the same intercepts. Thus the function $f_\gamma(r) = \frac{1}{\lambda_\gamma}(1-r/R)$ is simply the straight line between the intercepts of the wage-profit curve $\frac{w}{p_\gamma}(r)$.

Direction of Price-Value Deviations 2/10/84

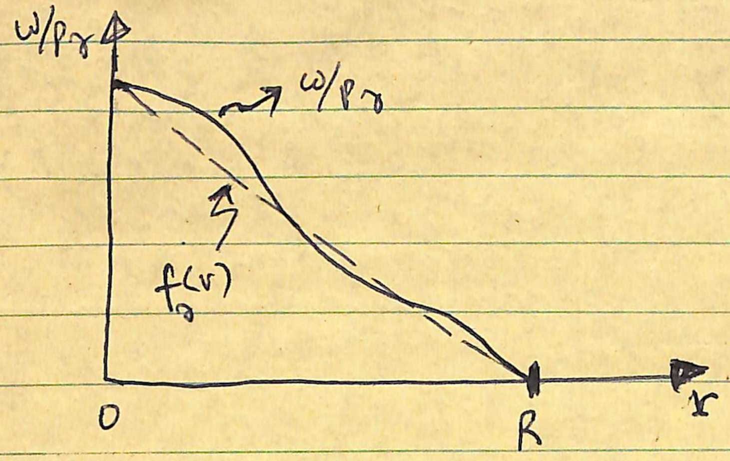
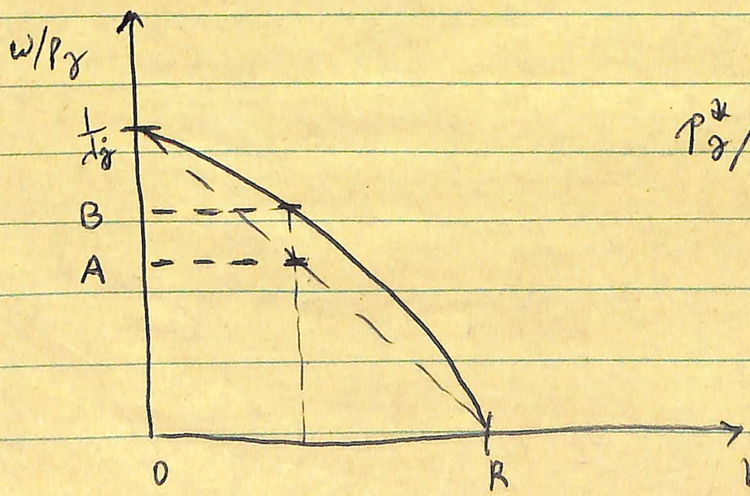


Figure 1

(iv) It follows that the standardized price-value ratio at any rate of profit r is simply the ratio of the height of the straight line between the intercepts of the w/p_γ curve and the height of the curve itself.

Therefore, the non-linearity of the curve is itself a reflection and measure of the standardized price-value deviation.



$$p^*_\gamma / d_\gamma = \frac{OA}{OB}$$

Figure 2

2/15/84

(v) It should be noted that the geometric interpretation of price-value deviations does not imply that p^*/λ_γ approaches ~~gets~~ ~~smaller~~ one (standard prices approach values) once ~~again~~ again as $r \to R$.

a) At $r = 0$, $p^* = \lambda_\gamma$, ~~But~~ and this is reflected by intersection of w/p_γ + $f_\gamma(r)$ — i.e. by equality of OA and OB at $r = 0$. But at $r = R$, OA and OB are once again equal. But in this case it ~~This~~ does not imply that $p^*_\gamma = \lambda_\gamma$, because both OA and OB are equal to zero. Thus their ratio approaches a determinate number [as we know from the strict positivity of $\hat{p}^*(R)$] which is in general ~~is~~ different from one.

b) What this says is that the geometric treatment is useful to discuss Marx-"reswitching", ~~and~~ [i.e. price-value direction switching] and ~~also~~ Sraffa-reswitching, ~~but not useful for directly seeing~~ and for the direction of price-value deviations, but not for directly ~~visualizing~~ size ~~of~~ these deviations.

c) To visualize size ~~of~~ deviations, we need to geometrically map ratio OA/OB onto another curve.
[First, use $w/(p_\gamma/\lambda_\gamma)$ instead of w/p_γ? This is good here but bad for geometric treatment of Sraffa reswitching, in which size of λ_γ matters

direction of initial deviation to can be shown depend on r - (g/λ) [illegible]

d) Note that further question of why Marx reswitching is not probable ~~if~~ unless w/p_γ is close to w^*/λ_γ is not obvious geometrically

Directions of Price-Value Deviations 2/15/84

4. We ~~can~~ now address the ~~formal~~ mathematics of standardized ~~question of the number of times~~ ~~the~~ price-value deviations, ~~can switch~~ directions ~~as we vary the rate of profit~~

The price system can we written as

$$(1) \qquad \hat{p} = w\hat{l} + \hat{p}A + r\hat{p}K$$

where $\hat{p}$ = row vector of unit prices
$\hat{l}$ = " " " unit (direct) labor requirements
A = input-output coefficients matrix
K = capital coefficients matrix
w = money wage
r = rate of profit

Rewriting

$$\hat{p}[I-A] = w\hat{l} + r\hat{p}K$$

$$(1') \qquad \boxed{\hat{p}(r) = w\hat{\lambda} + r\hat{p}(r)H}$$

where $\hat{\lambda} \equiv l[I-A]^{-1}$ = vector of unit labor values

$H \equiv K[I-A]^{-1}$ = Matrix of vertically integrated capital coefficients

2/10/84

Notice that at $r = R$, $w=0$ and the price system becomes

(III) $\hat{p}_R = R\hat{p}_R H$

Defining a standard commodity $\hat{X}_S$ as the quantity dual of (III), we have

(IV) $\hat{X}_S \equiv RH\hat{X}_S \longrightarrow \frac{1}{R}\hat{X}_S = H\hat{X}_S$

and applying this to the price system in (II), we get

$$\hat{p}\hat{X}_S = w\hat{\lambda}\hat{X}_S + r\,\hat{p}[H X_S] = w\lambda\hat{X}_S + r\,\hat{p}[\tfrac{1}{R}\hat{X}_S]$$

(V) $$w = \frac{\hat{p}\hat{X}_S}{\hat{\lambda}X_S}\left[1 - \frac{r}{R}\right]$$

The term $\hat{p}\hat{X}_S$ = <u>sum of prices</u> of the <u>standard</u> commodity $\hat{X}_S$, and $\hat{\lambda}\hat{X}_S$ = <u>sum of values</u> of the <u>standard</u> commodity. So if we normalize unit prices $\hat{p}$ by taking the <u>standard sum of prices equal</u> to the standard <u>sum of values</u>, we get the <u>standardized</u> wage rate w^*:

(VI) $\boxed{w^* = (1 - r/R)}$ when $P_S \equiv \hat{p}\hat{X}_S = \lambda_S \equiv \hat{\lambda}\hat{X}_S$

It also follows from the above definition that the ratio $\frac{\hat{\lambda}\hat{X}_S}{\hat{p}\hat{X}_S} \equiv \frac{\text{standard sum of values}}{\text{standard sum of prices}}$ = <u>STANDARD VALUE OF MONEY.</u>

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Thus, deflating the money wage w by the standard value of money will give a linear relation between the standardized (deflated) wage rate and the rate of profit.

Deflating unit prices $\hat{p}$ by the standard value of money then yields $\hat{p}^* \equiv \hat{p}\left[\frac{\hat{\lambda}\hat{X}_s}{\hat{p}\hat{X}_s}\right]$

Then, from (IV) $\hat{X}_s = RH\hat{X}_s$

$$\hat{p}^*\hat{X}_s = R\hat{p}^* H\hat{X}_s$$

$$\frac{\hat{p}^*\hat{X}_s}{\hat{\lambda}\hat{X}_s} = R\frac{\hat{p}^* H X_s}{\hat{\lambda}\hat{X}_s}$$

But by definition of $\hat{p}^*$, $\frac{\hat{p}^*\hat{X}_s}{\hat{\lambda}X_s} = \frac{\hat{p}\hat{X}_s\left(\frac{\hat{\lambda}\hat{X}_s}{\hat{p}\hat{X}_s}\right)}{\hat{\lambda}\hat{X}} = 1$

So,

(VII) $\frac{\hat{p}^* H X_s}{\hat{\lambda}\hat{X}_s} = \frac{1}{R}$ = standardized price of the vertically integrated capital stock of the standard system $\div$ the labor value of the gross output of the standard system

I.E.

(VIII) $\boxed{\frac{1}{R} = \bar{h}^*_s}$ = deflated (standardized) vertically integrated capital-labor ratio of the standard system.

Note that $\bar{h}^*_s$ is a constant for any given technology

2/15/84

We can now consider the standardized unit price vector. From (11), $\hat{p} = w\hat{\lambda} + r\hat{p}H$

$$\hat{p}^* = w^*\hat{\lambda} + r\hat{p}^*H = (1 - r/R)\hat{\lambda} + r\hat{p}^*H$$

$$\text{(ix)} \qquad \hat{p}^* - \hat{\lambda} = r\left(\hat{p}^*H - \frac{1}{R}\hat{\lambda}\right)$$

The j^{th} components of (ix) are

$$p_j^* - \lambda_j = r\left(\hat{p}^*\hat{H}_{(J)} - \frac{1}{R}\lambda_j\right)$$, where $\hat{H}_{(J)} = j^{th}$ column of H

$$\therefore \text{(x)} \quad d_j^* \equiv \frac{p_j^* - \lambda_j}{\lambda_j} = r\left[\frac{\hat{p}^*\hat{H}_{(J)}}{\lambda_j} - \frac{1}{R}\right]$$ = percentage deviation of the standardized j^{th} price from the j^{th} value

But the matrix H is the matrix of vertically integrated capital goods required per unit output, so that $\hat{p}^*\hat{H}_{(J)}$ is the standardized price of the capital stock per unit output in the j^{th} industry. Since λ_j is the labor value of the unit output in the j^{th} industry, we see that

$$\text{(xi)} \qquad \frac{\hat{p}^*\hat{H}_{(J)}}{\lambda_j} = h_j^*(r)$$ = standardized vertically integrated capital-labor ratio in the j^{th} industry

2/15/84

It is useful to note ~~that~~ at this juncture that the j^{th} industry's standardized vertically integrated capital-labor ratio $h^*_j(r)$ is a function of r. When $r = 0$, unit standardized prices $\hat{p}^*(0)$ equal unit labor values $\hat{\lambda}$, so that $h^*_j(0)$ is simply the ~~Marxian~~ ~~ratio~~ ~~ratio of~~ vertically integrated counterpart of Marx's ratio of dead-to-living labor [sometimes also known as the value composition of capital $C/\ell = C/(v+s)$]. We will therefore call $h^*_j(0)$ the j^{th} industry's vertically integrated value composition of capital.

For ~~a~~ $r > 0$, we will in general have $h^*_j(r)$ different from the corresponding vertically integrated value composition $h^*_j(0)$. Moreover, as $r \to R$, $\hat{p}^*(r) \to \hat{p}(R)$, and $h^*_j(r)$ will approach a limiting value $h^*_j(R)$. To find this limiting value, we note that from equation (III) (expressed in deflated prices $\hat{p}^*$)

$$\hat{p}^*_R = R\,\hat{p}^*_R H \qquad (\text{when } r = R)$$

so that the j^{th} terms

$$R\,\hat{p}^*_R H_{(j)} = p^*_{R_j}$$

$$\hat{p}^*_R H_{(j)} = \frac{1}{R}\,p^*_{R_j}$$

The limiting value of $h^*_j(r)$ at $r = R$ is therefore

$$\text{(XII)} \quad \therefore\ h^*_j(R) \equiv \frac{p^*_R H_{(j)}}{\lambda_j} = \frac{1}{R}\,\frac{p^*_{R_j}}{\lambda_j} = \bar{h}^*_s\,\frac{p^*_{R_j}}{\lambda_j}$$

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Since from (vii), $\frac{1}{R} = \overline{h^*_s}$ = constant with respect to variations in ω, r

Thus

$$d^*_\gamma = \frac{p^*_\gamma - \lambda_\gamma}{\lambda_\gamma} = r\left[h^*_\gamma(r) - \overline{h^*_s}\right]$$

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Recall that from (viii) we have $\frac{1}{R} = \bar{h}^*_s$ = standardized vertically integrated capital-labor ratio of the <u>standard system</u>. With this in mind, we can substitute (viii) and (ix) into (x), to get

$$d^*_\gamma \equiv \frac{p^*_\gamma - \lambda_\gamma}{\lambda_\gamma} = r\left[h^*_\gamma(r) - \bar{h}^*_s\right]$$

where $h^*_\gamma(0)$ = γ^{th} industry's vertically integrated value composition of capital

$h^*_\gamma(R) = (p^*_{R\gamma}/\lambda_\gamma)\,\bar{h}^*_s$

$$\text{(xii)} \qquad \boxed{d^*_\gamma = \frac{r}{R}\left[\frac{h^*_\gamma(r) - \bar{h}^*_s}{\bar{h}^*_s}\right]}$$

Equation (xii) says that the percentage (standardized) price-value deviation of the γ^{th} commodity ~~depends~~ is a fraction $\left(\frac{r}{R}\right)$ of the percentage (standardized) deviation of the γ^{th} industry's vertically integrated capital-labor ratio $[h^*_\gamma(r)]$ from the standard vertically integrated capital-labor ratio $(\bar{h}^*_s)$.

As $r/R \to 1$, the weight applied to the vertically integrated capital-labor ratio deviation gets larger. But this latter deviation is itself a function of r, ~~and~~ since ~~this~~ $h^*_\gamma(r)$ varies with r,

(~~xiii~~) Note that we can rewrite (xii) as

$$\text{(xiii)} \qquad d^*_\gamma = \frac{r}{R}\left[\frac{h^*_\gamma(0) - \bar{h}^*_s}{\bar{h}^*_s}\right] + \frac{r}{R}\left[\frac{h^*_\gamma(r) - h_\gamma(0)}{h_\gamma(0)}\right]\frac{h_\gamma(0)}{\bar{h}^*_s}$$

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In (xiii), the first term measures the contribution of the deviation of the j^{th} vertically integrated value composition of capital from the standard one. This corresponds to the emphases in Marx on the determinants of organic compositions, so I will call it the <u>Marx</u> ~~Factor~~ <u>Effect</u>.

The second term, on the other hand, measures the contribution of the difference between the j^{th} vertically integrated capital-labor ratio in (standard) prices and the corresponding ratio in values. This measures the effect of price-value deviations on the vertically integrated capital-labor ratios, or, from an orthodox point of view, <u>it measures the Wicksell Effect</u>

The important question is: what are the respective sizes of the two effects, and particularly, is it not true that the Wicksell Effect only causes price-value deviations to switch directions when the Marx Effect is small — i.e. when the vertically integrated value composition in the j^{th} sector is close to the standard one?

Direction of Price-Value Deviations 11/18/83

1.

(ii) we have already seen that the extent of the deviation ~~from~~ of the curve from the straight line through its origins measures the size (and direction) of the individual standard price-value deviation.

2. Now consider two different technologies, each of which produces the j^{th} commodity.

(i) These technologies will have two different standard commodities $\hat{X}_{S_I}$ and $\hat{X}_{S_{II}}$, respectively.

(ii) within each technology, the individual wage-profit curves can be analyzed as above: their curvature and variability in each case will ~~signify their~~ measure the size deviation, direction, and variability of the standard price-value ratio of the j^{th} commodity — measured in terms of the standard commodity of its own technology.

(iii) Switches and reswitches between the two alternate methods of production of this j^{th} commodity can be ~~visualized~~ seen by superimposing the two curves on one another. Since the numeraire

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p_γ represents the money price of the same commodity, ~~and~~ the commodity-wage w/p_γ has the same units for both curves.

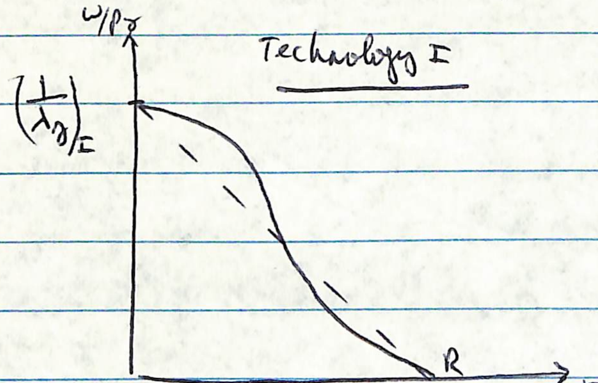

Figure 3

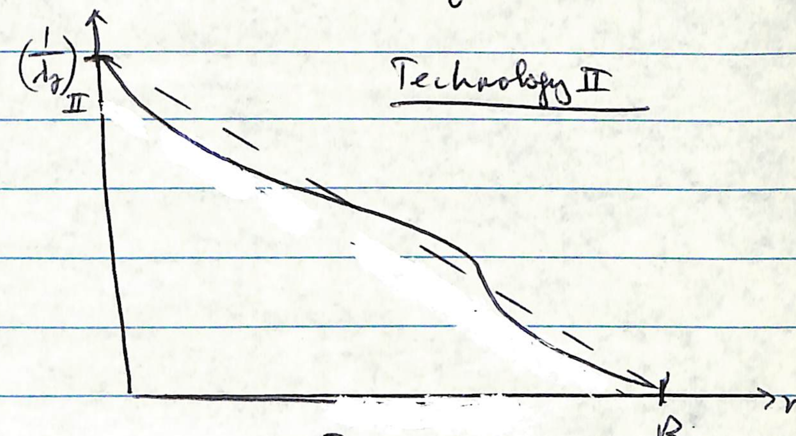

Figure 4

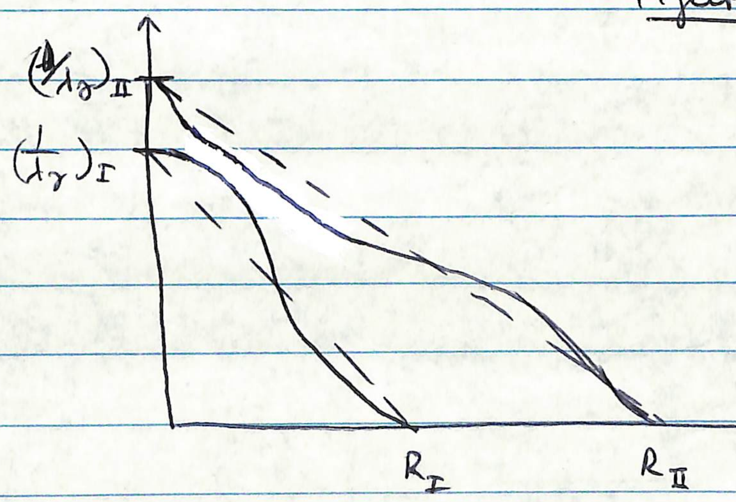

Figure 5

(iv) It is obvious from the above figures that the probability of intersection <u>at all</u>, ~~but above~~ depends primarily on the movements of the two intercepts $\left(\frac{1}{\lambda_\gamma}\right)$ and R.

a) Assuming technology II is the more advanced one, we can generally assume $\lambda_{\gamma_{II}} < \lambda_{\gamma_I} \Rightarrow \left(\frac{1}{\lambda_\gamma}\right)_{II} > \left(\frac{1}{\lambda_\gamma}\right)_I$. This simply says that more "advanced" technologies are characterized by lower unit values; in

Scratch

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} a_{11}(A_{11}) + a_{12}(A_{22}) & \\ & \end{bmatrix}$$

$\gg [0]$ if $A \gg [0]$ & at least one element of each row of $a > 0$

$$p(r) = \lambda\left[A_1 + \left(1 - \frac{r}{R_1}\right)\left[\frac{A_2}{1 - r/R_2} + \frac{A_3}{1 - r/R_3}\right]\right]$$

$$p(r)H - \lambda\left(\frac{1}{R_1}\right)I = \lambda\left[\left(A_1 H - \frac{1}{R}I\right) + \left(1 - \frac{r}{R_1}\right)\left[\frac{A_2 H}{1 - r/R_2} + \frac{A_3 H}{1 - r/R_3}\right]\right]$$

$$\frac{p(r) - \lambda}{(1 - r/R_1)} = \lambda\left[\frac{(A_1 - I)}{(1 - r/R_1)} + \left(\frac{A_2 H}{1 - r/R_2} + \frac{A_3 H}{1 - r/R_3}\right)\right]$$

Note; $\lambda A_1 H = \frac{1}{R}\lambda = p(R_1)H - \frac{1}{R}\lambda = \left[\frac{K(R_1)_i}{\lambda_i} - \frac{1}{R}\right]\lambda_i$

$$(\lambda A_1 - \lambda) = p(R_1) - \lambda$$

$$p(r) = \underbrace{\lambda A_1}_{\to\, p(R_1)} + \lambda\left[A_2 \frac{(1 - r/R_1)}{1 - r/R_2} + A_3 \frac{(1 - r/R_1)}{(1 - r/R_3)}\right]$$

since $R_1 < R_2 < R_3$

$\frac{r}{R_1} > \frac{r}{R_2} > \frac{r}{R_3}$

$1 - r/R_1 < 1 - r/R_2 < 1 - r/R_3$

$$p(r) - \lambda = \underbrace{[p(R_1) - \lambda]}_{(\lambda A_1 - \lambda)} + \lambda\left[A_2 \frac{(1 - r/R_1)}{(1 - r/R_2)} + A_3 \frac{(1 - r/R_1)}{1 - r/R_3}\right]$$

where $\frac{1 - r/R_1}{1 - r/R_i}$, $i \neq 1$, declines steadily as $r \to R_1$

X

since $\Sigma A_i = I$, $\lambda = \lambda \Sigma A_i$

$\therefore \lambda - p(r) = \lambda[\cancel{A_1} + A_2 + A_3 - A_1 - f_2(r)A_2 - f_3(r)A_3]$ ~~$- A_1 - A_2 - A_3$~~

$$\lambda - p(r) = \lambda\left[A_2 \frac{(r/R_1 - r/R_2)}{(1 - r/R_2)} + A_3 \frac{(r/R_1 - r/R_3)}{(1 - r/R_3)}\right]$$

$\rightarrow$ $\boxed{\hat{\lambda} - \hat{p}(r)} = \hat{p}(R_2)\left[\frac{x_1 - x_2}{1 - x_2}\right] + \hat{p}(R_3)\left(\frac{x_1 - x_3}{1 - x_3}\right) +$

$0 \le x_1 \le 1$

$0 \le x_2 \le$ [illegible]

$$\Delta(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)$$

$$\Delta^{(1)}(\lambda) = (\lambda - \lambda_2) + (\lambda - \lambda_1) = \quad ^{n} \prod (\lambda - \lambda_i)$$

$$\Delta^{(1)}(\lambda_1) = (\lambda_1 - \lambda_2) = (-1)(\lambda_2 - \lambda_1)$$

$$\Delta(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)$$

$$\Delta^{(1)}(\lambda) = (\lambda - \lambda_2)(\lambda - \lambda_3) + (\lambda - \lambda_1)(\lambda - \lambda_3) + (\lambda - \lambda_1)(\lambda - \lambda_2)$$

$$\Delta^{(1)}(\lambda_1) = (\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3) = (-1)^2 \prod_{2} (\lambda_2 - \lambda_1)$$

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0. $p(r) \equiv (1 - r/R_1)\lambda + r p(r) H \to (p-\lambda) = r\left[p(r)H - \frac{1}{R_1}\lambda\right]$

1. $\boxed{p(r) = \lambda\left(\frac{1}{r} - \frac{1}{R_1}\right)\left(\frac{1}{r} I - H\right)^{-1} = \lambda\left[\frac{\left(1-\frac{r}{R_1}\right)}{r}\frac{F(\frac{1}{r})}{h(r)}\right]}$ (Elementary Matrices; Frazer, Duncan & Collar) (78)

where $h(\frac{1}{r}) \equiv \left(\frac{1}{r} - \frac{1}{R_2}\right)\left(\frac{1}{r} - \frac{1}{R_3}\right)\cdots$, $\frac{1}{R_1}$ = dominant root

$$\boxed{F(\tfrac{1}{r})} = \frac{A_1}{\frac{1}{r}-\frac{1}{R_1}} + \frac{A_2}{\frac{1}{r}-\frac{1}{R_2}} + \cdots, \quad A_i = \text{coefficient matrices}$$

$$A_i = \text{coeff. matrices} = \frac{F(\frac{1}{R_i})}{\Delta^{(1)}(\frac{1}{R_i})} \quad (78)$$

$$\Delta(\tfrac{1}{r}) \equiv \left|\left(\tfrac{1}{r} I - H\right)\right| = \left(\tfrac{1}{r} - \tfrac{1}{R_1}\right)\left(\tfrac{1}{r} - \tfrac{1}{R_2}\right)\left(\tfrac{1}{r} - \tfrac{1}{R_3}\right)\cdots \quad (73)$$

$$\to \Delta^{(1)}(\tfrac{1}{r}) \equiv \frac{d}{d(\frac{1}{r})}\left[\Delta(\tfrac{1}{r})\right] = -\left(\tfrac{1}{r}-\tfrac{1}{R_2}\right)\left(\tfrac{1}{r}-\tfrac{1}{R_3}\right)\cdots + \left(\tfrac{1}{r}-\tfrac{1}{R_1}\right)\left(\tfrac{1}{r}-\tfrac{1}{R_3}\right) - \left(\tfrac{1}{r}-\tfrac{1}{R_1}\right)\left(\tfrac{1}{r}-\tfrac{1}{R_2}\right)$$

$$\therefore \boxed{\Delta^{(1)}(\tfrac{1}{R_i}) = \prod_{\gamma \neq i}\left(\tfrac{1}{R_i} - \tfrac{1}{R_\gamma}\right)}$$

$$\boxed{F(\tfrac{1}{R_i}) = \underset{(m\times 1)}{\hat{X}S_i} \cdot \underset{(1\times n)}{\hat{p}(R_i)}} \ (61) \longleftrightarrow \boxed{F(\tfrac{1}{R_i})} = (-1)^{n-1}\prod_{\gamma \neq i}\left(\tfrac{1}{R_i} I - H\right) \quad (75)$$

$$\boxed{A_i = \frac{(-1)^{n-1}\prod_{\gamma\neq 1}\left(\frac{1}{R_i}I - H\right)}{\prod_{\gamma \neq 1}\left(\frac{1}{R_i} - \frac{1}{R_\gamma}\right)}}$$

2. $\therefore \boxed{\hat{p}(r) = \hat{\lambda}\cdot\left[A_1 + \left(1-\frac{r}{R_1}\right)\left(\frac{A_2}{1-\frac{r}{R_2}} + \frac{A_3}{1-\frac{r}{R_3}} + \cdots\right)\right]}$

$$A_i \equiv \frac{(-1)^{n-1}\prod_{\gamma\neq i}\left(\frac{1}{R_\gamma}I - H\right)}{\prod_{\gamma\neq i}\left(\frac{1}{R_\gamma} - \frac{1}{R_i}\right)} \quad \left(A_i \equiv Z_0(\tfrac{1}{R_i}),\ \text{p. } 78\right)$$

Note that $\sum_{i=1}^{n} A_i = I$ (79)

and $A_i \cdot A_\gamma = 0$ if $i \neq \gamma$ (79)

and $A_i^m = A_i$ for any $m > 0$ (79)

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$$p(r) = w\lambda + rpH = \cancel{w}\lambda\,(\cancel{H}1 - r/R_1) + rpH$$

$$p - \lambda = -r/R_1\,\lambda + r(p-\lambda)H + r\lambda H$$

$$d = r\hat{\lambda}(H - \tfrac{1}{R_1}I) + r\,d\,H = \hat{r}\,1\langle\lambda_i\rangle(H - \tfrac{1}{R}I) + r\,dH$$

$$d\langle\lambda_i\rangle^{-1} = \hat{\delta} = r\,1\langle\lambda_i\rangle(H - \tfrac{1}{R_1}I)\langle\lambda_i\rangle^{-1} + r\,d\langle\lambda_i\rangle^{-1}\langle\lambda_i\rangle H\langle\lambda_i\rangle^{-1}$$

$$\delta[I - rH_\lambda] = r\,1[H_\lambda - \tfrac{1}{R_1}I]$$

$$\hat{\delta} = r\,1[H_\lambda - \tfrac{1}{R_1}I][I - rH_\lambda]^{-1}$$

$$\delta_2 \equiv [\delta_1, \delta_2, \delta_3]\begin{bmatrix}0\\1\\0\end{bmatrix} = \hat{\delta}\,\hat{e}_2 = \hat{r}$$

$d = \hat{\kappa} r[I - rH]^{-1} = \hat{\kappa}\, r[I + rH + r^2H^2 + \cdots] = r\hat{\kappa} + r^2\kappa H^2 + r^3\kappa H^3$

$f(r) \equiv \frac{d_\gamma}{r} = \kappa_\gamma + r[\hat{\kappa} H_{(\gamma)}] + r^2[\hat{\kappa}(H^2)_{(\gamma)}] + \cdots = 0 \quad \text{if } d_\gamma = 0$

$\frac{r}{R}\,\|\hat{\kappa} RH\|$? $\leq \|\hat{\kappa}\|$

$\|H\|$

$p = (1 - r/R)\lambda + rpH \rightarrow p = (1 - r/R)\lambda[I - rH]^{-1}$

$p = (1 - r/R)\lambda + (1 - r/R)\lambda rH + (1 - r/R)\lambda r^2H^2 + \cdots$

$p - \lambda = -r/R\,\lambda + (1 - r/R)[r\lambda H + r^2\lambda H^2 + \cdots]$

$= -r/R\,\lambda + (1 - r/R)r\lambda H[I + rH + r^2H^2 + \cdots]$

$(p - \lambda)_\gamma = -r/R\,\lambda_\gamma + (1 - r/R)r\lambda H_{(\gamma)} + (1 - r/R)r^2\lambda(H^2)_{(\gamma)} +$

$= -r/R\,\lambda_\gamma + r\lambda H_{(\gamma)} - \frac{r^2}{R}\lambda H_{(\gamma)} +$

$p = (1 - r/R)\lambda + rpH \rightarrow d \equiv p - \lambda = -r/R\,\lambda + rdH + r\lambda H$

$\boxed{d} = r\lambda[H - \frac{1}{R}I][I - rH]^{-1} = \boxed{\lambda[H - \frac{1}{R}I][\frac{1}{r}I - H]^{-1}}$

$= r\hat{\kappa}[I - rH]^{-1}$ where $\hat{\kappa} \equiv \lambda[H - \frac{1}{R}I] = -\lambda f(\frac{1}{R})$

$\hat{\delta} \equiv d\langle\lambda_\gamma\rangle^{-1} = I[H_\lambda - \frac{1}{R}I][\ldots]$

~~$d = r\lambda(H - \frac{1}{R}I)[I + rH + r^2H + \cdots]$~~

But $[\frac{1}{r}I - H]^{-1} = \underbrace{\sum_{i=1}^{n} \frac{F(\frac{1}{R_i})}{\Delta^{(1)}(\frac{1}{R_i})\cdot(\frac{1}{r} - \frac{1}{R_i})}}_{(+)}$, $F(\frac{1}{R_i}) = (-1)^{n-1}\prod_{\gamma \neq i}(\frac{1}{R_\gamma}I - H)$

and $(H - \frac{1}{R}I) = -f(\frac{1}{R_1})$

$\Delta(\frac{1}{r}) = (\frac{1}{r} - \frac{1}{R_1})(\frac{1}{r} - \frac{1}{R_2})(\ldots)$

$\Delta^{(1)}(\frac{1}{r}) = (-\frac{1}{R_1})(\frac{1}{r} - \frac{1}{R_2})(\frac{1}{r} - \frac{1}{R_3})$

$- \frac{1}{R_2}(\frac{1}{r} - \frac{1}{R_1})(\frac{1}{r} - \frac{1}{R_3})$

$- \frac{1}{R_3}(\frac{1}{r} - \frac{1}{R_1})(\frac{1}{r} - \frac{1}{R_2})$

$r = .2 \quad \frac{1}{r} = 10$

$R_1 = .5 \quad \frac{1}{R_1} = 2$

$R_2 = -1$

$\frac{1}{R_3} = +3/2$

$F(\frac{1}{R_1}) = (-1)^2[(\frac{1}{R_2}I - H)(\frac{1}{R_3}I - H)]$

$F(\frac{1}{R_2}) = +[(\frac{1}{R_1}I - H)(\frac{1}{R_3}I - H)]$

$F(\frac{1}{R_3}) = +[(\frac{1}{R_1}I - H)(\frac{1}{R_2}I - H)]$

If $0 < r < R_1 \rightarrow \frac{1}{r} > \frac{1}{R_1}$

and $|\frac{1}{R_1}| > |\frac{1}{R_\gamma}|$, $\gamma \neq 1$ then $\frac{1}{r} - \frac{1}{R_\gamma} > 0$ for all γ

~~But only $F(\frac{1}{R_1}) >> 0$, since~~ $\equiv f(\frac{1}{R_i})\, F(\frac{1}{R_i}) = 0$

$\boxed{\text{But } (\frac{1}{R_1}I - H)(\frac{1}{R_2}I - H)(\frac{1}{R_3}I - H) = 0}$ $\therefore \Delta^{(1)}(\frac{1}{r}) > 0$

and $\Delta^{(1)}(\frac{1}{R_\gamma}) > 0$

also, since $[\frac{1}{r}I - H]^{-1} >> [0]$, it must be true that $F(\frac{1}{R_i}) >> [0]$ also?

$(\frac{1}{r} - \frac{1}{R_1}) > 0$

$(\frac{1}{r} - \frac{1}{R_2}) = 10 - (-1) > 0$

$(\frac{1}{r} - \frac{1}{R_3}) = 10 - 3/2$

$(\frac{1}{R_1} - \frac{1}{R_2}) = 2 - (-1) > 0$

$(\frac{1}{R_1} - \frac{1}{R_3}) = 2 - 3/2 > 0$

$\|Ax\| \leq \|x\| \cdot \mathrm{lub}(A)$

and for some special x^*, $\|A \cdot x^*\| = \|x^*\| \cdot \mathrm{lub}(A)$

For consistent norms, $\|Ax\| \leq \|A\| \cdot \|x\|$

$\mathrm{lub}(A) \leq \|A\|$

~~For some~~ $\mathrm{lub}_K(A) = \rho(A) =$ spectral radius of A

$$d = \lambda \left[-f\left(\tfrac{1}{R_1}\right) \right] \left[\frac{F\left(\tfrac{1}{R_1}\right)}{Z_1\left(\tfrac{1}{R_1}\right)} + \cdots \right.$$

$$= \lambda \left[0 - \underline{f\left(\tfrac{1}{R_1}\right) F\left(\tfrac{1}{R_2}\right)} - f \cdots \right.$$

$$= \lambda \left[-f\left(\tfrac{1}{R_1}\right) \right] \left[\underline{F\left(\tfrac{1}{R_2}\right)} + \underline{F\left(\tfrac{1}{R_3}\right)} \right]$$

Let $Q^{-1} H Q = \left\langle \tfrac{1}{R_i} \right\rangle$

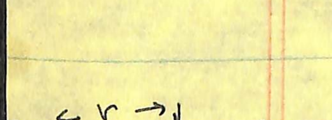
$$d = \lambda \left[H - \tfrac{1}{R_1} I \right] \left[\tfrac{1}{r} I - H \right]^{-1} \cancel{= \lambda Q^{-1} \left[Q H Q^{-1} - \tfrac{1}{R} I \right] Q \left[\tfrac{1}{r} I - H \right]}$$

$$dQ = \lambda Q \, Q^{-1} \left[H - \tfrac{1}{R_1} I \right] Q \, Q^{-1} \left[\tfrac{1}{r} I - H \right]^{-1} Q$$

$$dQ = \lambda Q \left[\left\langle \tfrac{1}{R_i} \right\rangle - \left\langle \tfrac{1}{R_1} \right\rangle \right] \left[\left\langle \tfrac{1}{r} \right\rangle - \left\langle \tfrac{1}{R_i} \right\rangle \right]^{-1}$$

$$dQ = \lambda Q \left[\left\langle \tfrac{1}{R_i} - \tfrac{1}{R_1} \right\rangle \right] \left[\left\langle \tfrac{1}{r} - \tfrac{1}{R_i} \right\rangle \right]^{-1}$$

$$dQ = \lambda Q \left[\left\langle \frac{\left(\tfrac{1}{R_i} - \tfrac{1}{R_1}\right)}{\tfrac{1}{r} - \tfrac{1}{R_i}} \right\rangle \right]$$



$$dQ = -\lambda Q \left\langle \frac{\left(\tfrac{1}{R_1} - \tfrac{1}{R_i}\right)}{\left(\tfrac{1}{r} - \tfrac{1}{R_i}\right)} \right\rangle$$

$(+)$

where ~~$\tfrac{1}{R_1} > \tfrac{1}{R_i}$~~

$\tfrac{1}{r} - \tfrac{1}{R_i} > 0$ and

$\tfrac{1}{R_1} - \tfrac{1}{R_i} > 0$ so

$\tfrac{1}{R_i} - \tfrac{1}{R} < 0$

where $dQ = d_1 \left[\hat{x}_{S_1}, \hat{x}_{S_2}, \hat{x}_{S_3} \right] = \lambda \left[\hat{x}_{S_1}, \hat{x}_{S_2}, \hat{x}_{S_3} \right] \left\langle \frac{R_i - R_1}{R_i - r} \right\rangle$

1. $p = w\lambda + rpH = (1 - r/R)\,\lambda + rpH$, where $\boxed{px_s = \lambda x_s} \Leftrightarrow w = 1 - \frac{r}{R}$

2. $\boxed{d(r) \equiv p - \lambda = r\lambda(H - \tfrac{1}{R}I) + r\,d\,H}$

(i) Note that $\hat{d}(r) = \hat{\phi}$ for $r \neq 0$ requires $\lambda H = \frac{1}{R}\Lambda \rightarrow C_\gamma^T/\Lambda_\gamma = \frac{1}{R}$ for all γ
so $\hat{d}(r) \neq \phi$ except in equal OCC case.

(ii) In two dimensions, $d_1(r) = \hat{0}$ implies $\hat{d}_2(r) = \hat{0}$, since
$px_s = \lambda x_s \rightarrow d\,x_s = 0 \rightarrow d_1 x_{s_1} + d_2 x_{s_2} = 0$, so
that $d_1 = 0$ implies $d_2 = 0 \rightarrow d(r) = \phi \rightarrow$ not possible

(iii) Note that $d x_s = r\lambda(\cancel{Hx_s - \tfrac{1}{R}x_s})^{0} + r\,d\,Hx_s = r\,d\,x_s \rightarrow dx_s = 0$
$\rightarrow px_s = \lambda x_s$ ✓.
$\rightarrow \lambda H x_s = \frac{1}{R}\lambda x_s \Leftrightarrow \frac{C_s^T}{\Lambda_s} = \frac{1}{R}$ ✓

(iv) Since $\hat{d}\hat{x}_s = [d_1, \hat{d}_2]\begin{bmatrix} x_{s_1} \\ \hat{x}_{s_2} \end{bmatrix} = 0$, it follows that

$d_1 x_{s_1} + \hat{d}_2 \hat{x}_{s_2} = 0 \rightarrow$ so (if) $d_1(r) = 0$, then $\hat{d}_2(r)\hat{x}_{s_2} = 0$

~~But~~ $\cancel{\hat{d}\hat{x}_s = (d_1 x_{s_1} + \hat{d}_2 \hat{x}_{s_2}) = r\epsilon}$

$d = [\underset{1\times1}{d_1}, \underset{1\times(m-1)}{\hat{d}_2}] = r[\underset{1\times1}{\lambda_1}, \underset{1\times(m-1)}{\hat{\lambda}_2}]\left(\begin{bmatrix} H_{11} & \\ H_{21} & \end{bmatrix} - \left(\frac{1}{R} \quad 0\right.\right.$

$[d_1 x_{s_1}, \hat{d}_2 \hat{x}_{s_2}] = \hat{d}\begin{bmatrix} x_{s_1} & 0 \\ [0] & \hat{x}_{s_2} \end{bmatrix} =$

$d_\gamma x_{s_\gamma} = r[C_\gamma - \frac{1}{R}\lambda_\gamma]x_{s_\gamma} + r\,d\,H_\gamma x_{s_\gamma} = r[C_\gamma - \frac{1}{R}\lambda_\gamma]x_s + rpH_{(\gamma)}x_{s_\gamma} - r\cancel{C_\gamma^T x_{s_\gamma}}$
$= r[pH_{(\gamma)} - \frac{1}{R}\lambda_\gamma]x_{s_\gamma}$

6/16/93

(I) $$p_\gamma(0) = \lambda_\gamma = \underbrace{\omega\lambda_\gamma}_{\text{wages}} + \underbrace{(1-\omega)\lambda_\gamma}_{\text{profits}} = \omega\lambda_\gamma + \frac{r_0}{R}\lambda_\gamma^T = \omega\lambda_\gamma + r_{0\gamma} C_\gamma^T$$

where $$r_{0\gamma} \equiv (1-\omega)\frac{\Lambda_\gamma}{C_\gamma^T} = \frac{1-\omega}{(C_\gamma^T/\Lambda_\gamma)} = \frac{r/R}{C_\gamma^T/\Lambda_\gamma} = r\frac{(C^T/\Lambda)_s}{(C^T/\Lambda)_\gamma}$$

(II) But price of production need $r_\gamma = r$. Thus if $(C^T/\Lambda)_\gamma > (C^T/\Lambda)_S \equiv \frac{1}{R}$, $r_{0\gamma} < r$

so either $p(r)\uparrow$, or price of m.o.p of γ falls enough to make this $K_\gamma^T/\Lambda_\gamma$ equal to the standard. ~~prices implies that K_γ^T will also change~~. Thus, in the end

$$p_\gamma(r) = \omega\lambda_\gamma + rK_\gamma^T$$

$$p_\gamma(r) - p_\gamma(0) = (rK_\gamma^T - r/R\,\Lambda_\gamma) = r\Lambda_\gamma\left[\frac{K_\gamma^T}{\Lambda_\gamma} - \frac1R\right].$$

$$\boxed{\frac{p_\gamma(r)-p_\gamma(0)}{p_\gamma(0)} = \left(\frac{r}{R}\right)\frac{\left[\frac{K_\gamma^T}{\Lambda_\gamma} - \frac1R\right]}{(1/R)} = \frac{r}{R}\frac{\left[\underbrace{\left(\frac{K_\gamma^T}{C_\gamma^T}\right)}_{p_{K\gamma}/\lambda_{K\gamma}}\left(\frac{C_\gamma^T}{\Lambda_\gamma}\right) - \frac1R\right]}{1/R}}$$

– By assumption, $\boxed{C_\gamma^T/\Lambda_\gamma > \frac1R}$. Thus the only way the price of γ can remain unchanged is if the price (price/value ratio) of its vertically integrated means of prod ($p_{K\gamma} \equiv (K_\gamma^T/C_\gamma^T)\lambda_{K\gamma}$) can fall to the point where it frees up enough income to make up the initial profit-deficit.

$$\rightarrow p_{K\gamma} = \frac{\frac1R}{(C/\Lambda)_\gamma}\lambda_\gamma < 1 \rightarrow \boxed{\frac{p_{K\gamma}-\lambda_{K\gamma}}{\lambda_{K\gamma}} = \frac{\frac1R - (C/\Lambda)_\gamma}{(C/\Lambda)_\gamma}} < 0$$ → NOTE THAT THIS IS independent of r!

But
$$p_{K\gamma}(r) = \omega\lambda_{K\gamma} + rK_{K\gamma}^T$$
$$p_{K\gamma}(0) = \omega\lambda_{K\gamma} + r/R\,\lambda_{K\gamma}$$

$$\frac{p_{K\gamma}(r)-p_{K\gamma}(0)}{p_{K\gamma}(0)} = \frac{r}{R}\frac{\left[\frac{p_{KK\gamma}}{\lambda_{KK\gamma}}\left(\frac{C_{K\gamma}}{\Lambda_{K\gamma}}\right) - \frac1R\right]}{\frac1R}$$

(−) (+) (+)

$$\rightarrow \frac{\frac1R - (C^T/\Lambda)_\gamma}{(C^T/\Lambda)_\gamma} = \frac{r}{R}\left[\frac{\left(\frac{C_K}{\Lambda_K}\right)_\gamma - \frac1R}{\frac1R}\right]\left(\frac{p_{KK\gamma}}{\lambda_{KK\gamma}}\right)$$

(−) (−)

→ for $\frac{p_{KK\gamma}}{\lambda_{KK\gamma}} > 0$ must have $(\ ) < 0$

$$\boxed{\left(\frac{C_K}{\Lambda_K}\right)_\gamma < \frac1R}$$

$p(r) = \lambda(\frac{1}{r} - r/R_1)\ [(I\frac{1}{r} - H)^{-1}] = \lambda[(\frac{1}{\frac{1}{r} - \frac{1}{R_i}})$ 3

$p(r) - \lambda = \lambda$

λ 6/18/93

3. Price - Value Deviations

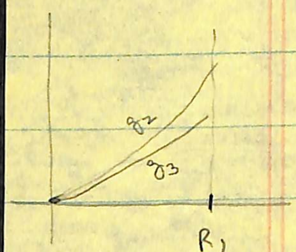
$(1)\ p(r) = \lambda[A_1 + f_2(r) A_2 + f_3(r) A_3]$

where $f_\gamma(r) = \frac{1 - r/R_1}{1 - r/R_\gamma}$ $\begin{cases} r = 0 \rightarrow f_\gamma(0) = 1 \\ r = R_1 \rightarrow f_\gamma(R_1) = 0 \end{cases}$

and $f_\gamma(r)$ declines steadily as $r \rightarrow R_1$

(11) Since $\Sigma A_i = I$, $\lambda = \lambda \Sigma A_i$

$\therefore \lambda - p(r) = \lambda[(1 - f_2(r)) A_2 + (1 - f_3(r)) A_3]$



$$\boxed{\lambda - p(r) \equiv \lambda[g_2(r) A_2 + g_3(r) A_3]}$$

where $g(r) = \frac{r/R_1 - r/R_\gamma}{1 - r/R_\gamma}$ $g_\gamma(0) = 0$, $g_\gamma(R_1) = 1$, & $g_\gamma(r) \uparrow$ as $r \rightarrow R_1$

But $\lambda A_1 = \hat{p}(R_1)$, $\lambda A_2 = \hat{p}(R_2)$ etc. [verify this (prove this ✓)]

verify this

$$\therefore \boxed{(\lambda - p(r)) = g_2(r) \cdot \hat{p}(R_2) + g_3(r) \cdot \hat{p}(R_3)}$$

$$0 \leq \boxed{g_\gamma(r) = \frac{(\frac{1}{R_1} - \frac{1}{R_\gamma})}{(\frac{1}{r} - \frac{1}{R_\gamma})}} \leq 1$$

convex combination of nondominant eigenvectors (with weights between 0 & 1)

According to Sraffa, $p(r) >> 0$ for $0 \leq r \leq R_1$. But for $r > R_1$, some prices must go negative to make $w < 0$

$$P(R_2)\begin{bmatrix} - \\ + \\ + \end{bmatrix} + P(R_3)\begin{bmatrix} - \\ - \\ + \end{bmatrix}$$

note that $g_\gamma(r)$ rises to 1, and $g_2(r) > g_3(r)$ for all r

Only need to focus on those with sign changes

So, if for some $r \cong 0$, ~~sign is +~~ balance of +, - is (say) +, then as $r \uparrow$, weight of (+) $\uparrow$ faster than weight of (−)

~~But if initial balance is negative, can switchover?~~

Since ~~we can~~ initial balance depends on rel size of $p(R_2)$ & $p(R_3)$

$$\hat{d}(r) \equiv \lambda - p(r) = x\left[\left(\frac{1 - \frac{R_1}{R_2}}{1 - x\frac{R_1}{R_2}}\right)\hat{p}(R_2) + \left(\frac{1 - \frac{R_1}{R_3}}{1 - x\frac{R_1}{R_3}}\right)\hat{p}(R_3)\right]$$

Although $(I-A)^{-1}$ is expandable since $\mu(A)<1$, or $[I-A]^{-1}$

$(I-rH)^{-1} = [I - r A(I-A)^{-1}]^{-1}$ need $\frac{\mu(A)}{1-\mu(A)} < 1$

NOT BE since $\mu(H) = \frac{\mu(A)}{1-\mu(A)}$

p. 78 f ranger etc

Resolvent of a square matrix

$$p = w\lambda + rpH \rightarrow p = \lambda\left[(1-r/R)(I-rH)^{-1}\right]$$

$$d = w\,1 + r\,d\,H, \quad d_\gamma = \frac{p_\gamma}{\lambda_\gamma}$$

$$\boxed{p = \lambda\left[\left(\frac{1}{1-r/R}\right)I - \left(\frac{r/R}{1-r/R}\right)RH\right]^{-1}}$$

Thus is invertible even at $r=R$! (see prec. mcd)

$$p = \lambda(1-r/R)\left[I + rH + r^2H^2 + \cdots\right] = \lambda - r/R\,\lambda + (1-r/R)\lambda\, r\, H\,[I - rH]$$

$$p - \lambda = -r/R\,\lambda + r(1-r/R)\lambda H\left[I + rH + r^2H^2 + \cdots\right]$$

$$p_\gamma = \left[(1-r/R)\left(\lambda_\gamma + r\lambda H_\gamma + r^2\lambda(H^2)_\gamma + \cdots\right)\right]$$

$$p = \lambda\underline{(1-r/R)}\left[\left(\frac{1}{r/R} - RH\right)(r/R)\right]^{-1} = \frac{1}{R}\left(\frac{1-r/R}{r/R}\right)\lambda\cdot\left[\left(\frac{1}{r/R} - RH\right)\right]^{-1}$$

Suppose $x_{s_1}, x_{s_2} \equiv x_{s_3}\ldots$ = right hand eigenvectors of H, all of distinct eigenvalues R_1, R_2, R_3

let $\hat\lambda \equiv \alpha_1\hat p_{R_1} + \alpha_2\hat p_{R_2} + \alpha_3\hat p_{R_3}$ = linear comb of x_{s_k}

where $\frac{1}{R_k}p_{R_k} = p_{R_k}H$, $\frac{1}{R_k^2}p_{R_k} = p_{R_k}H^2$, etc

$\hat p_1 \equiv \hat\lambda$

$$\hat p_\gamma = \bar w\lambda_\gamma + \bar r\frac{H^*}{\lambda_\gamma}$$

$$\frac{p_\gamma}{\lambda_\gamma}$$

Since $p = (1-r/R)\cdot\lambda\left[I + rH + r^2H^2 + \cdots\right]$

~~$p = (1-r/R)\left[\alpha_1 p_{R_1} + \alpha_2 p_{R_2} +\right.$~~

$$p = (1-r/R)\left[\underbrace{\sum\alpha_k p_{R_k}}_{\lambda} + \underbrace{r\sum\frac{\alpha_k p_{R_k}}{R_k}}_{r\lambda H} + r^2\sum\frac{\alpha_k p_{R_k}}{R_k^2} + \cdots\right]$$

$$p = (1-r/R_1)\left[\alpha_1 p_{R_1}\left(1 + r/R_1 + (r/R_1)^2 + \cdots\right)\right.$$
$$\left. + \alpha_2 p_{R_2}\left(1 + r/R_2 + (r/R_2)^2 + \cdots\right) + \cdots\right]$$

$$p = (1-r/R_1)\left[\underbrace{(\alpha_1 p_{R_1} + \alpha_2 p_{R_2} + \alpha_3 p_{R_3})}_{\lambda} + \frac{\alpha_1(r/R_1)p_{R_1}}{1-r/R_1} + \frac{\alpha_2(r/R_2)p_{R_2}}{1-r/R_2} + \frac{\alpha_3(r/R_3)p_{R_3}}{1-r/R_3}\right]$$

~~$$\hat p = \alpha_1\hat p_{R_1} + \frac{\alpha_2(1-r/R_1)}{(1-r/R_2)}\hat p_{R_2} + \frac{\alpha_3(1-r/R_1)}{(1-r/R_3)}\hat p_{R_3}$$~~

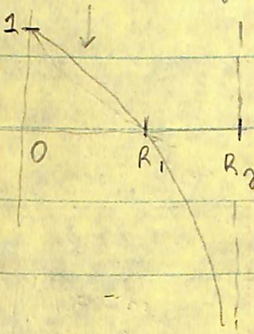
$$\hat p = (1-r/R_1)\left[\hat\lambda + \alpha_1\left(\frac{r}{R_1}\right)\hat p_{R_1} + \alpha_2\frac{(r/R_2)(1-r/R_1)}{(1-r/R_2)}\hat p_{R_2} + \alpha_3\frac{(r/R_3)(1-r/R_1)}{(1-r/R_3)}\hat p_{R_3}\right.$$

$\lambda_1\ \lambda_2\ \lambda_3 \ldots + (r/R_1)(p_{R_1}, p_{R_2}, p_{R_3}) +$

6/18/93

5. <u>Several properties follow</u> ~~from:~~ $\boxed{\hat{p}(r) = \hat{\lambda}\left[A_1 + (1-\tfrac{r}{R_1})\left(\frac{A_2}{1-r/R_2} + \frac{A_3}{1-r/R_3} + \ldots\right)\right]}$

$h_i(r) \equiv \frac{1-r/R_1}{1-r/R_g}$



$h_i'(r) = \frac{(1-r/R_2)(-\frac{1}{R_1}) - (1-r/R_1)(-\frac{1}{R_g})}{(1-r/R_g)^2}$

$= -\left[\frac{1}{R_1(1-r/R_g)} + \frac{1}{R_g(1-r/R_g)}\right]$

$h_i'(r) < 0$ for $r < R_g$

$h_i''(r) = + \frac{1}{R_1 R_g (1-r/R_g)^2} + \frac{1}{R_1 R_g (1-r/R_1)^2}$

$h_i''(r) > 0$

(i) At $r = 0$, $\hat{p}(r) = \hat{\lambda}\left[A_1 + A_2 + A_3 + \ldots\right] = \hat{\lambda}$ ✓

since $\Sigma A_i = I$

(ii) At $r = R_1$, $\hat{p}(R_1) = \hat{\lambda} A_1$ <u>alone</u>! (verified in MathCad, PRICE.MCD, 6/93)

— This is interesting, because $A_1 = \frac{(\frac{1}{R_2}I - H)(\frac{1}{R_3}I - H)\ldots}{(\frac{1}{R_2} - \frac{1}{R_1})(\frac{1}{R_3} - \frac{1}{R_1})\ldots}$

where $H = A(I-A)^{-1}$ (or, more generally, $H = K(I-A)^{-1}$)

note that R_i are determined by H itself.

Similarly, $\lambda = \ell(I-A)^{-1}$. It follows that

$\boxed{\hat{p}(R_1) = f(\hat{\ell}, H, A)}$ alone $\longrightarrow$ <u>$\hat{p}(R_1)$ depends on $\hat{\ell}$?!</u>

— We know from the original form of the equations $p(r) = w\lambda + r p(r) H$, that $p(R_1)$ is the eigenvector of H, and its <u>proportions</u> are determined by H alone. Since the two expressions for $\hat{p}(R_1)$ give the <u>same</u> proportions, the dependence of "standardized prices of production" $\hat{p}(r)$ on the labor coeff. vector ℓ <u>must</u> be due to the <u>scaling</u> peculiar to such standardized pop's.

(iii) If we postmultiply $\hat{p}(r)$ by A_1, then <u>since</u> $A_i A_g = 0$ for $i \neq g$, and since $A_i^m = A_i$ (79), it follows that

$\boxed{p(r) A_1} = \lambda A_1^2 = \boxed{\lambda A_1}$ (!) where A_1 is <u>singular</u> + hence <u>not</u> <u>invertible</u>.

But $\lambda A_1 = \hat{p}(R_1) \longrightarrow \boxed{\hat{p}(r) A_1 = \hat{p}(R_1)}$ <u>for?</u> all r

[Actually, given any ~~nonzero~~ vector y, $y A_1 = p(R_1)$]

(see p. 2A, next)

$$A_1 = \frac{\left(\frac{1}{R_2} I - H\right)\left(\frac{1}{R_3} I - H\right)}{\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\left(\frac{1}{R_3} - \frac{1}{R_1}\right)} = \frac{F_2 F_3}{h_{21} h_{31}}$$

$$A_2 = \frac{\left(\frac{1}{R_1} I - H\right)\left(\frac{1}{R_3} I - H\right)}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(\frac{1}{R_3} - \frac{1}{R_2}\right)} = \frac{F_1 F_3}{-h_{21} h_{32}}$$

$$A_3 = \frac{\left(\frac{1}{R_2} I - H\right)\left(\frac{1}{R_3} I - H\right)}{\left(\frac{1}{R_1} - \frac{1}{R_3}\right)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)} = \frac{F_2 \cdot F_3}{(-h_{31})(-h_{32})}$$

3. Several properties follow

(1) At $r = R_1$, $\boxed{\rho(R_1) = \lambda A_1}$, where $A_1 = (-1)^{n-1} \dfrac{\left[(\frac{1}{R_2}I - H)(\frac{1}{R_3}I - H)\cdots\right]}{(\frac{1}{R_2} - \frac{1}{R_1})(\frac{1}{R_3} - \frac{1}{R_1})\cdots}$

– Since $H = A(I-A)^{-1}$ only [or $K(I-A)^{-1}$ is more generally]

$A_1 = f(R_j, K, A)$ – only A, not

– Similarly, $\lambda = \ell(I-A)^{-1} = g(\ell, A)$ only – A[illegible]

$\rightarrow$ changing ℓ should change $\rho(R_1)$?!

1. $p(r) = \lambda(\frac{1}{r} - \frac{1}{R_1})(\frac{1}{r}I - H)^{-1} = \dfrac{\lambda F(\frac{1}{r})}{\underset{(+)}{(\frac{1}{r}-\frac{1}{R_2})}\underset{(+)}{(\frac{1}{r}-\frac{1}{R_3})}\cdots} = \dfrac{\lambda F(\frac{1}{r})}{h(r)}$

for $\boxed{r > 0}$

where $F(\frac{1}{r}) = H^{n-1} + (\frac{1}{r}+p_1)H^{n-2} + [(\frac{1}{r})^2 + (\frac{1}{r})p_1 + p_2]H^{n-3}$
$+ [(\frac{1}{r})^{n-1} + (\frac{1}{r})^{n-2}p_1 + \cdots + p_{n-1}]\cdot I$

where $H >> [0]$

2. $p(r) = \left[\dfrac{1}{(\frac{1}{r}-\frac{1}{R_2})(\frac{1}{r}-\frac{1}{R_3})\cdots}\right]\left[\underbrace{\lambda H^{n-1}}_{C^{n-1}} + (\frac{1}{r}+p_1)\lambda H^{n-2} + \cdots\right]$, $r \neq 0$

Since $H >> [0]$, we have positive coeff's $\lambda H^k_{(\gamma)} \equiv C_\gamma^k$

(1) $p_\gamma(r) = C_\gamma^{n-1} + (\frac{1}{r}+p_1)C_\gamma^{n-2} + [(\frac{1}{r})^2 + (\frac{1}{r})p_1 + p_2]C_\gamma^{n-3} + \cdots + [(\frac{1}{r})^{n-1} + (\frac{1}{r})^{n-2}p_1 + \cdots + p_{n-1}]\lambda_\gamma$

$\rightarrow \boxed{\dfrac{p_\gamma(r)}{\lambda_\gamma} - 1 = d_\gamma(r) = \dfrac{C_\gamma^{n-1}}{\lambda_\gamma} + (\frac{1}{r}+p_1)\dfrac{C_\gamma^{n-2}}{\lambda_\gamma} + [\sim]\dfrac{C_\gamma^{n-3}}{\lambda_\gamma} + [(\frac{1}{r})^{n-1} + (\frac{1}{r})^{n-2}p_1 + \cdots + (p_{n-1} - 1)]}$

and for any one price i to equal value, $d_\gamma^i(r) = 0$ is nec.

(ii) Is it true that $C_{\gamma_i}^{n-1} > C_\gamma^{n-2} > C_\gamma^{n-3} \cdots$? (Since $H^k > H^{k-1}$?)

What about p_γ, since all the roots $(\frac{1}{R_i}) > 0$?

$$f(r)^{-1} = (\tfrac{1}{r}I - H)^{-1} = \frac{\sum_{i=1}^{n} F(\frac{1}{R_i})}{\Delta'(\frac{1}{R_i})(\frac{1}{r} - \frac{1}{R_i})}$$

$$f(r) = (\tfrac{1}{r}I - H)$$

$$f(\lambda)^{-1} = \frac{F(\lambda)}{\Delta(\lambda)} = \frac{A_1}{\lambda - \lambda_1} + \frac{A_2}{\lambda - \lambda_2}$$

$$F(\tfrac{1}{R_i}) = (-1)^{n-1} \prod_{\gamma \neq i} f(\tfrac{1}{R_\gamma})$$

$$= \frac{A_1}{\lambda - \lambda_1} + \frac{A_2}{\lambda - \lambda_2}$$

3 X 3 $\quad F(\frac{1}{R_1}) = (+1)^n \left[f(\frac{1}{R_2}) \cdot f(\frac{1}{R_3})\right] = (\frac{1}{R_2}I - H)(\frac{1}{R_3}I - H)$.

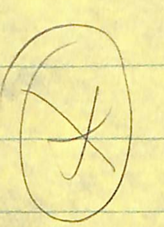
Note that since $H = A(I-A)^{-1} = A(I + A + A^2 + \cdots) = \gg 0$ ✓

~~[and diagonal element > 1 ??]~~

and $\left|\frac{1}{R_1}\right| > \left|\frac{1}{R_2}\right|, \left|\frac{1}{R_3}\right|$, etc.

[Thus if $R_1 = .5$, $\frac{1}{R_1} = 2$, $\frac{1}{R_2} = -1$, $\frac{1}{R_3} = 1.5$, etc., + $\boxed{\frac{1}{r} > \frac{1}{R_1}}$

If $H \equiv A(I-A)^{-1}$, then $H = A(I + A + A^2 + \cdots) \gg [0]$



But $(I - H)^{-1} = \left[I - A(I-A)^{-1}\right]^{-1} = \left[[(I-A) - A](I-A)^{-1}\right]^{-1} = (I-A)[I + 2A]^{-1}$

$(I + H)^{-1} = \left[I + A(I-A)^{-1}\right]^{-1} = \left(\{(I-A) + A\}(I-A)^{-1}\right)^{-1} = (I-A)A$

$R_1 =$

$R_2 = e_2$

$R_3 = e_3$

$$(\tfrac{1}{r}I - H)^{-1} = \frac{F(\frac{1}{r})}{(\frac{1}{r} - \frac{1}{R_1})(\frac{1}{r} - \frac{1}{R_2}) \cdots} = \frac{A_1}{\frac{1}{r} - \frac{1}{R_1}} + \frac{A_2}{\frac{1}{r} - \frac{1}{R_2}} + \frac{A_3}{\frac{1}{r} - \frac{1}{R_3}} + \cdots$$

$p(r) = \lambda A_1 + \lambda A_2 \frac{(\frac{1}{r} - \frac{1}{R_1})}{(\frac{1}{r} - \frac{1}{R_2})} + \lambda A_3 \cdots$

$p(r) = \lambda A_1 + \sum \lambda A_{k} \frac{(\frac{1}{r} - \frac{1}{R_1})}{(\frac{1}{r} - \frac{1}{R_k})}$

$$\boxed{p(r)} = \lambda \cdot (\tfrac{1}{r} - \tfrac{1}{R_1})(\tfrac{1}{r}I - H)^{-1} = \frac{\lambda \cdot F(1/r)}{\underset{(+)}{(\frac{1}{r} - \frac{1}{R_2})}\underset{(+)}{(\frac{1}{r} - \frac{1}{R_3})} \cdots}$$

where $\frac{1}{r} - \frac{1}{R_i} > 0$ for $i = 2, \ldots, N$ and $r \leq R_1$,

and $(\frac{1}{R_1} - \frac{1}{R_i}) > 0$ for $i = 2, \ldots, N$

$$F(\tfrac{1}{r}) = H^{n-1} + (\tfrac{1}{r} + p_1)H^{n-2} + \left[(\tfrac{1}{r})^2 + p_1(\tfrac{1}{r}) + p_2\right]H^{n-3} + \cdots + \left[(\tfrac{1}{r})^{n-1} + p_1(\tfrac{1}{r})^{n-2} + \cdots + p_{n-1}\right]I$$

where $\Delta(\frac{1}{r}) =$ ch. equation $= \left[(\frac{1}{r})^n + p_1(\frac{1}{r})^{n-1} + p_2(\frac{1}{r})^{n-2} + \cdots + p_n\right] = 0$

→ SINCE ALL ROOTS ARE POSITIVE (Roots $= \frac{1}{R_i}$, + Sraffa says all $R_i > 0$, and $0 < R_1 < R_2 < R_3 \cdots$) (Sraffa 30, 31)

we can say something about p_1's p_2, etc?

$$\hat{p}(r) = \hat{\lambda}(\tfrac{1}{r} - \tfrac{1}{R_1})(\tfrac{1}{r}I - H)^{-1} = \lambda H^{n-1} + (\tfrac{1}{r} + p_1)\lambda H^{n-2} + \left[(\tfrac{1}{r})^2 + (\tfrac{1}{r})p_1 + p_2\right]\lambda H^{n-3} + \cdots [\;\_\_\;]\lambda$$

and in the limit, $\hat{p}(R_1) = \frac{\lambda \cdot F(\frac{1}{R_1})}{\underset{+}{(\frac{1}{R_2} - \frac{1}{R_1})}\underset{+}{(\frac{1}{R_3} - \frac{1}{R_1})}} = \frac{\lambda \cdot (\frac{1}{R_2}I - H)(\frac{1}{R_3}I - H)}{(\frac{1}{R_2} - \frac{1}{R_1})(\frac{1}{R_3} - \frac{1}{R_1})} \; ?\; p$

works! MCD ✓

see proof

$$x^{(n-1)} + \left(\tfrac{1}{r} + p_1\right)x^{(n-2)} + \left[\left(\tfrac{1}{r}\right)^2 + \left(\tfrac{1}{r}\right)p_1 + p_2\right]x^{n-3} + \left[\left(\tfrac{1}{r}\right)^3 + \left(\tfrac{1}{r}\right)^2 p_1 + \left(\tfrac{1}{r}\right)p_2 + p_3\right]x^{n-4}$$

$$+ \dots + \left[\left(\tfrac{1}{r}\right)^{n-2} + \left(\tfrac{1}{r}\right)^{n-3} p_1 + \left(\tfrac{1}{r}\right)^{n-4} p_2 + \dots + p_{n-2}\right]x^{(2)} + \left[\left(\tfrac{1}{r}\right)^{n-1} + \left(\tfrac{1}{r}\right)^{n-2} p_1 + \left(\tfrac{1}{r}\right)^{n-3} p_2 + \left(\tfrac{1}{r}\right)^{n-4} p_3 + \dots + p_{n-1}\right]$$

$$\left(\tfrac{1}{r}\right)^{n-1} + \left(\tfrac{1}{r}\right)^{n-2}\left(p_1 + x^{(2)}\right) + \left(\tfrac{1}{r}\right)^{n-3}\left(p_2 + p_1 x^{(2)} + x^{(3)}\right) + \left(\tfrac{1}{r}\right)^{n-4}\left(p_3 + p_2 x^{(2)} + p_1 x^{(3)} + x^{(4)}\right) + \dots$$

Now, if $x^{(2)} >$

$p_0' = 1$	$p_1' = (p_1 + x^{(2)})$	$p_2' = p_2 + p_1 x^{(2)} + x^{(3)}$	$p_3' = p_2$	p_{n-1}'
$R_0 = p_0 = 1$	$R_1 = p_1 + x^{(2)}$	$R_2 = \dfrac{p_2' - p_0 p_3'}{p_1}$		

$$A_1 = x \cdot \left(\tfrac{1}{R_2} I - A(I + A)^{-1}\right)\left(\tfrac{1}{R_3} I - A(I + A)^{-1}\right)$$

6/13/93

1. $p = (1 - r/R)\lambda + rpH \cdot H \rightarrow \frac{1}{r} p = \left[\frac{(1-r/R)}{r/R}\right]\frac{1}{R}\lambda + pH$

$$\boxed{p = \left(\tfrac{1}{r} - \tfrac{1}{R}\right)\lambda\left[\left(\tfrac{1}{r}\right)I - H\right]^{-1}}$$, $\cancel{z(r) = \frac{1}{R}\left(\frac{1-r/R}{r/R}\right) > 0}$ for all $0 < r < R$, R = dominant eigenvalue

$= \lambda\left[\left(\frac{1}{r}I - H\right)/\left(\frac{1}{r} - \frac{1}{R}\right)\right]^{-1}$ $z(r) = \frac{1}{r} - \frac{1}{R} > 0$

2. $f(r) \equiv \left(\frac{1}{r}I - H\right)$ = characteristic matrix of H (64) [Frazer, Duncan, Collar, Elementary Matrices, p. 64)

= lambda - matrix (57)

(i) $f\left(\frac{1}{r}\right) \equiv \left(\frac{1}{r}I - H\right) = A_0\left(\frac{1}{r}\right) + A_1$ where $A_0 = I$, $A_1 = -H$ (57)

Thus $f\left(\frac{1}{r}\right)$ is of order n, degree 1, rank n (since $\left(I\frac{1}{r} - H\right)$ exists for $\frac{1}{r} < \frac{1}{R}$) (58)

(ii) $\left|f\left(\frac{1}{r}\right)\right| \equiv \det\left[\left(\frac{1}{r}I - H\right)\right]$

and the characteristic eq. $\left|f\left(\frac{1}{r}\right)\right| = 0 \Leftrightarrow \left|f\left(\frac{1}{r}\right)\right| = \left(\frac{1}{r} - \frac{1}{R_1}\right)\left(\frac{1}{r} - \frac{1}{R_2}\right) = 0$ (68)

(iii) Adjoint $F\left(\frac{1}{r}\right)$ has the property $F\left(\frac{1}{r}\right)f\left(\frac{1}{r}\right) = f\left(\frac{1}{r}\right)F\left(\frac{1}{r}\right) = \left|f\left(\frac{1}{r}\right)\right| \cdot I$ (61)

If ~~roots~~ are distinct, then for root $\frac{1}{R_i}$ & characteristic vectors

$\hat{p}(R_i)$ and $\hat{x}_{Si}$, $\boxed{\underset{n \times n}{F\left(\tfrac{1}{R_i}\right)} = \underset{(n \times 1)}{\hat{x}_{Si}} \cdot \underset{(1 \times n)}{\hat{p}_{Ri}}}$ (61)

also $\boxed{F\left(\tfrac{1}{R_i}\right) = (-1)^{n-1}\prod_{j \neq i} f\left(\tfrac{1}{R_j}\right)}$ (75)

(iv) let k = matrix formed by characteristic row vectors = $\begin{bmatrix} p_{R_{11}} & p_{R_{12}} & \\ p_{R_{21}} & p_{R_{22}} & \\ p_{R_{31}} & p_{R_{32}} & \end{bmatrix}$

k = " " " col. vector = $\begin{bmatrix} (x_{S_1})_1 & \\ (x_{S_1})_2 & \\ (x_{S_1})_3 & \end{bmatrix}$

Then since $H\hat{k}_i = \frac{1}{R_i}\hat{k}_i \rightarrow Hk = \left\langle\frac{1}{R_i}\right\rangle k$

Then $\boxed{k^{-1}Hk = \left\langle\frac{1}{R_i}\right\rangle}$ (66) $\rightarrow \boxed{H = k\left\langle\frac{1}{R_i}\right\rangle k^{-1}}$

and $k^{-1}H^2k = \left\langle\frac{1}{R_i^2}\right\rangle$ (66) $\rightarrow \boxed{H^n = k\left\langle\frac{1}{R_i^n}\right\rangle k^{-1}}$

and $kHk^{-1} = \left\langle\frac{1}{R_i}\right\rangle$, etc.

(v) $\boxed{\left(\frac{1}{R_1}I - H\right)\left(\frac{1}{R_2}I - H\right)\left(\frac{1}{R_3}I - H\right) \cdots = [0]}$ [$f(\lambda)$ satisfies its own characteristic eq.] (70)

(*) (vi) $\left(\frac{1}{r}I - H\right)^{-1} = \frac{F\left(\frac{1}{r}\right)}{\left|f\left(\frac{1}{r}\right)\right|} = \frac{F\left(\frac{1}{r}\right)}{\left(\frac{1}{r} - \frac{1}{R_1}\right)\left(\frac{1}{r} - \frac{1}{R_2}\right)\cdots} = \sum_{i=1}^{n} \frac{F\left(\frac{1}{R_i}\right)}{\Delta^{(1)}\left(\frac{1}{R_i}\right) \cdot \left(\frac{1}{r} - \frac{1}{R_i}\right)}$ (78)

where $\left(\frac{1}{r} - \frac{1}{R_1}\right) \geq 0$ for $\leq R$

and $\left(\frac{1}{R_1} - \frac{1}{R_i}\right) > 0$, so $\left(\frac{1}{r} - \frac{1}{R_i}\right) > 0$ for $r < R$ (where $\Delta^{(1)}(\lambda_i)$ = first derivative of $\left|f\left(\frac{1}{r}\right)\right|$, evaluated at $\frac{1}{r} = \frac{1}{R_i}$

$\leftrightarrow \quad P(R_2) + P(R_3) = \lambda - P(R_1)$

$x_2 P(R_2) + x_3 P(R_3) =$

$x_2 = \dfrac{1 - R_1/R_2}{1 - \cancel{r}/R_2} \qquad x_3 = \dfrac{1 - R_1/R_3}{1 - r/R_3}$

$= \dfrac{R_2 - R_1}{R_2 - r} \qquad = \dfrac{R_3 - R_1}{R_3 - r}$

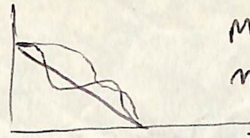
Suppose $R_2 = -a + bi$, $R_3 = -a - bi$ $\Big\}$ $R_2 + R_3 = -2a$, $R_2 R_3 = a^2 + b^2$, $R_2 - R_3 = 2bi$

$x_2 = \dfrac{-a + bi - R_1}{-a + bi - r} \qquad x_3 = \dfrac{-a - bi - R_1}{-a - bi - r}$

$x_2 - x_3 = \dfrac{(R_2 - R_1)(R_3 - r) - (R_3 - R_1)(R_2 - r)}{(R_2 - r)(R_3 - r)}$

$= \cancel{R_2 R_3} - R_1 R_3 - rR_2 + \cancel{rR_1} - \cancel{R_2 R_3} + R_1 R_2 + rR_3 - \cancel{rR_1}$

$= R_1 (R_2 - R_3)$



$\rightarrow d^* = 0$

Marx-reswitching amounts to multiple solutions to $\left(P/P_S\right)_\gamma = \lambda_\gamma$

$$p = w\lambda + rpH \quad ; \quad w^* \equiv w/p_S = 1 - r/R$$

$$p^* \equiv \frac{p}{p_S} = w^*\lambda + rp^*H = (1 - r/R)\lambda + rp^*H$$

Define $\delta^* \equiv \left[\dfrac{\lambda_\gamma - p_\gamma^*}{p_\gamma^*}\right] = \left[\dfrac{\lambda_\gamma}{p_\gamma^*} - 1\right] = \left[\lambda_\gamma^* - 1\right]$

$$p^*\langle p_\gamma^*\rangle^{-1} = 1 = (1 - r/R)\lambda\langle p_\gamma^*\rangle^{-1} + r\underbrace{1\langle p_\gamma^*\rangle H\langle p_\gamma^*\rangle^{-1}}_{}$$

$$1 = (1 - r/R)\lambda^* + r1H^*_{(r)}$$

$$\lambda^* - 1 \equiv \delta^* = r/R\,\lambda^* - r1H^*$$

$$\delta^* = r/R(\lambda^* - 1) + r/R\,1 - r1H^*$$

$$\delta^* = r/R\,\delta^* + \frac{r}{R}1(I - RH^*_{(r)})$$

$$\boxed{\delta^* = \frac{r/R}{(1 - r/R)}1\left[I - RH^*_{(r)}\right]}$$

where $H^* = \left[\dfrac{p_{i(r)}^* H_{i\gamma}}{p_{\gamma(r)}^*}\right] = \left[K_{i\gamma(r)}^{T*}\right]$

This can be used for approx & for iterative solutions

$$\boxed{\delta^* = \left(\frac{r/R}{1 - r/R}\right)\left(\hat{1} - \frac{\hat{k}^*_{(r)}}{R}\right)}$$ where $k_\gamma^{*(r)} \equiv \dfrac{K_\gamma^{T*}(r)}{K_S^{T*}}$

and $K_S^{T*} = \dfrac{1}{R}$

If $\delta_\gamma^* = 0 \rightarrow k_\gamma^*(r) = 1 \rightarrow K_\gamma^{*T}(r) = \frac{1}{R} \rightarrow \dfrac{K_\gamma^{*T}(r)}{C_\gamma^{T}(r)} \equiv \dfrac{p^*_{K_\gamma^T}}{\lambda^*_{K_\gamma^T}} = \dfrac{C_S^T}{C_\gamma^T} \rightarrow \delta_{K_\gamma^{*T}} \equiv \dfrac{\lambda_{K_\gamma^{*T}}}{p^*_{K_\gamma^*}} - 1 = \dfrac{C_\gamma^T}{C_S^T} - 1$

So value-price ~~price-value~~ deviations will switch signs

$= \left(\frac{r/R}{1-r/R}\right)\left(1 - K_{K_\gamma}^{*T}/C_S^T\right) \rightarrow \left(\frac{C_\gamma^T}{C_S^T} - 1\right)(1 - r/R) = \left(1 - \frac{K_K^{T*}}{C_S^T}\right)(r/R)$

as $\frac{r}{R}$ goes from 0 to 1 only if the vertically

Sraffa?

integrated capital output ratios switch magnitudes w/r to the standard one.

– H is strictly positive? $p^*_\gamma(r/R) > \phi$ also

H^* is $\langle p_\gamma\rangle H\langle p_\gamma^*\rangle^{-1}$ and $1/R$ = eigenvalue of $H \equiv K_S^{T*}$

$$\begin{array}{r} 89 \\ 89 \\ \hline 801 \\ 712x \\ \hline 7921 \end{array}$$

$$p = (pA + wl)(1+r) = pA + wl + rpK$$

$$p = w\lambda + rpH \quad , \quad H = K[I-A]^{-1}$$
$$\lambda = l[I-A]^{-1}$$

$$w^* = 1 - r/R \equiv \frac{w}{p_s}$$

$$\cancel{p_\gamma = w\lambda_\gamma + rpH^{(\gamma)}}$$

$$\cancel{\frac{p_\gamma}{p_s} = p^*_\gamma = \left(\frac{w}{p_s}\right)\lambda_\gamma}$$

$$\boxed{p^*} \equiv \frac{p}{w^*} \equiv \left[\frac{p_\gamma}{p_s}\right] = \boxed{(1-r/R)\lambda + rp^* H}$$

$$(p^* - \lambda) = -r/R\,\lambda + r(p^* - \lambda)H + r\lambda H$$

$$\cancel{d^* \equiv (p^* - \lambda)\langle\lambda_\gamma\rangle^{-1} = -r/R\,1 + r\,d^* H_v + r\,1 H_v}$$

$$\boxed{d^* = \frac{r}{R}1(RH_v - I) + \frac{r}{R}d^* H_v}$$

$$= \cancel{\ldots}$$

$$\delta^*_\gamma \equiv \frac{\lambda_\gamma - p^*_\gamma}{p^*_\gamma} = \ldots$$

$$p^*\langle p^*_\gamma\rangle^{-1} = (1-r/R)\lambda\langle p^*_\gamma\rangle^{-1}$$

$$+ rp^*\langle p^*_\gamma\rangle^{-1}\underbrace{\langle p^*_\gamma\rangle H\langle p^*_\gamma\rangle^{-1}}_{H^*}$$

$$\lambda\langle p^*_\gamma\rangle^{-1} - 1 = \frac{r}{R}\lambda\langle p^*_\gamma\rangle^{-1} - r/R\,1$$

$$- r1H^* + r/R\,1$$

$$\delta^* = \frac{r}{R}\delta^* + r1\left(\tfrac{1}{R}I - H\right)$$

$$d^* = x1(RH_v - I)(I - xRH_v)^{-1} \quad , \quad x \equiv r/R$$

$$d^* = [1(RH_v - I)]\cdot\left[\left(\frac{1}{x}I - RH_v\right)^{-1}\right]$$

"Basic deviations vector", indep. of r/R, with +/- components

"distribution effect" term

DO IN MCAD

$$A \to H \to (H - \tfrac{1}{R}I)\cdot(\tfrac{1}{r}I - H)^{-1}$$

Note that if $d^*_\gamma = 0$ is ~~...~~ excluded for any ~~...~~ $0 < r < R$, then no Marx-reswitching. → any ONE element

Also, as ~~...~~ $r \to R$, $(H - \frac{1}{R}I)(\frac{1}{r}I - H)^{-1} \longrightarrow$??

Restrictions on Vertically Integrated DCC

12/3/85

I

1. $\hat{p} = w\hat{\lambda} + r\hat{p}H$

$\rightarrow p_\gamma = w\lambda_\gamma + rH_\gamma$

$\rightarrow$ ① $\boxed{\frac{w}{(p_\gamma/\lambda_\gamma)} = 1 - rh_\gamma}$ where $h_\gamma \equiv \frac{H_\gamma}{p_\gamma} = \frac{\hat{p}He_\gamma}{\hat{p}e_\gamma}$

2. we know from other considerations that

$1 \geq \frac{w}{(p_\gamma/\lambda_\gamma)} \geq 0$ as $0 \leq r \leq R$

$\Rightarrow 1 \geq 1 - rh_\gamma \geq 0 \Rightarrow 0 \leq rh_\gamma \leq 1$ as $0 \leq r \leq R$

$\therefore$ ① $\boxed{h_\gamma(r) < \frac{1}{r}}$ for $0 < r < R$

② $\boxed{h_\gamma(R) = \frac{1}{R}}$ at $r = R$

②' $\boxed{h_\gamma(0) = h_{0\gamma} = (C^T/\lambda)_\gamma}$ at $r = 0$

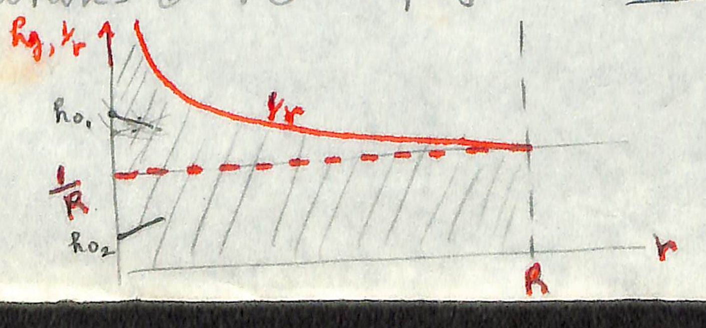
Finally, $d(rh_\gamma) > 0$ for $0 \leq r \leq R$ [since $d\left(\frac{w}{(p_\gamma/\lambda_\gamma)}\right) < 0$ $\rightarrow -d(rh_\gamma) < 0$]

$\rightarrow \frac{dr}{dr}h_\gamma + r\frac{dh_\gamma}{dr} > 0 \rightarrow \frac{dh_\gamma}{dr} > -\left(\frac{h_\gamma}{r}\right)$

③ $\boxed{\frac{dh_\gamma}{dr} > -\frac{h_\gamma}{r}}$ for $0 \leq r \leq R$

* ③' $\boxed{\frac{dh_\gamma}{dr}\frac{r}{h_\gamma} > -\frac{1}{r}}$ $\Rightarrow$ what family of curves have elasticity $= -\frac{1}{r}$?

3. Conditions ① + ② imply that h_γ must be in the shaded space below



II 1. Constraint is $\frac{dh_\gamma}{dr}\frac{1}{h_\gamma} > -1/r$ — this is not elasticity 2

12/3/85

But $-\frac{1}{r}$ = elasticity of curve $f(r) = \frac{1}{r}$

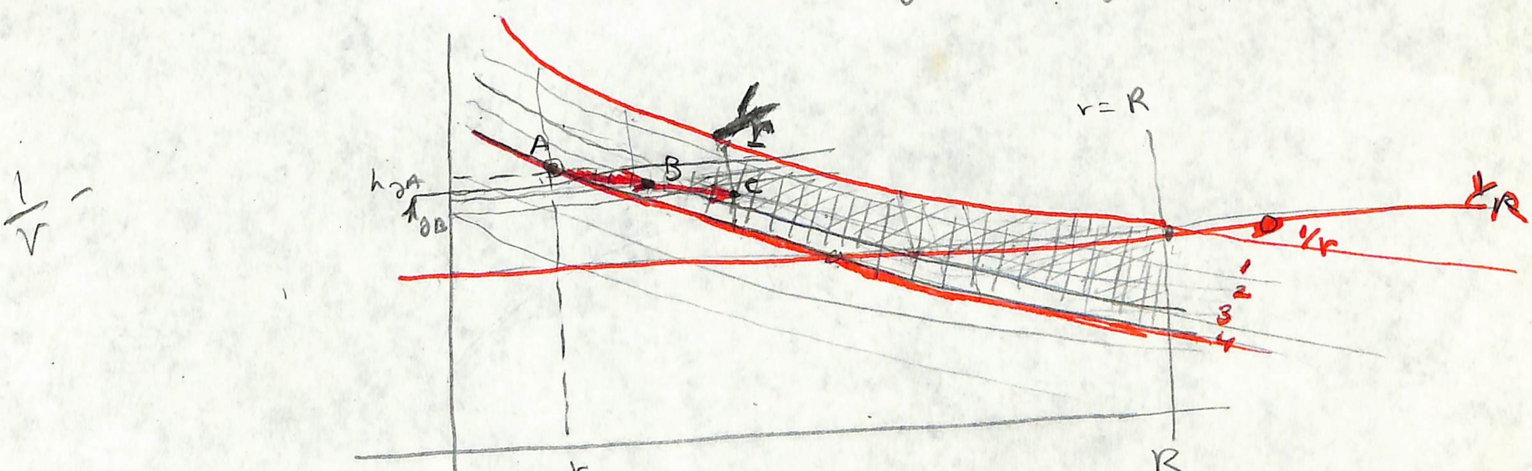
since $\frac{df(r)}{dr} = -\frac{1}{r^2}$, $\frac{df(r)}{dr}\frac{1}{f(r)} = -\frac{1}{r}$

∴ Constraint is $\cancel{\varepsilon_{h_\gamma}}\ \frac{dh_\gamma}{dr}\frac{1}{h_\gamma} > \varepsilon_{1/r}$

2. Note that ~~Any~~ curve parallel to 1/r has same elasticity
→ family of isoclines

(i) h_γ must be below 1/r

(ii) We are concerned only with falling ~~h_γ's~~ ranges of h_γ's



3 (a) At point A, if h_γ is to fall, it must be ~~between~~ within hatched space — i.e. between isocline 4 and 1/r and r = R and $h_\gamma = h_{\gamma A}$

(ii) Now, if h_γ moves to any point B in this space, then it must move to a higher isocline 3. This then defines a new, narrower feasible space (double hatched)

But from B, any feasible movement is again towards line 1/r, since any ~~movement~~ point between 1/r and an ~~parallel~~ isoclines moves towards 1/r!

→ CANNOT CROSS OVER LINE of 1/R!

12/3/85

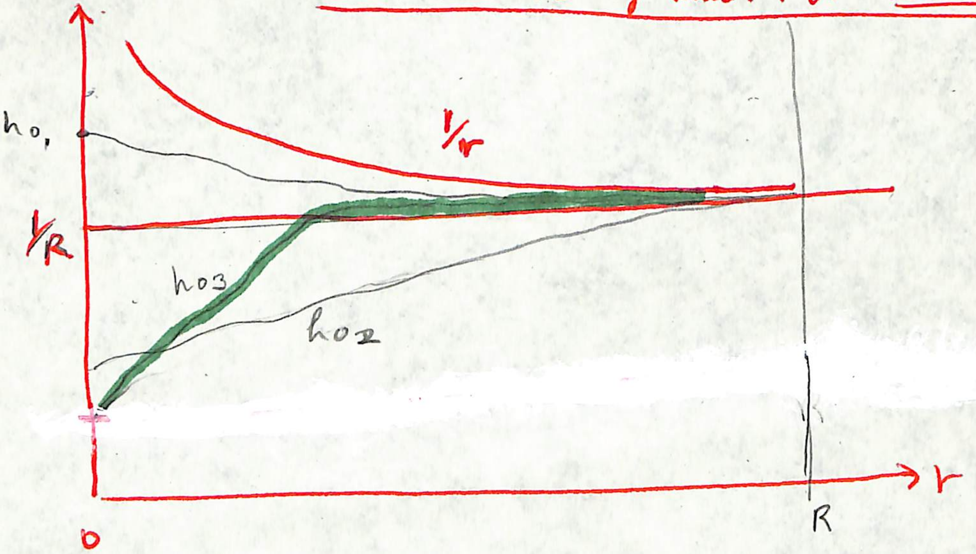
III

X numerical example 6/30/93 (Mathcad) has $h_\gamma = .873 > h_s = .773$ + still get one crossing

1. It follows from the above proof that if $h_{\gamma n}$ starts above $1/R$, it cannot cross over; and if it ~~starts~~ starts below $1/R$, it can cross over at most once!

X

This is because h_γ must move towards $1/r$ at all times.



??

2. It further follows from this that ~~we~~ the γ^{th} wage profit curve can cross over at most once, and that only from above to below? (or vice versa?)

— because if $h(0)_\gamma < 1/R$, then initially price is ~~[illegible]~~ ...

3. ~~This raises an interesting possibility: there ~~exists~~ an isocline which is tangent to the peak of the crossover prices. This isocline then defines an implicit ~~[illegible]~~ commodity which~~

* 3. Note that the above result does not preclude two commodity ~~[illegible]~~ vertically-integrated price OCC's from ~~intersecting~~ intersecting since only $1/r$ is the ceiling for a rise in h_γ. Thus reswitching is perfectly compatible with NO (or only few) price-value crossovers

Sheetz

This file contains examples from Chapter IV of Ed Ochoa's dissertation : Labor-Values and Prices of Production: An interindustry Study of the U.S. Economy, 1947-1972. Equation numbers correspond to those in Chapter IV.

✓ $A := \begin{pmatrix} .265 & .968 & .00681 \\ .0121 & .391 & .0169 \\ .0408 & .808 & .165 \end{pmatrix}$ $I := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

✓ $D := \begin{pmatrix} 0 & 0 & 0 \\ .00568 & .0267 & .0028 \\ .00265 & .0147 & .00246 \end{pmatrix}$

l ✓ $a_0 := (\,.193 \quad 3.562 \quad .616\,)$

x $q := (\,26530 \quad 18168 \quad 73840\,)$

P_m $m := (\,4 \quad 60 \quad 7\,)$

Equation 4 is given:

✓ (4) $v := a_0 \cdot (I - (A + D))^{-1}$ ✓ $v = (\,0.519 \quad 8.309 \quad 0.941\,)$

As Ochoa notes, the values above are not commensurable with market prices because the former are expressed in units of unskilled labor time and the latter in dollars. So the following normalization procedure is employed.

(7) $\kappa := \dfrac{\sum \overrightarrow{(m \cdot q)}}{\sum \overrightarrow{(v \cdot q)}}$ $\kappa = 7.314$

Program note: The normalization procedure requires vector multiplication elementwise, and the summation of those values. To achieve this in Mathcad, utilize the "vectorize" operator **[CTRL]-** and the "Sum" operator **[CTRL]4**

Labor-values can be normalized to Direct-prices as follows:

(12) $d := \kappa \cdot v$ $d = (\,3.798 \quad 60.778 \quad 6.881\,)$

The MAWD is calculated as follows:

(13) $\text{MAWD} := \left(\sum \overrightarrow{(m \cdot q)}\right)^{-1} \cdot \sum \overrightarrow{\left[\left(\left|m^T - d^T\right|\right) \cdot q^T\right]}$

(14) $\text{MAWD} \cdot 100 = 1.651$

Program note: Mathcad does not return the absolute value of a row vector. In **(13)** we have transposed the row vectors to column vectors, carried out the element manipulations and summed the results. Their are alternate methods available to arrive at the same results.

The MAD is calcultaed as follows:

(15) $$\mathrm{MAD} := \left(\frac{1}{3}\right)\cdot\left[\sum\left(\overrightarrow{\frac{\left|m^{T} - d^{T}\right|}{m^{T}}}\right)\right]$$

(16) $$\mathrm{MAD}\cdot 100 = 2.684$$

Marxian Prices of Production

(25) $b := (.0109 \quad .0275 \quad .296)$ b(1000 units/worker-year)

$$K := \begin{pmatrix} 0 & 0 & 0 \\ .120 & .791 & .096 \\ .037 & .251 & .043 \end{pmatrix}$$ Capital Stock

$$Z := \xcancel{\begin{pmatrix} .170 & 1.34 & .086 \\ .341 & 3.91 & .379 \\ .217 & 3.00 & .340 \end{pmatrix}}$$

$$t := \begin{pmatrix} .317 & 0 & 0 \\ 0 & .099 & 0 \\ 0 & 0 & .187 \end{pmatrix}$$ Turnover time

The solution of interest is $[(1/\pi)\,(p_1\ p_2\ p_3) = (p_1\ p_2\ p_3)\,(Z)]$ where Z is a matrix given below. The rate of profit is given as $1/\pi$ (maximum eigenvalue). $(p_1\ p_2\ p_3)$ is obtained by the eigenvector corresponding to the maximum eigenvalue,

$$Z := \left[K + \left(A + b^T \cdot a_0\right)\cdot t\right]\cdot\left(I - A - b^T \cdot a_0 - D\cancel{Z}\right)^{-1} = \begin{pmatrix} 0.143 & 0.678 & 0.045 \\ 0.349 & 3.963 & 0.388 \\ 0.217 & 2.573 & 0.316 \end{pmatrix}$$

$$\text{eigenvals}(Z) = \begin{pmatrix} 4.278 \\ 0.092 \\ 0.051 \end{pmatrix} \qquad \text{eigenvec}(Z, 4.278) = \begin{pmatrix} 0.141 \\ 0.827 \\ 0.545 \end{pmatrix} \qquad \pi := \max(\text{eigenvals}(Z))$$

$$\rho := \frac{1}{\pi} \qquad p := \text{eigenvec}(Z, 4.278)^T$$

$\rho \cdot 100 = 23.375$ $p = (0.141 \quad 0.827 \quad 0.545)$ Compared to (37) in Ochoa: 23.2% p = (.221 2.53 .246)

Because Mathcad normalizes the corresponding eigenvector to unit length the result given for the price vector is inconsistant. Purely on a hunch we redefined p by post multiplying it by Z (thinking it would re-weight each term to its proper relative magnitude?) p_o is Ochoa's result.

$p := p \cdot Z$

$p = (0.427 \quad 4.773 \quad 0.499)$ $p_o := (.221 \quad 2.53 \quad .246)$

$\frac{.427}{4.773} = 0.089$

$\frac{.221}{2.53} = 0.087$

We computed price ratios for the first two prices in each set and came up with similar ratios. The discrepency we feel is attributable to rounding error between our results and Ochoa's results. Note the difference between Z and Z_o (Ochoa's result). We then proceeded to normalize p and came up with similar results to Ochoa's.

$$\kappa := \frac{\sum\overrightarrow{(m\cdot q)}}{\sum\overrightarrow{(p\cdot q)}} \qquad \kappa = 12.699$$

$$p := \kappa\cdot p \qquad p = (\ 5.427 \quad 60.615 \quad 6.336\ )$$

$$MAWD := \left(\overrightarrow{\left(\sum(m\cdot q)\right)}\right)^{-1}\cdot\sum\overrightarrow{\left[\left(\left|m^T - p^T\right|\right)\cdot q^T\right]}$$

$$MAWD\cdot 100 = 5.724$$

$$MAD := \left(\frac{1}{3}\right)\cdot\left[\sum\left(\overrightarrow{\frac{\left|m^T - p^T\right|}{m^T}}\right)\right]$$

$$MAD\cdot 100 = 15.391$$

The MAWD (5.72%) and MAD (15.4%) compare to Ochoa's MAWD (8.44%) and MAD (17.5%), eq. (39) p. 60.

$$L^* = V^* + S^*$$
$$pY^* = \omega^* L^* + \pi^*$$

Since $pY^* = \lambda Y^* = L^*$

$$V^* - \omega L^* = \Pi^* - S^*$$
$$v - \omega^* = \pi^* - s$$

By definition $k^T = pH \rightarrow [k_i^T / p_i] = pH \langle p_i \rangle^{-1} = 1 \langle p_i \rangle H \langle p_i \rangle$

and $\frac{1}{R}$ is between biggest & smallest $\frac{k_i^T}{p_i}$ (column sums)

for any p_i $\rightarrow$ hence also between biggest & smallest c_i^T / λ_i

(since $\frac{1}{R}$ is a convex comb. of $\frac{c_i^T}{\lambda_i}$ or $\frac{k_i^T}{\lambda_i}$, because $k^T Y^* = pY^* = \lambda Y^*$

$$p = (1-s)\lambda + r p H \rightarrow pH = (1-s)\lambda H + r p H^2 \quad \text{[Hyperintegration]}$$

$$pH_i = k_i^T = (1-s) C_i^T + r k_i^{T2}$$

$$k_i^T \equiv \frac{k_i^T}{p_i} = (1-s)\frac{C_i^T}{\lambda_i}\left(\frac{\lambda_i}{p_i}\right) + r k_i^{T2} = (1-s) k_i^T(0)\left[\frac{1 - (s k_i^T/p_i)/(k^*/p^*)}{1-s}\right] + r k_i^{T2}$$

$$k_i^T = k_i^T(0)\left[1 - sRk_i^T\right] + \overbrace{\left(\frac{r}{R}\right)}^{s} R k_i^{T2} = k_i^T(0) - sR k_i^T k_i^T(0) + sR k_i^{T2}$$

$$k_i^T = \frac{k_i^T(0) + sR k_i^{T2}}{(1+sR)}$$

$\rightarrow k_i^T(0) = k_i^T(0)$ ✓

$\rightarrow$ $\boxed{k_i^T(1) = \frac{1}{R} = \frac{k_i^T(0) + R k_i^{T2}(1)}{1+R}}$

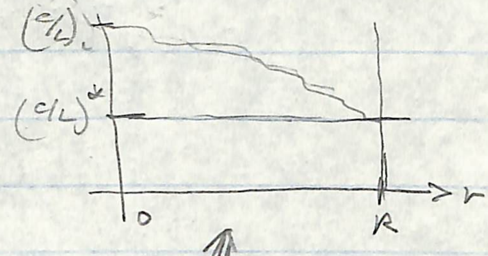
$$\rightarrow \left(\frac{1}{R} + 1\right) - k_i^T(0) = R k_i^{T2}(R)$$

$$k_i^{T2}(R) = \left[\frac{k^{T*} - k_i^T(0)}{R}\right] + \frac{1}{R}$$

$$X^* = (1+R)AX^*$$
$$y^* = RAX^*$$

At $r=0$ $\quad \left(\frac{K}{L}\right)_i = \left(\frac{C}{L}\right)_i, \quad \left(\frac{K}{Y_i}\right) = \left(\frac{C}{L}\right)_i$

At $r=R$ $\quad \left(\frac{K}{L}\right)_i = ?, \quad \left(\frac{K}{Y}\right)_i = \left(\frac{C}{L}\right)^* = \left(\frac{K}{Y}\right)^*$



IF, at any $0 < r < R$, $\left(\frac{K}{Y}\right)_i$ is always between $\left(\frac{C}{L}\right)_i$ and $\left(\frac{C}{L}\right)^*$ [no more reswitching]

$p_i \equiv Y_i + K_i = Y_i\left[1 + \frac{K_i}{Y_i}\right]$. So, is p_i between

Y_i is ~~not~~ a variable $Y_i(\pi_i)$ $\quad p_{i1} \equiv Y_i\left[1 + \frac{C_i}{L_i}\right]$ and $p_{i2} \equiv Y_i\left[1 + \left(\frac{C}{L}\right)^*\right]$?

$\rightarrow$ IF K

$\rightarrow$ ~~Increasing~~ Since $Y_i \equiv wL_i + rK_i + K_i = wL_i + (1+r)K_i$,

~~increasing $\frac{K_i}{Y_i}$ or lowering it is equivalent~~ moving from p_i to p_{i1} or p_{i2} is equivalent to lowering or raising profit margin π_i ? If so, then we can show that $(C/L)_i$ and $(C/L)^*$ form bounds, from results of 9/11/94

$$p_i = K_i + wL_i + \pi_i = Y_i\left(1 + \frac{K_i}{Y_i}\right) = wL_i\left(1 + \frac{K_i}{Y_i}\right) + \pi_i\left(1 + K_i/Y_i\right)$$
$$p_i = wL_i + K_i(1+r_i)$$

$$\boxed{p = w\lambda + rk^T \overset{?}{=} v^*\lambda + rk^T = (v^* + s^*)\lambda + rk^T - s\lambda = \lambda + rk^T - s\lambda}$$

$$p_i = \lambda_i + \frac{r}{R}(k_i^T R) - (s\lambda_i) = \lambda_i + r/R\,\lambda_i\left(\frac{k_i^T/\lambda_i - s}{k^*/\lambda^*}\right) = \lambda_i\left[1 + r/R\left(\frac{k_i^T/\lambda_i - s}{k^*/\lambda^*}\right)\right]$$

$$\cancel{R = k^T/p^* = k^T/\lambda^*}$$

$$1 = \frac{\lambda_i}{p_i} + r/R\left(\frac{k_i^T/p_i}{k^*/p^*}\right) - \left(s\frac{\lambda_i}{p_i}\right)$$

$$(1-s)\frac{\lambda_i}{p_i} = 1 - r/R\left(\frac{k_i^T/p_i}{k^*/p^*}\right) \rightarrow \frac{\lambda_i}{p_i} = \frac{1 - (r/R)\left[\frac{k_i^T/p_i}{k^*/p^*}\right]}{v}$$

where $(k_i^T/p_i)_{s=0} = \left(\frac{c_i^T}{\lambda_i}\right)$

Since $r/R = \frac{\Pi^*/K^*}{Y^*/K^*} = \frac{\Pi^*}{Y^*} \overset{?}{=} s \rightarrow \frac{\lambda_i}{p_i} = \frac{1 - s\left[\frac{k_i^T/p_i}{k^*/p^*}\right]}{1-s}$

$(k_i^T/p_i)_{s=1} = k^*/p^*$

$$\frac{\lambda_i}{p_i} = \frac{1 - s - s[\sim - 1]}{1-s} = 1 - \left(\frac{s}{v}\right)\left[\frac{k_i^T/p_i}{k^*/p^*} - 1\right]$$

Determinants of Price-Value Deviations in Marx

11/18/73

1. In Marx's procedure, $\boxed{P_i^{(1)} = (C_i+V_i) + \rho C_i}$

$$P_i^{(1)} = (C_i+V_i) + \frac{S}{C} C_i = (C_i+V_i+S_i) + sL\frac{C_i}{C} - S_i$$

$$P_i^{(1)} = \Lambda_i + sL\frac{C_i}{C} - sL_i = \Lambda_i + sL_i\left(\frac{C_i/L_i}{C/L} - 1\right)$$

$$\boxed{(P_i^{(1)} - \Lambda_i) = sL_i\left(\frac{C_i/L_i}{C/L} - 1\right) = sL_i\left(\frac{C_i/V_i}{C/V} - 1\right)}$$

(i) So, even here, the absolute price-value deviations ~~will be~~ $P_i^{(1)} - \Lambda_i$ will be correlated with % organic composition deviations $\frac{C_i}{V_i} / \frac{C}{V}$ only if the employment L_i is the same in each sector

~~and~~ $\frac{C_i}{V_i}$ are equivalent to $\frac{C_i}{S_i}$ and $\frac{C_i}{L_i}$ only if $\left(\frac{S}{V}\right)_i = (S/V)_j = \left(\frac{\bar{S}}{V}\right)$

$$\frac{P_i^{(1)} - \Lambda_i}{\Lambda_i} = \text{\% Price-value Deviations}$$

$$= \frac{sL_i}{\Lambda_i}\left(\frac{C_i/L_i}{C/L} - 1\right)$$

$$\frac{P_i^{(1)} - \Lambda_i}{\Lambda_i} = \frac{s}{\left(\frac{C_i}{L_i}\right)+1}\left(\frac{C_i/L_i}{C/V} - 1\right)$$

(ii) So % price-value deviations depend on % organic (or value) composition deviations and the ratio $\frac{L_i}{\Lambda_i} = \frac{L_i}{C_i+L_i}$

2. Note that here the relevant "organic compositions" are Stocks of const. capital to flow of living labor or flow of variable capital — NOT VARIABLE CAPITAL ADVANCED

Determinants of Price-Value Deviation in Marx 11/18/87

2.

(1) We found that $\dfrac{C_i/V_i}{C/V}$ is an important determinant of price-value deviations. But note that here

$$C_i = \bar{C}_{f_i} + \frac{1}{n_i}(C_{m_i} + V_i) = \text{STOCK OF CAP ADVANCED} \quad \cancel{\text{and } V_i = vL_i = \text{flo}}$$

$$C_i = \bar{C}_{f_i} + \bar{C}_{m_i} + \bar{V}_i$$

But $V_i = vL_i =$ FLOW of variable capital used up

Thus
$$C_i/V_i = \frac{\bar{C}_{f_i} + \bar{C}_{m_i} + \bar{V}_i}{V_i} = \left(\frac{\bar{C}_{f_i} + \frac{1}{n_i}C_{m_i}}{V_i}\right) + \frac{1}{n_i}$$

That is, the relevant organic composition here is really
$$\frac{\bar{C}_{f_i} + \bar{C}_{m_i}}{V_i} = \frac{\text{Stock of const cap}}{\text{Flow of variable capital}}$$

and not
$$\frac{C_i}{\bar{V}_i} = \left(\frac{C_i}{V_i \frac{1}{n_i}}\right) = \left(\frac{C_i}{V_i}\right) n_i$$

(*) Impossibility of "Marx-Reswitching" 6/6/93

j^{th} column of H

1. Initially, $w = 0$, $r = R$ $\rightarrow$ (1) $\boxed{p_j^*(R) = R H_j^*(R)}$, $H_j^*(R) \equiv p^*(R) H_j$

2. Now let std wage ~~share~~ $w > 0$, $w = 1 - r/R$, but hold prices constant

$\rightarrow$ (2) $\boxed{p_j^*(R) = w\lambda_j + (R H_j^*(R) - w\lambda_j)}$ =

$\rightarrow$ ~~ex~~ interim profit $\Pi_j' \equiv R H_j(R) - w\lambda_j$

" rop $r_j' = \frac{\Pi_j'}{H_j^*(R)} = R - w\frac{\lambda_j}{H_j^*(R)} = \frac{1}{\ldots} - \frac{w}{\left(\frac{H_j^*(R)}{\lambda_j}\right)}$

Clearly, r_j' will vary across industries as long as $\lambda_j / H_j^*(R)$ varies, & this latter ratio is independent of unit (since both H_j^* & λ_j change)

$\rightarrow$ so prices, & hence p_j^* & H_j^*, must change [since $H_j^* \equiv \hat{p}^* \cdot H_j$]

(If $\frac{H_j^*(R)}{\lambda_j} = \frac{1}{R}$, then $r_j' = R(1-w) = r$, so $p_j^*(R)$ is the appropriate price & prices ~~do~~ not have to change)

3. Now assume new appropriate prices have been established, so as to make $r_j = r$, at same given std. wage share

If $\frac{H_j^*(R)}{\lambda_j} > \frac{1}{R}$ then $r_j < r$

(3) $\boxed{p_j^*(r) = w\lambda_j + r H_j(r)}$ $\rightarrow$ suppose $p_j^*(r) = \lambda_j$. Then since $w = 1 - r/R$, $\lambda_j = (1 - r/R)\lambda_j + r H_j^*$

$p_j^*(r) = \lambda_j + r H_j^*(r) - \frac{r}{R}\lambda_j = \lambda_j + r \ldots/\lambda_j$

~~Then $[p_j(r) - p_j(R)] = w\lambda_j + [r H_j(r) - \ldots]$~~ $\rightarrow \frac{H_j^*(r)}{\lambda_j} = \frac{1}{R} \rightarrow \frac{\hat{p}(r) H_j}{\lambda_j} = \frac{1}{R}$

~~Now (if) $\frac{H_j^*(r)}{p_j^*(r)} = \frac{1}{R}$ also, then $[p_j(r) - p_j(R)] = \ldots \lambda_j [\ldots]$~~

~~$p_j^*(r) = w\lambda_j + r/R\, p_j(r) \rightarrow p_j^*(r)[1 - r/R] = w\lambda_j = (1 - r/R)\lambda_j$~~

so, ~~if $\frac{H_j^*(r)}{p_j(r)} = \frac{1}{R}$~~ $\rightarrow \boxed{p_j^* = \lambda_j}$ ~~& this does ... contradicts~~

4. But Sraffa says that if $\left[\frac{H_i^*(0)}{\lambda_i}\right] \neq \left[\frac{H_j^*(0)}{\lambda_j}\right]$, then $\frac{H_i^*(r)}{\lambda_i} \neq \frac{H_j^*(r)}{\lambda_j}$ at any $0 < r < R$ (ff 1, p. 13). By implication, $\frac{H_j^*(r)}{\lambda_j} \neq \frac{H_{std}^*(r)}{\lambda_j} = \frac{H_{std}^*(r)}{p_{std}(r)}\,\frac{p_{std}}{\lambda_{std}} = \frac{1}{R}$

Thus $\frac{H_j^*(r)}{\lambda_j} \neq \frac{1}{R}$ for any $0 < r < R$ $\rightarrow$ But then $p_j^* \neq \lambda_j$ at any $0 < r < R$.

$P_\gamma(0) = \lambda\theta$

① Initially $P_\gamma(R) = R H^*_\gamma(R)$; $w = 0$

② Interim, $P_\gamma = P_\gamma(R)$ But $w = 1 - r/R$, $r = r$

$\rightarrow \quad P^*_\gamma(R) = w\lambda + (R H^*_\gamma(R) - w\lambda) \rightarrow \boxed{r'_\gamma \gtrless r \text{ as } \frac{H^*_\gamma(R)}{\lambda_\gamma} \lesseqgtr \frac{1}{R}}$

$= w\lambda + r'_\gamma H^*_\gamma(R)$

③ Final $\quad P_\gamma(r) = w\lambda + r H^*_\gamma(r)$

$$\Delta P_\gamma(r) = P_\gamma(r) - P^*_\gamma(R) = R H^*_\gamma(R) - w\lambda - r H^*_\gamma(r)$$

$$= R H^*_\gamma(R) - \lambda_\gamma - \frac{r}{R}(R H^*_\gamma(r) - \lambda_\gamma)$$

$$\frac{\Delta P_\gamma(r)}{\lambda_\gamma} = (R H^*_\gamma/\lambda_\gamma - 1) - r/R\left(\frac{R H^*_\gamma(r)}{\lambda_\gamma} - 1\right)$$

$+, 0, -$

So if $H^*_\gamma/\lambda_\gamma = \frac{1}{R}$, then $\frac{\Delta P_\gamma(r)}{\lambda_\gamma} = 0$ since

$$(RH - I)(I - xRH)^{-1} = (\cancel{RH} - I)(I - \cancel{x}RH)^{-1} + (1-x)RH(I - xRH)^{-1}$$

$= -I +$

$$d^* = x\,1(B - I) + d^* B \quad \text{where } B = RH_\lambda$$

$$(d^*_1, d^*_2, d^*_3) = x[1, 1, 1]\begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{32} \\ B_{31} & B_{32} & B_{32} \end{bmatrix} - x1I + (d^*_1, d^*_2, d^*_3)\Big[\; B \;\Big]$$

If some $d^*_\gamma = 0$, say d^*_2

$$\cancel{d^*_2} = x(B_{12} + B_{22} + B_{32}) - x + (d^*_1 B_{12} + \cancel{d^*_2} B_{22} + d^*_3 B_{32})$$

$$\cancel{\to \quad 0 = B_{12}(x + d^*_1)}$$

$+ \quad + \quad +/-$

$$d^*_\gamma \equiv \frac{p_\gamma - \lambda_\gamma}{\lambda_\gamma} = \frac{p_\gamma}{\lambda_\gamma} - 1 \to d^* = \pi - 1$$

$$d^* = (x1 + d^*)B - x1I =$$

$$\boxed{p^*_\gamma = \lambda_\gamma + r'(p^* H_\gamma - \tfrac{1}{R}\lambda_\gamma)} \quad \text{so if } p^*_\gamma = \lambda_\gamma, \text{ then}$$

$$\boxed{p^*(r') H_\gamma = \tfrac{1}{R}\lambda_\gamma = \tfrac{1}{R} p_\gamma(r')} \to \text{This particular commodity } \gamma \text{ has the standard VOCC for this } r' < R$$

But $p^*(R) H = \frac{1}{R} p^*(R)$

$\to p^*(R) H_\gamma = \frac{1}{R} p^*_\gamma(R) \to \boxed{\dfrac{p^*(r') H_\gamma}{p_\gamma(r')} = \dfrac{p^*(R) H_\gamma}{p^*_\gamma(R)}}$

$$\left[\left(\frac{p^*(r')}{p_\gamma(r')}\right) - \left(\frac{p^*(R)}{p^*_\gamma(R)}\right)\right] H_\gamma = \phi$$

But $\underbrace{p^*(r')\, H_\gamma}$ $= (1 - r'/R)\lambda H_\gamma + r'(p^*(r') H) H_\gamma$

$$\tfrac{1}{R}\lambda_\gamma = (1 - r'/R)\lambda H_\gamma + r'$$

$$\tfrac{1}{R}\lambda_\gamma = (1 - \tfrac{r'}{R})\, C^T_\gamma + r'$$

$$1 = (R - r')\frac{C^T_\gamma}{\lambda_\gamma} + r\,\frac{p(r) H H_\gamma}{\tfrac{1}{R}(\lambda_\gamma)}$$

But from sum of prices condition

$p^*(r')x = p^*(R)x$

(This is for absolute level)

Note that ~~But~~ Both elements are positive if $0 \le r' < R$

two sets of relative prices applied to the same H_γ

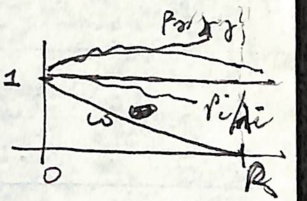
In Sraffa reswitching

$$p^{(1)}_\gamma \equiv \bar{w}\lambda^{(1)}_\gamma + \bar{r}p^{(1)} H^{(1)}_\gamma = p^{(2)}_\gamma = \bar{w}\lambda^{(2)}_\gamma + \bar{r}p^{(2)} H^{(2)}_\gamma \to \bar{w}[\lambda^{(1)}_\gamma - \lambda^{(2)}_\gamma] + \bar{r}(p^{(1)}_{(r)} H^{(1)}_\gamma - p^{(2)}_{(r)} H^{(2)}_\gamma) = \phi$$

$\to$ two sets of rel. prices applied to different H_γ's and diff λ_γ's

MAX Stock-Price – Value Deviation

6/02/93



(1) $p_\gamma = w\lambda_\gamma + rH_\gamma^*(r) = (1 - r/R)\lambda_\gamma + rH_\gamma^*(r)$

$d_\gamma \equiv \frac{p_\gamma}{\lambda_\gamma} = 1 - \frac{r}{R} + r\frac{H_\gamma^*(r)}{\lambda_\gamma} = 1 + r\left[\frac{H_\gamma^*}{\lambda_\gamma} - \frac{1}{R}\right]$

(2) $\frac{d\delta_\gamma}{dr} =$

$\to r\frac{H^{*\prime}(r)}{\lambda_\gamma} + \left(\frac{H_\gamma^*(r)}{\lambda_\gamma} - \frac{1}{R}\right) = 0 \to \frac{H^{*\prime}(r)}{\lambda_\gamma} + \left(\frac{p_\gamma - \lambda_\gamma}{\lambda_\gamma}\right)\frac{1}{r} = 0$

$\to H_\gamma^{*\prime}(r) + \left(\frac{p - \lambda_\gamma}{\lambda_\gamma}\right)\frac{1}{r^2} = 0$

so if such a point exists, then

(3) $\frac{d^2\delta_\gamma}{dr^2} = r\frac{H_\gamma^{*\prime\prime}(r)}{\lambda_\gamma} + \frac{H_\gamma^{*\prime}(r)}{\lambda_\gamma} + \frac{H^{*\prime}(r)}{\lambda_\gamma} < 0$

$p_\gamma = (1 - r/R)\lambda_\gamma + rH_\gamma^*(r)$

$d_\gamma = p_\gamma/\lambda_\gamma = \frac{1 - r/R}{1 - r\left(\frac{H_\gamma^*(r)}{p_\gamma(r)}\right)}$. But as $r \to R$, top + bottom both $\to 0$ since $\frac{H_\gamma^*}{p_\gamma} \to \frac{1}{R}$

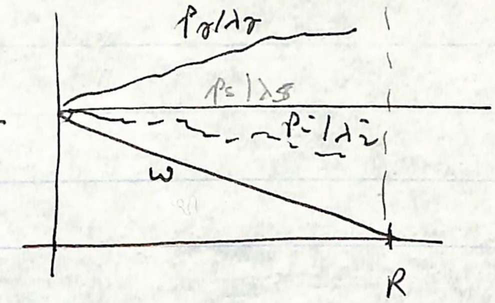
If there is no capital-output reswitching, then if $\frac{H_\gamma^*(0)}{p_\gamma^*(0)} > \frac{1}{R}$, ~~it falls~~ $\frac{H_\gamma^*(r)}{p_\gamma^*(r)}$ **falls** toward $\frac{1}{R}$ and if ~ $< \frac{1}{R}$, then $H_\gamma^*(r)/p_\gamma(r)$ rise towards $\frac{1}{R}$.

d

1. $p_\gamma = w\lambda_\gamma + r H_\gamma^* = (1 - r/R)\lambda_\gamma + rH_\gamma^*$

$d_\gamma \equiv p_\gamma / \lambda_\gamma = 1 + r\left(\frac{H_\gamma^*}{\lambda_\gamma} - \frac{1}{R}\right)$

$\delta_\gamma \equiv \frac{p_\gamma - \lambda_\gamma}{\lambda_\gamma} = r\left[\frac{H_\gamma^*(r)}{\lambda_\gamma} - \frac{1}{R}\right]$



2. $\frac{d}{dr}(d_\gamma) = r\frac{H_\gamma^{*\prime}}{\lambda_\gamma} + \left(\frac{H_\gamma^*(r)}{\lambda_\gamma} - \frac{1}{r}\right) = r\frac{H^{*\prime}(r)}{\lambda_\gamma} + \left(\frac{p_\gamma - \lambda_\gamma}{\lambda_\gamma}\right)\frac{1}{r}$

(1) At $r = 0$, $\frac{p_\gamma - \lambda_\gamma}{\lambda_\gamma} = 0$ and $\frac{H_\gamma^*(0)}{\lambda_\gamma} \gtrless \frac{1}{R}$, If $H^{*\prime}(r) > 0$, then at some $r \cong \varepsilon$, $\frac{p_\gamma - \lambda_\gamma}{\lambda_\gamma} > 0$. Then

$p^* = w\lambda + r p^* H$, $H_{i\gamma} > 0$, $\theta_\gamma > 0$ for all $0 \le r \le R$

$p^*(R) = R p^*(R) H$

$p^*(r) - p^*(R) = w\lambda + [r p^*(r) - R p^*(R)] \cdot H$

1. $\rho = \omega\lambda + r\rho H \begin{cases} \rho(0) = \lambda \\ \rho(R) = R\rho(R)H \end{cases}$

$\quad = \lambda_\gamma + r\left(\rho H - \frac{1}{R}\lambda\right)$

2. $\rho_\gamma = \omega\lambda_\gamma + r\rho H_\gamma \quad = \lambda_\gamma + r\left[\rho \hat{H}_\gamma - \frac{1}{R}\lambda_\gamma\right]$

3. If $\rho_\gamma = \lambda_\gamma$, then $\rho\hat{H}_\gamma = \frac{1}{R}\lambda_\gamma$ & vice-versa

(i) But $\rho\hat{H}_\gamma = \omega\lambda\hat{H}_\gamma + r\left[\rho H\cdot\hat{H}_\gamma - \frac{1}{R}\lambda\hat{H}_\gamma\right]$

$\therefore \frac{1}{R}\lambda_\gamma = C^T_\gamma + r\left[\rho H\cdot\hat{H}_\gamma - \frac{1}{R}C^T_\gamma\right]$

By assumption, $C^T_\gamma/\lambda_\gamma \gtrless \frac{1}{R}$ (otherwise $\rho_\gamma = \lambda_\gamma$ for all r)

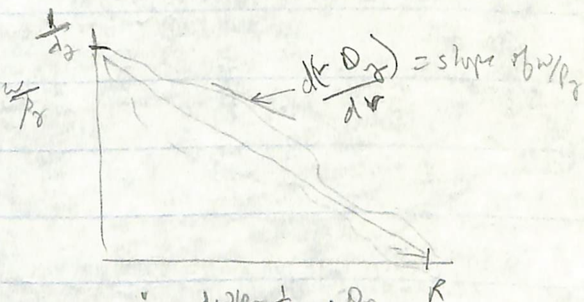
$\therefore r\left[\rho(R)H\cdot\hat{H}_\gamma - \frac{1}{R}C_\gamma^T\right] \lessgtr 0$ as $C_\gamma^T/\lambda_\gamma \lessgtr \frac{1}{R}$

$\omega = 0$ 1. $\rho_0 \equiv \lambda_0$

$\omega=0, \rho = \rho_{0\gamma}$ 2. $\rho_{0\gamma} = \lambda_{0\gamma} = \omega\lambda_\gamma + \left(\frac{r}{R}\right)\lambda_\gamma \quad \left(= r H_\gamma(0)\left[\frac{\lambda_\gamma}{R H_\gamma(0)}\right]\right) \rightarrow \rho_\gamma = \frac{\frac{r}{R}\lambda_\gamma}{H_\gamma(0)} = r\left[\frac{\frac{1}{R}}{(C_\gamma^T/\lambda_\gamma)}\right] \gtrless r$

$1 = \left(\frac{\omega}{\rho_{0\gamma}}\right)\lambda_\gamma + r\frac{H_\gamma(0)}{\rho_{0\gamma}}\left[\frac{\lambda_\gamma}{R H_\gamma(0)}\right]$ as $C_\gamma^T/\lambda_\gamma \lessgtr \frac{1}{R}$

3. $\rho_\gamma = \omega\lambda_\gamma + r H_\gamma(r)$



$1 = (\omega/\rho_\gamma)\lambda_\gamma + r\frac{H_\gamma(r)}{\rho_\gamma(r)}$

$(\omega/\rho_\gamma) = \frac{1}{\lambda_\gamma} - \frac{1}{\lambda_\gamma}\underbrace{\left[r\frac{H_\gamma(r)}{\rho_\gamma(r)}\right]}_{\text{slope}}$

$-\frac{1}{\lambda_\gamma}\theta_\gamma = \frac{d\omega/\rho_\gamma}{dr}\frac{1}{r} + \frac{\theta_\gamma}{\lambda_\gamma}\frac{1}{r}$

$\frac{d(\omega/\rho_\gamma)}{dr} = -\frac{1}{\lambda_\gamma}\left[r\theta_\gamma' + \theta_\gamma\right] < 0$ for all r

$\therefore r\theta_\gamma' + \theta_\gamma > 0$

$\theta_\gamma'/\theta_\gamma > -\frac{1}{r}$ $\rightarrow \theta_\gamma' \geq 0$ ✓

$r\frac{\theta_\gamma'}{\theta_\gamma} + 1 > 0$ $\rightarrow \theta_\gamma' < 0 \rightarrow \theta_\gamma' <$

and $\frac{d^2(\omega/\rho_\gamma)}{dr^2} = -\frac{1}{\lambda_\gamma}\left[r\theta_\gamma'' + \theta_\gamma'\right] = -\frac{1}{\lambda_\gamma}\left[r\theta_\gamma''\right] + \underset{(-)}{\frac{d\omega/\rho_\gamma}{dr}\frac{1}{r}} + \underset{(+)}{\frac{\theta_\gamma}{\lambda_\gamma}\frac{1}{r}}$

$p = w\lambda + r\, pH$

$\cancel{p \langle p_\gamma \rangle^{-1} = w \lambda \langle p_\gamma \rangle^{-1}} + r\, pH \langle p_\gamma^{-1} \rangle$

$\cancel{\hat{1}} = w[\lambda_\gamma / p_\gamma] + r\, 1\, H^*(r)$, $\quad H^*(r) \equiv \langle p_\gamma \rangle H \langle p_\gamma \rangle^{-1}$

$\hat{p}/w = \hat{\lambda} + r\, \hat{p}/w \cdot H$

$\hat{p}/w = \lambda \underbrace{[I - rH]^{-1}}_{B(r)}$; and $\hat{p}/w \uparrow$ as $r \uparrow$ $[w/p_\gamma \downarrow$ as $r\uparrow]$

$\dfrac{\hat{p} H_\gamma}{w} = \lambda\, B(r) \cdot H_\gamma = \dfrac{H_\gamma^*}{w} \uparrow$ as $r \uparrow$

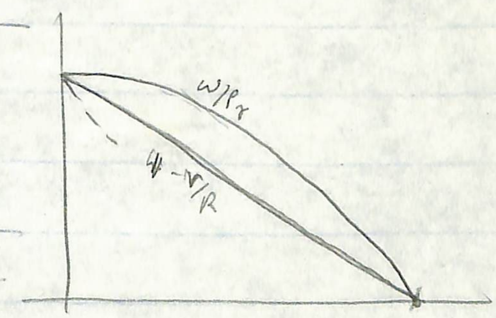
so $\dfrac{H_\gamma^*}{\lambda_\gamma w} \uparrow$ as $r \uparrow$

$p_\gamma = w \lambda_\gamma + r H_\gamma^* = \cancel{w\lambda_\gamma \left[1 + \frac{r H_\gamma^*}{w \lambda_\gamma} \right] = w\lambda_\gamma [}$ $= \lambda_\gamma + r[$

At $\begin{matrix} w = 1 \\ r = 0 \end{matrix}$, $p_\gamma = \lambda_\gamma$

Now let $w > 0$ by some small amount $\rightarrow r\uparrow$, $H_\gamma^* / w\lambda_\gamma \uparrow$ and $w \downarrow$,

and $p_\gamma / w \uparrow$



$\dfrac{p_\gamma / w}{\lambda_\gamma} = 1 + r \dfrac{\lambda H_\gamma}{\lambda_\gamma} + r^2 \dfrac{(\lambda H) H_\gamma}{\lambda_\gamma} + \cdots$

$p_{std}/w = \dfrac{1}{1 - r/R}$

$\dfrac{\left(\frac{p_\gamma}{\lambda_\gamma}\right)}{w} = \dfrac{\frac{p_{std}}{\lambda_{std}}}{w} \rightarrow 1 + r\left[\dfrac{\lambda H_\gamma}{\lambda_\gamma}\right] + r^2 \left[\dfrac{\lambda H \cdot H_\gamma}{\lambda_\gamma}\right] + \cdots$

$= \dfrac{1}{1 - r/R} = 1 + \dfrac{r}{R} + \dfrac{r^2}{R} + \cdots$

$\rightarrow r\left[\dfrac{\lambda H_\gamma}{\lambda_\gamma} - \dfrac{1}{R}\right] + r^2 \underbrace{\left[\dfrac{\lambda H \cdot H_\gamma}{\lambda_\gamma} - \dfrac{1}{R^2}\right]}_{-} + \cdots = 0$

$+$

BLA

$\frac{H_\gamma^\alpha(r)}{\omega} \uparrow$ as $r \uparrow (\omega \downarrow)$

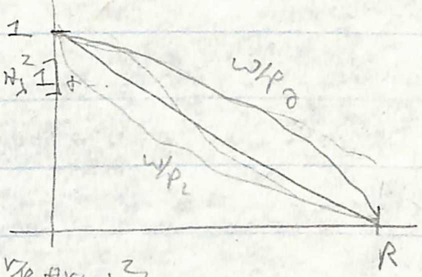
$\hat{p}/\omega = \hat{\lambda} + r\,\frac{\hat{p}}{\omega}(H) \rightarrow p_{\gamma/\omega} = \lambda_\gamma + \left(r \cdot \frac{H_\gamma^\alpha(r)}{\omega}\right)$ and $r \cdot \frac{H_\gamma^\alpha(r)}{\omega} \uparrow$ as $r \uparrow$

$\hat{d}/\omega = \hat{1} + r\frac{\hat{d}}{\omega} \cdot H_\lambda$ where $\hat{d} = [p_\gamma/\lambda_\gamma]$, $H_\lambda \equiv \langle \lambda_\gamma \rangle H \langle \lambda_\gamma \rangle^{-1}$

$\hat{d}/\omega = \hat{1}[I - rH_\lambda]^{-1} = 1 + r\hat{1}H_\lambda + r^2\hat{1}H_\lambda^2 \gamma$

$d_{\gamma/\omega} = 1 + r\frac{H_\gamma^\alpha}{\omega\lambda_\gamma}$ and $d_{\gamma/\omega} \uparrow$ as $r \uparrow$

$= 1 + r\,1(H_\lambda)_\gamma + r^2\,1(H_\lambda)^2_\gamma + \cdots \equiv 1 + r[1H_\lambda 1] + r^2[1H_\lambda^2 1] + \cdots$



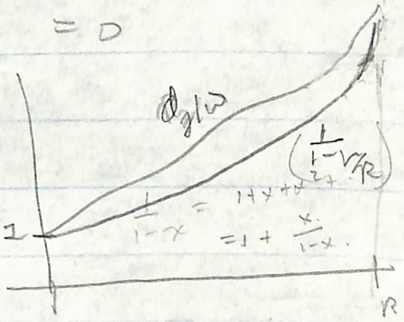
For $p_\gamma = \lambda_\gamma$, need $d_{\gamma/\omega} = \frac{1}{\omega}$

But for $p_{std} = \lambda_{std}$, $d_{std} = 1 \rightarrow \frac{d_{std}}{\omega} = \frac{1}{\omega} = \frac{1}{1 - r/R} = 1 + r/R + (r/R)^2 + \cdots$

$\therefore$ Need $r\underbrace{\left[1(H_\lambda)_\gamma - \frac{1}{R}\right]}_{(\hat{c}\gamma/\lambda_\gamma - 1/R)} + r^2\left[1(H_\lambda)^2_\gamma - \frac{1}{R^2}\right] + \cdots = 0$

$r\left[1H_\lambda e_\gamma - \frac{1}{R}\right] + r^2\left[1H_\lambda^2 e_\gamma - \frac{1}{R^2}\right] + \cdots$

should have



$\hat{d}/\omega = 1 + r(1H_\lambda) + r^2\,1(H_\lambda)^2 + \cdots = 1[I - rH]^{-1} = \hat{1} + 1rH[I - rH]^{-1}$

$\frac{d}{dr}(\hat{d}/\omega) = 1H_\lambda + 2r\,1(H_\lambda)^2 + 3r^2\,1(H_\lambda)^3 + \cdots$

$\cancel{H_\lambda} = H_1\ \frac{r}{R}\left[\hat{1}H_\lambda\hat{e}_\gamma - \hat{e}_\gamma\right] + r/R^2\left[1H_\lambda^2 e_\gamma - e_\gamma\right] + \cdots$

$+$

or $(=)$

$(r/R)^2\,1[H_\lambda^2 - I]e_\gamma + (r/R)^3[H_\lambda^3 - I]e_\gamma + \cdots$

$\therefore$ ~~$\frac{r^2}{R}[H_\lambda^2]$~~

$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix}$

Suppose $y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots = a + \sum_{i=1}^{N} \beta_i y_{t-i}$

But now suppose we have varying parameters
$$a = a_0 + \sum_{k=1}^{M} a_k y_{t-k}$$
$$\beta_i = \beta_{i0} + \sum_{j=1}^{L} \beta_{ij} y_{t-j}$$

$$y_t = a_0 + \sum^{M} a_k y_{t-k} + \sum_{i=1}^{N} \beta_{i0} y_{t-i} + \sum_{i=1}^{N} \left[\sum_{j=1}^{L} \beta_{ij} y_{t-j} \right] y_{t-i}$$

E. G. $N = M = 2$

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \beta_{10} y_{t-1} + \beta_{20} y_{t-2} + \cancel{\sum_{i=1}^{N}[\beta_{i1} y_{t-1} + \beta_{i2} y_{t-2}}$$
$$+ \sum_{i=1}^{N} [\beta_{i1} y_{t-1} + \beta_{i2} y_{t-2}] y_{t-i}$$
$$= a_0 + (a_1 + \beta_{10}) y_{t-1} + (a_2 + \beta_{20}) y_{t-2} + \beta_{11} y_{t-1} y_{t-1} + \beta_{12} y_{t-2} y_{t-1}$$
$$+ \beta_{21} y_{t-1} y_{t-2} + \beta_{22} y_{t-2} y_{t-2}$$
$$= A + B_1 y_{t-1} + B_2 y_{t-2} + B_3 y_{t-1}^2 + B_4 y_{t-1} y_{t-2} + B_5 y_{t-2}^2$$

~~$y_t = a + \gamma x$~~ $\quad y_t = \alpha + \beta_1 y_{t-1} + \cdots + \gamma x_t + \gamma_1 x_{t-1} + \cdots$

$\alpha =$

β_1

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A more detailed and systematic analysis of these issues was conducted by Ochoa (1984). Ochoa calculates and compares labor values, prices of production, and market prices from U.S. input-output tables for 1947, 1958, 1961, 1963, 1967, 1968, 1968, 1970, 1972, at a 71-order level. His results, some of which are summarized below, strongly confirm our general findings.

Table 1 shows the general cross sectional relation between (prices proportional to) labor values and prices of production. It also shows the empirical relation between (partially tranformed) prices of production as Marx calculates them in Volume III of Capital, and fully transformed prices of production a la Bortkiewicz-Sweezy-Sraffa-Samuelson-etc. These two comparisons are at the very heart of the tranformation debate. As we can see, the absolute average deviation of labor values from (fully transformed) prices of production is roughly 15%, which is very modest. Even more interestingly, Marx's partially transformed prices of production, which can be shown to be a first step in an iterative procedure which always converges on the "correct" prices of production (Shaikh, 1977), turn out to be *empirically* excellent approximations of the latter: fully transformed prices of production (full pop) deviate by less than 6% from Marx's approximations (Mx's pop), on average. Both of these results indicate a strong inner connection between labor values and prices of production.

Table 1

Absol Avg Devtn	1947	1958	1963	1967	1972	AVG. 1947-72
Value/full pop	.198	.123	.147	.159	.139	.153
Mx's pop/full	.062	.045	.055	.056	.057	.055

Table 2 looks at another extremely interesting aspect of Ochoa's results. Namely, that values, Marx's approximations to prices of production, and fully transformed prices of production are all roughly equally close to market prices, with mean absolute deviations averaging between 12-14%.

Table 2

Absol Avg Devtn	1947	1958	1963	1967	1972	AVG. 1947-72
Value/mkt pr	.199	.118	.119	.108	.106	.120
Mx's pop/mkt pr	.193	.131	.121	.131	.137	.134
Full pop/mkt pr	.185	.131	.126	.137	.139	.137

Ochoa also provides us with three different measures of the general rate of profit: the Marxian value rate of profit r_0, the uniform rate of profit r (Bortkiewicz-Sraffa), and the actual average market rate of profit r^m, for each of the input-output years. Table 3 shows that these three different estimates are virtually indistinguishable from one another. On average, the deviation between the Marxian value rate r^0 and the Sraffian uniform rate r is a mere 3.7%, and in no year does it exceed 5%. Interestly enough, of the two the Marxian value rate turns out to be closer to the market rate, on average (none of them are adjusted for variations in capacity utilization, and therefore cannot be used to judge the trend of the rate of profit. Chapter 6 will adress these issues).

Table 3

Profit Rate	1947	1958	1963	1967	1972	AVG. 1947-72
Value rate	.235	.175	.205	.221	.184	.204
Uniform rate	.247	.179	.212	.232	.188	.212
Market rate	.236	.176	.210	.229	.181	.206

Recent investigations provide yet further confirmation of the arguments in this section. Using Yugoslav input-output tables, Petrovic finds strong empirical support for "Ricardo's empirical proposition that relative production prices are mainly determined by labour-value ratios", with root-mean-percentage-error measures of the deviations between value and production price ratios of averaging around 6% (Petrovic, 1987, pp. 209 and Table 1, p. 202). In an elegant paper, Bienenfeld develops a quadratic approximation to (fully transformed) prices of production, which he then shows to be more than 99.75% accurate for U.S. data. This quadratic approximation may be thought of as a improvement on Marx's own linear approximation (which is itself over 94% accurate, as indicated in Table 1). What makes it even more powerful is that the Bienenfeld's is 100% accurate not only at a zero rate of profit (where all price measures are proportional to labor values), but also at the maximum rate of profit. Since full prices of production are in principle so complex that many authors have despaired of saying anything about their individual determinants, Bienenfeld's result is a real theoretical breakthrough because it enables us to capture virtually all of this complexity in a quadratic expression. For analytical work in which we look at subsectors within the whole (sectors within the nation, nations within the world economy), good linear or quadratic approximation are crucial. We will see the significance of this in Chapter 5, where we take up the question of international values, prices of production, and the patterns of trade.

TABLE 6.10 (Ochoa, 1984, p.128)

Values/Mkt Prices CROSS-SECTIONAL RESULTS FOR DIRECT PRICES

YEAR	1947	1958	1961	1963	1967	1968	1969	1970	1972	MEAN
MAD	.199	.118	.121	.119	.108	.107	.102	.103	.106	.120
MAWD	.160	.118	.127	.125	.118	.111	.115	.111	.115	.122
NVD	.173	.120	.136	.134	.132	.112	.132	.127	.123	.133
R^2	.957	.978	.975	.974	.975	.974	.977	.978	.979	.974

TABLE 7.3 (Ochoa, 1984) p.143

Full POP/Mkt Prices

PRICES OF PRODUCTION AND MARKET PRICES

CROSS-SECTIONAL STATISTICS

Year	1947	1958	1961	1963	1967	1968	1969	1970	1972	MEAN
MAD	.185	.131	.127	.126	.137	.132	.128	.125	.139	.137
MAWD	.168	.134	.141	.143	.150	.145	.141	.131	.158	.146
NVD	.196	.155	.164	.167	.174	.168	.161	.153	.188	.169
R^2	.963	.987	.986	.987	.983	.983	.984	.986	.980	.982

TABLE 7.6 (Ochoa, 1984, p.151)

Values/Full P.O.P.

PRICES OF PRODUCTION AND DIRECT PRICES

CROSS-SECTIONAL STATISTICS

Year	1947	1958	1961	1963	1967	1968	1969	1970	1972	MEAN
MAD	.198	.123	.133	.147	.159	.162	.160	.152	.139	.153
MAWD	.143	.134	.142	.153	.164	.164	.164	.157	.141	.151
NVD	.125	.150	.162	.174	.191	.190	.194	.189	.156	.170
R^2	.972	.979	.976	.972	.967	.965	.966	.971	.979	1972

TABLE 7.10 (Ochoa, p.162)

Marx's P.O.P./Full/P.O.P. MARX'S SOLUTION AND PRICES OF PRODUCTION

CROSS-SECTIONAL STATISTICS

Year	1947	1958	1961	1963	1967	1968	1969	1970	1972	MEAN
MAD	.062	.045	.050	.055	.056	.055	.056	.054	.057	.055
MAWD	.059	.057	.062	.067	.072	.070	.071	.069	.072	.067
NVD	.061	.074	.081	.088	.099	.094	.095	.092	.109	.088
R^2	.994	.995	.995	.994	.994	.994	.994	.995	.996	.995

Finally, it should be noted that the small size of price-value deviations implies that wage-profit curves are not likely to have much curvature. Indeed, Ochoa finds that the relation between the real wage and the profit rate is striking in its near linearity for all available years, and this in spite of the fact that the actual output proportions of the economy are very different from standard proportions in all years (Ochoa, 1988, pp. 20-24, and Figure 6 which is reproduced below).

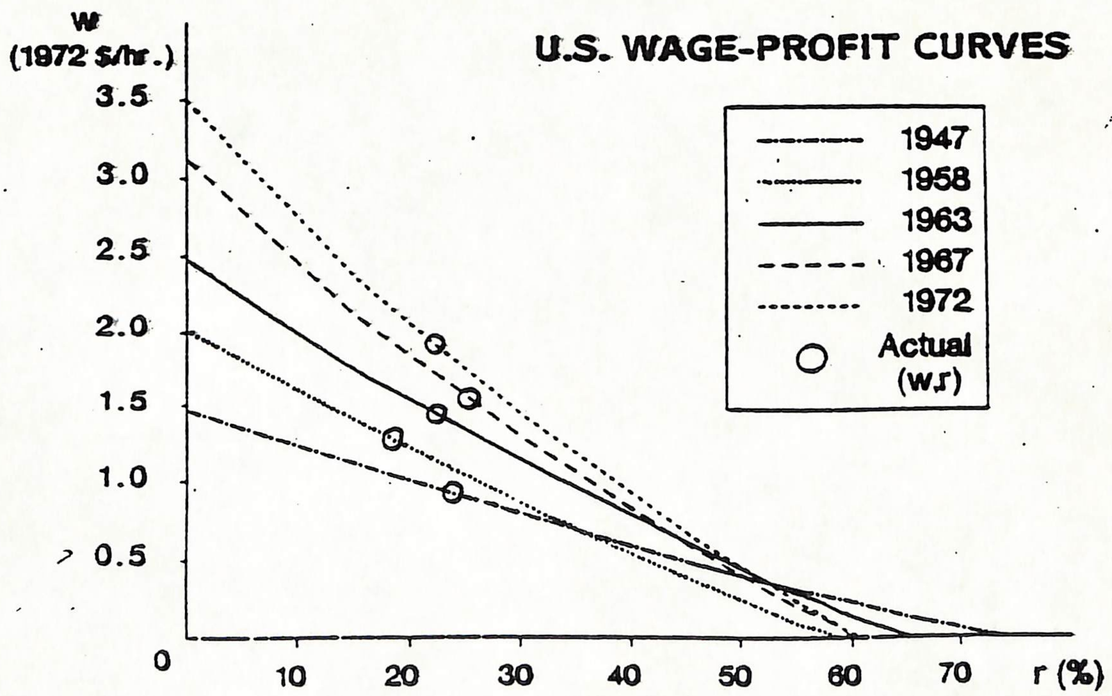

Figure 6

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Ochoa, E. "Labor Values and Prices of Production: An interindustry study of the U.S. economy, 1947-1972", Unpublished dissertation, New School for Social Research, 1984, pp.124-25.

Petrovic, P. "The deviation of prices of production from values: Some methodology and empirical evidence" Cmbridge Journal of Economics, v.11, 1987, pp.207-8.

Shaikh, A. "Marx's theory of value and the transformation problem" in Schwartz, J (ed) The Subtle Anatomy of Capitalism, Goodyear Publishing Company 1977.

From Shaikh (1984) "Transformation from Marx to Sraffa"

Lastly, the nature of the expressions for cross-sectional and inter-temporal correlations of relative prices and relative values (equations (16) and (17) respectively) suggests that we can rewrite them in the following useful forms:

$$\ln p_{ij} = \ln \lambda_{ij} + \ln z_{ij} \tag{18}$$

$$\ln (p_{ij})_{\Delta t} = \ln (\lambda_{ij})_{\Delta t} + \ln (z_{ij})_{\Delta t}. \tag{19}$$

When written in the above form, we can see that the relation between relative prices and relative values is a log-linear one, in which the terms $\ln z_{ij}$ and $\ln (z_{ij})_{\Delta t}$ play the parts of a 'disturbance' term. This in turn suggests that we can picture the extent of price–value deviations by drawing up a scatter diagram of the log of relative prices versus the log of relative values. Moreover, it also suggests that a natural form for cross-sectional and inter-temporal hypotheses is that empirical correlation between relative prices and relative values is log-linear.

Cross-Sectional Hypothesis H_0:

$$\ln p_{ij} = \alpha + \beta \ln \lambda_{ij} + u_{ij} \tag{20}$$

Inter-Temporal Hypothesis H_0:

$$\ln (p_{ij})_{\Delta t} = \alpha + \beta \ln (\lambda_{ij})_{\Delta t} + u_{ij}. \tag{21}$$

A. Marzi and Varri Data

Cross-Sectional ($r = 0.40$)

$$\textit{1967:}\quad \ln p_{ij} = \underset{(0.23)}{0.0095} + \underset{(16.60)}{0.8470} \ln \lambda_{ij} \tag{22}$$

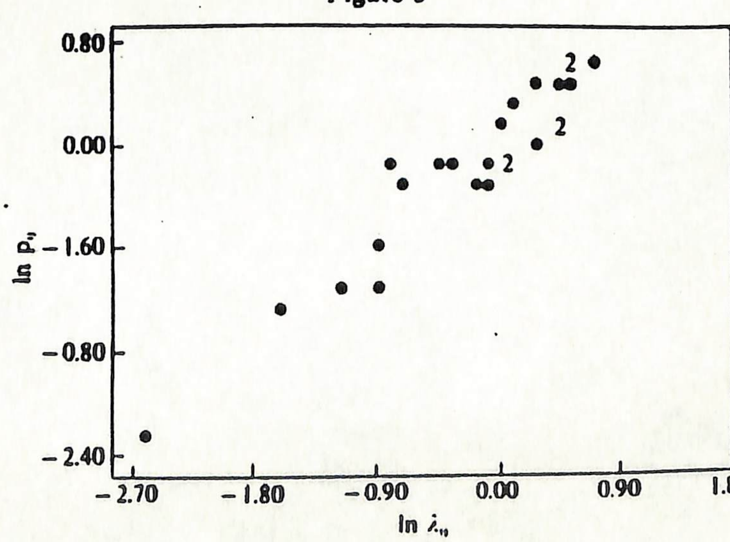
$R^2 = 0.920$ (adjusted for degrees of freedom)

$$\textit{1959:}\quad \ln p_{ij} = \underset{(-0.20)}{-0.0096} + \underset{(12.48)}{0.8717} \ln \lambda_{ij}$$

$R^2 = 0.866$ (adjusted for degrees of freedom).

(see appendix B for the actual data). In figure 3 below, the vertical axis represents the natural logarithm of the ratios of individual prices of production to the average price of production, at $r = 0.40$. The

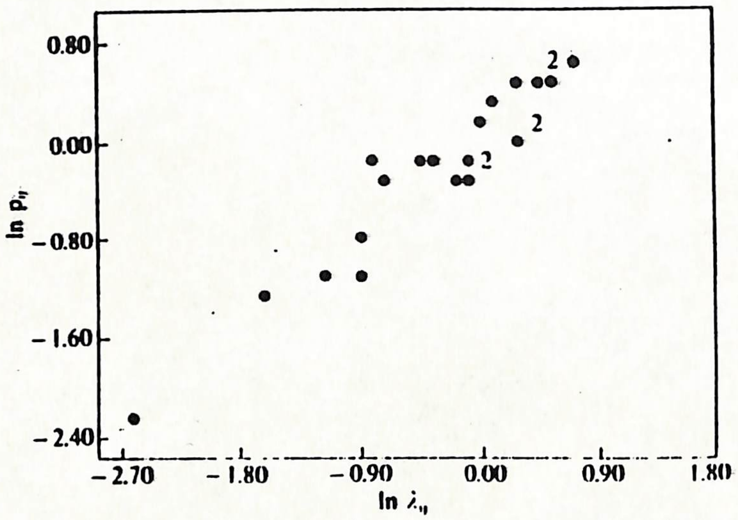
Figure 3



horizontal axis, on the other hand, represents the natural log of the ratios of individual values to the average value, which as I explained above can be calculated from the prices of production at $r = 0$. Lastly, this particular data refers to 1967. The corresponding data for 1959 gives virtually the same picture, though, with only a slightly lower correlation (see equation (22) below).

Because the data covers two different time periods, we can also use it to test the inter-temporal correlation between changes in relative prices and changes in relative values. Figure 4 below pictures $\ln(p_{ij})_{\Delta t}$ and $\ln(\lambda_{ij})_{\Delta t}$ on the vertical and horizontal axes, respectively, where both are in terms of 1959 prices relative to 1967 prices.

Figure 4



Inter-Temporal (r = 0.40)

$$\textit{1959/1967:}\quad \ln (p_{ij})_{\Delta t} = \underset{(-1.90)}{-0.0298} + \underset{(16.08)}{1.008} \ln (\lambda_{ij})_{\Delta t} \tag{23}$$

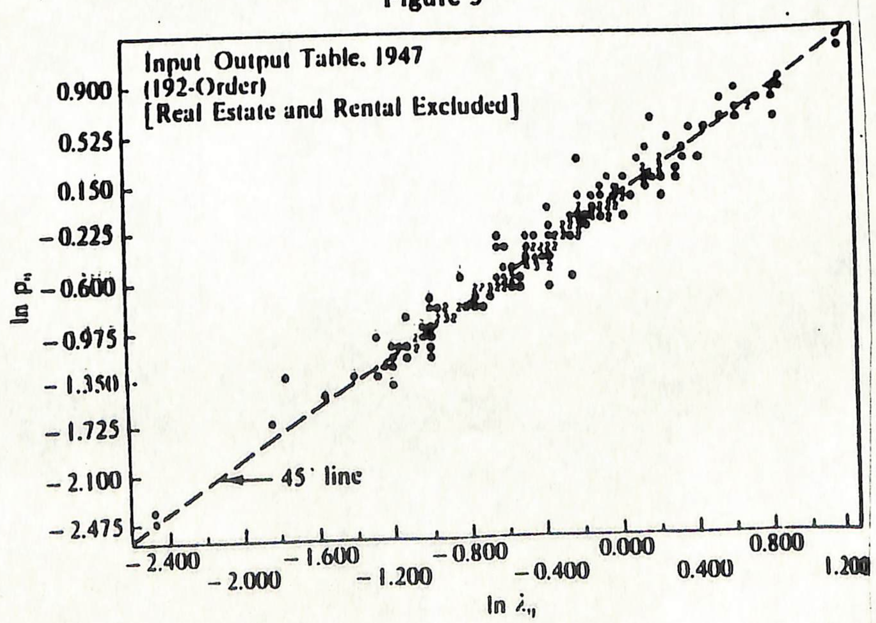
$R^2 = 0.915$ (adjusted for degrees of freedom)

B. The Leontief Data

The Marzi-Varri data pertains to prices of production and values calculated from a 25-order input output table. But for the relation of market prices to values, even more detailed data is available in some earlier work by Leontief.

market prices (sales). Figure 5 below is a graph of the natural log of relative market prices versus that of relative direct prices, for 190 sectors (the real estate and rental sector is excluded on theoretical grounds, since differential rent, though determined by surplus-value, is not expected to be proportional either to prices or to values).

Figure 5



The closeness of the correlation between market prices and direct prices is obvious. For this data, the typical deviation is about $\pm 20\%$, and, as indicated below, a log-linear regression yields excellent results

(standard errors are in parentheses below the coefficients. A parametric test indicates no significant heteroskedasticity in the data):[40]

Cross-Sectional: 1947

$$\ln (p_{ij}) = \underset{(0.0106)}{-0.00095} + \underset{(0.01498)}{0.96809} \ln (\lambda_{ij}) \tag{24}$$

$$R^2 = 0.95814$$

On the basis of data made available by Edward Wolff of New York University, I was able to repeat the preceding experiment for the 1967 input–output table, on 83-order data. The results are virtually identical to those for Leontief's data:

Cross-Sectional: 1963

$$\ln (p_{ij}) = \underset{(0.01457)}{0.01380} + \underset{(0.02602)}{0.99078} \ln (\lambda_{ij}) \tag{25}$$

$$R^2 = 0.94894$$

4. *Summary of the Empirical Evidence*

The results of the previous section can now be briefly summarized. In general, for both prices of production and for market prices, the typical percentage deviation (the sum of the absolute values of deviations divided by the sum of prices) is moderate: for the price of production data it is of the order of ± 17–19%; and for the market price data of the order of ± 20–25%. The fact that for an individual commodity a typical deviation is on the order of $\pm 20\%$ means that when we consider a bundle of commodities such as those consumed by capitalists, then the net deviation δ_t of this bundle is likely to be much smaller than $\pm 20\%$ because negative and positive deviations will tend to offset each other. This justifies the assumption that $\delta_t \cong 10\%$, which I used earlier (see p. 65) to estimate aggregate profit and profit-rate deviations from their corresponding value categories.

A typical deviation of $\pm 20\%$ of course implies that the typical non-deviation is on the order of $\pm 80\%$. In other words, it implies that the variations in prices are likely to be highly correlated with corresponding variations in values. And we find that this is just the case. For price of production data, the cross-sectional regression yields an $R^2 = 0.92$ for 1967 and $R^2 = 0.87$ for 1959, while the inter-temporal regression yields an $R^2 = 0.92$. For market price data, we get a cross-sectional $R^2 = 0.96$ for 1947 and $R^2 = 0.95$ for 1963. Finally, on the basis of the data utilized by Jacob Schwartz, we find that even under

the turbulent conditions of business cycle downturns, relative price variations are small enough (about 7%) for us to conclude that by far the major source of variations in relative prices over a period of several years will be the variations in the corresponding relative values. Ricardo, it seems, had a vastly superior grasp of these issues than the neo-Ricardians.

LTV II: Buffalo

1. Reprise: Law of Value = Abstract L.T. as Hidden Regulator

(i) Turbulent Reproduction ⟺ Equil.

(ii) $P_m \rightarrow P_{SS/DD} \rightarrow p^* \rightarrow \lambda, s$ [EACH LEVEL DIFFERENT BUT CONNECTED]

— See quotes, p.2, 5/22/78, "Eco205"

2. Price-Value Deviations & Transfers of Value

(i) ~~Array~~ price-value dev. = transfer between buyer/seller

(ii) Agg Reproduction = Interactions ⟨ Circuits of Cap & Revenue / Stocks & Flow

→ Conservation of Value = Transfers between ↑

(iii) Mystery of $P \neq p^0$ even when $\Sigma P = \Sigma P^0$ ~~Dept I~~

(iv) Transf. Problem as a specific application of general case

→ Simplest Case = Balanced Reprod (SR, Exp. Rep)

because in balanced reprod, stocks/flow rel. is isolated → Then P/S deviation due solely to Cap/Revenue circuit transfer (see "Transf. from Marx to Sraffa")

(v) Size & Direction of P.O.P/Value Deviations → Next time

3. Critique of NeoRicardians

II. Basic Structure

1. ~~Role of~~ In all societies, reprod. < use-values / social relation > — labor process as fundamental

In all class ", surplus labor ⟸ surplus prod. / class relation

So labor ~~process~~ fundamental to social reprod < social form / labor-time

2. Capital Prod. (See A 1) Bechow 2 (I) – 2 (IV)

(1) Generalized Commod. Prod.

3. Some Application of the Differences [See "Structure of Marxist vs. Neoclassical Eco. Analyses" (Columbia Urpe) Dec 1, 1978, p. 7]

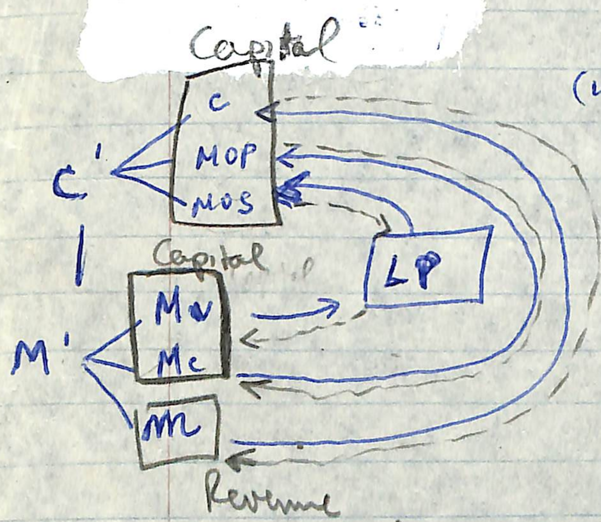
III

The above analysis was designed to show how the relation of value & price appear in Marx, based on his conception of prod & reprod

~~Now~~.

Now we look at other side of prod / reprod

(i) Individual circuits of capital M-C-M'
of revenue



Bold lines = Flow of Value in Money Form

Dotted lines = Flow of Value in Commodity Form.

(ii) Social circulation — subsumption of worker's cons. under circuit of capital

Not C'-M' but C'
M'

cons. of worker dictated by invest (Keynesian fallacy)

capitalist cons. the truly autonomous circuit of consumption

[We will use this in transf. problem]

Eco 205
Spring 1979

Section V.1 Equalization of the Rate of Profit 5/22/78

I **Production, Reprod & Exchange** (See NLR conf. typed notes)
(Establishes role of exchange & hence price phenom. in reprod, as well as

II **Value, Forms of Value** — Direct and Transformed Forms

1. $S \rightarrow \pi^{\circ} \xrightarrow{(Vol III, 43, 48)} \pi^{*}$ — 167: first transformation is with $\pi^{\circ} = S^{\circ}$

$V \rightarrow W^{\circ} \rightarrow W^{*}$

$\frac{S}{V} \rightarrow (\pi/W)^{\circ} \rightarrow (\pi/W)^{*}$

$\rho \rightarrow r^{\circ} \rightarrow r^{*}$

(i) Note that Marx (Vol III, 43) frequently will say: value of commodity is 100 hrs, or what is the same thing, £200. We will say value or direct price

2. We can derive determination of direct form of value as a simple transformation of value magnitudes. $TP = \frac{TV}{\lambda_G}$, $\pi = \frac{S}{\lambda_G}$

Citation Vol I →

(1) But even so < difference in units (hrs vs. \$ or og. of gold)
difference in movements (even direct price is a relative value)

3. Tranformation

(i) We know the general problem: with prices proportional to values, rates of profit are unequal. Thus prices will have to deviate from direct proportionality. Analytically prices of production are TRANSFORMED FORMS-OF-VALUE.

Citation MC comp →

(ii) General Task is two-fold < derive New forms
show that previous laws are still valid

sheets on transf. (Ross's course) →

4. First problem: Derivation < Calculation
Conception

~~Marx can $\Pi \neq S$ ⇒ Prices of Values~~

5. Second Problem: Still Valid Laws (DISPLACED REFLECTION = TRANSFORMED FORM OF VALUE

(i) Relationship between $\frac{S}{V}$ and r^{*} is same form as relationship between S/V and r° (Morishima, Marx's Economics, Cambridge, 1973, ch 5-6
Shaikh, Theories of Value & Theories of Distrib 1973, Ch IV, Sect. 4)

(ii) $r^{*} = \rho^{+}$ (PER)

Quote Marx on avg. ind.

In PER < $\pi^{+} = S^{+}$
$TP^{+} = TV^{+}$ } THIS IS "AVERAGE" INDUSTRY MARX REFERS TO

(Shaikh, 1973)

(iii) Direction of deviation of prices from values

a) Marx uses $\left(\frac{C}{V}\right)_{avg}$ as basis for direction of deviation: $P \gtrless \lambda$ as $\frac{C}{V} \gtrless \left(\frac{C}{V}\right)_{avg}$

b) In three-Dept, both $\left(\frac{C}{V}\right)_{avg}$ and (C/V) PER "work" — because both lie between $(C/V)_I$ + $(C/V)_{II}$

Seton Rest. Stud. June 1957

(V) Regulation of Indiv. Prices of Production

c) In general ? seems to be wrong

(IV) Size of Deviations
(See empirical evidence)
RAISE THEORETICAL QUESTION OF LEVEL OF ANALYSIS AT WHICH DEV... NOT IMPORTANT ... (Elbar)

Just two causes of changes in price of production P_j:
1. changes in r ⟷ changes in value of LP ⟷ changes in S/V
2. changes in λ_j ⟷ changes in l_j, in a_{ij} or in λ_i ... 206] [see also Ch IX, p. 166]

(ii) Profit & Surplus Value : Once again, the very formulation is alien to me

— On the surface, individual profit = surplus on balance of trade
$= p - (c+v) = \pi$

— But raises question: How is this possible for capital as a whole

— Meek notes that ~~[illegible]~~ long before Marx's time, it was well known in political economy that there were two basic sources, Marx discusses these several times (TSV I, ch 1)

Profit on Transfer into Sphere of Capital
= Profit on Alienation (Marx)
= "Unequal Exchange" (Buy cheap, sell dear)
"Mercantilist" Period
Neoclassical Eco.

Profit on Production of Surplus
= Profit in Surplus Value
= Gain on balance of trade with the workers, in which they pay in v hours worth and get e = v+s hours worth ⇒ length of working day, surplus l[illegible]

not a trade, but an Extraction of surplus labor time

– Thus Marx's *begins* story by isolating one from the other, by forbidding unequal exchange ⇒ exchange at values

If workers work long enough to just reproduce themselves, $\ell = v$, no surplus product, & no profit even when prices ≠ values.

If surplus labor time & consequent surplus product, then $\pi, s > 0$ but it does *not* follow that $\pi = s$ *once we allow for unequal exchange*

⇓

Transfers from/to circuits outside capital circuit
⇒ Profit on alienation
[This comes up for Marx – DR]

[PRINCIPLE OF CONSERVATION OF VALUE *WITHIN*

Deviations of Profit / Surplus Value 4/28/81

1. I have shown how the deviation arises, from the transfer of value between the circuit of capital and the circuit of revenue

2. I have also shown that it is greatest when in simple reproduction [and reproduction of some sort is presupposed for the actual existence of prices of production] NOT STRICTLY TRUE.

CONCEPTUALLY, I have not shown why outside of simple reprod Π ≠ S [i.e. what exactly gives rise to this difference]

3. What needs to be shown how big it is relative to surplus value itself: i.e., one must show how large price-value deviations can be in "Dept III", relative to surplus value as a whole → how big is the % transfer of value.

(i) ~~At~~ the most abstract level we can only posit the limits themselves. But a proper abstraction is one which understands the ~~nature of the~~ relative size & range of these deviations themselves, and of the necessity of this ~~relative~~ range

(ii) Note that to have a "model" in which the organic composition variation is infinite is to precisely ignore the fact that capitalist relations force the adoption of new methods and hence the force the generalization of technical change

(iii) Moreover, a two "industry" model is very different from a two sector model. Only the latter is relevant, precisely because it represents an abstract division in the whole world of industries (+ precisely a world of industries of hence commodities is necessary for generalized commodity production). Any such abstract division is therefore an aggregation of industries, and must reflect that.

Eco 204 11/4/87

1. Last Time $\frac{p_i^*}{p} = \frac{\lambda_i}{\lambda} z_i$ where $z_i = \left[\frac{1+\frac{r}{w} Q_i}{1+r/w \bar{Q}}\right]$

→ Two causes of changes in price: $w-r$, $\lambda g^{(n)}$'s $Q_i \equiv \left(\frac{K^{\$T}}{\Lambda}\right)_i$

→ Cross Section Hypotheses

→ Time Series Hypotheses

→ Data

p/λ

p vs r_m, $\bar{r}$

$w-r$ ~~Relatives~~ curves!

2. Reprise : (i) Tendential Regulation ≠ equal
Mkt price ≠ price of prod.
(ii) [Hence "choices" made
under turbulent & uncertain conditions
(iii) Domination of Mkt price by P.O.P as "c.g",
and domination of P.O.P by labor values as "principal regulator"

3. Steedman's Critique — <u>redundancy</u> of Marxian path through values to POP (production data; INCONSISTENCY; PRIMACY (calculation made in terms of POP) "determination"

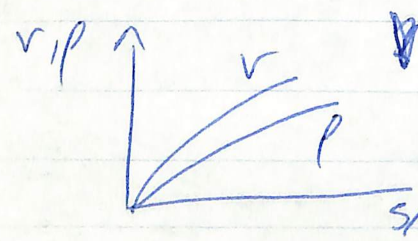
(i) Values irrelevant to ~~calculation~~ of POP

— Determination as Calculation

— Values as mental exercise" (idealism)

— Values in the real are the materiality of LT in production → output transformed by human labor into outputs → outputs sold at market prices → P.O.P. as C.G. of market prices and P.O.P. as expression (transformed) of structure of labor times → <u>real values as regulators of prices of prod</u>

(ii) $\pi \neq S$ → already shown to
$r \neq p$ be a <u>consequence</u> of conservation of value in exchange.



(iii) Choice of technique incorrect if done on basis of "values". Quite right. Also if done on basis of POP! → Apparatus is intra

Notes on LTV IV

4/29/81

I Method

1. Materialism implies priority of real over thought about real (Althusser, Colletti)
2. Abstraction is a process of simplification, typification (Colletti, 42)
   (i) ~~Abst~~ Marx: Concrete → Abstract → Concrete
   (ii) Numerical representation must be valid ~~of~~ representation
   (iii) Marx on Abstraction
   (iv) Real Determination versus calculation
   (v) Difference between ~~real~~ abstraction

3. Concept of Economy, Social Reprod
   (i) Mode of prod, labor, surplus labor (see notes on Modes of Prod)
   (ii) labor as objectification of the ideas & intentions of laborers (Colletti 67)

   (iii) Base / Superstructure; conditions / limits / . . .
   (Althusser, Colletti, .)

4. Commod. Prod, Cap. Commod Prod
   (i) General Contradictions, exchange, . . –
   One Commodity World
   (ii) Abstract labor: real abstraction process, not mental generalization (Colletti, 79-81)
   Dobb, Sweezy, Steedman,
   Implications for Steedman, et al (Bartwell too) on "determination"

Notes on LTV II 4/29/81

(iii) Capital, Competition, Prices of Prod: Real process, ~~actual~~ formation of prices of prod

a) Diff. between perfect competition as ideal and abstraction of real comp. and [illegible]

b) Steedman/Garegnani error here also of confusing real det. with calculcalculation

→ necessity of reprod proportions (Garegnani admits this...)

→ choice of technique as symptom of the diff.

→ Refusal to face implications of market price / price of prod. ~~complications~~ deviation (even if $P_M \cong p^*$, actual decisions are made in terms of market prices & may give "incorrect" results; Steedman paper)

Eco 205: Competition 4/13/88

1. Review of ~~[illegible]~~ Marxian Competition

(I) In Vol I/II, Marx assumes that mkt prices gravitate directly around values, & ~~[illegible]~~ examines implications of this < forms of value: Turnover. ROP, π/w etc. Schemes of Reprod (IMPLICIT here is mobility of capital)

(II) In Vol III, he considers further implications of mobility of capital → p.o.p. as c.g.'s of mkt prices ~~[illegible]~~
→ values as c.g.'s of prices of production

(III) Then, he takes up still more concrete implications < competition within industry
competition between industries, now in more detail

(IV) Technical change is now also concretized

a) given many capitals → what ~~[illegible]~~ determines "common" price within an industry → "Best" technique is cheapest generally available one → Regulating capital

b) Given many industries → ~~[illegible]~~ Tendential ~~[illegible]~~ Equalization of Regulating ROP's

(V) What is MONOPOLY (vs. Concentration & Centraliz.)?

2. Review of N.C.

(I) Equal markups & ROP's < within industry
Between industry
> hence in any subset

(II) Empirical ~~[illegible]~~ Hypothesis implicit → Non-Persistence of differences

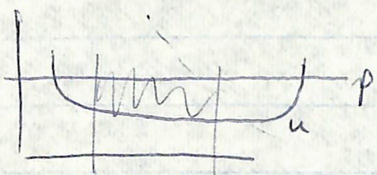
(III) Indications of failure of empirical hypothesis → Imperfect Comp, Monop., Olig., ~~[illegible]~~

Eco 205 4/13/88

→ Neoclassical notion of monopoly ...
oligopoly ...
imperfect competition ...

3. Kalecki

(i) For *avg firm* ~~p = ...~~ $p = \left(\frac{m}{1-n}\right)u$ (theory of price)
$m > 0$, $0 < n < 1$



→ Kalecki does *not* pursue the question of how the average level of output in the industry or firm is determined (he does say that it is in flat *range*)

(ii) Instead, he shifts to *aggregate* output which he determines in a way very similar to Keynes

→ Steindl takes up issue of *industry*

(iii)

(III)

(IV) What then does change in prices do?

It brings about transfers of value. And thus, it would seem, could only change individual shares but not totals.

But we shall see that problem has two aspects: transfers between capitalists
transfers from circuit of capital to revenue

(V) Return for a moment to distinction between prod / exchange (p. 3 of yellow sheets)

1. Price of Prod-Value Deviations

$$\mathbf{P} = pA + wl + rpK$$

total price ratios $P_i/P_j \cong \frac{\Delta_i}{\Delta_j}$ if

$\frac{w}{q} \sim 1$ *

z_i is small

Marx's Sol. as Iteration

Quadratic Approx (Bienenfeld)

2. Linearity of w-r curves (see notes)

3. Critique of Neo Ricardians

LTV III

1. $p = pA + wl + rpK \rightarrow p[I-A] = wl + rpK$

$\rightarrow p = w\lambda + rp\underbrace{H}$, $H \equiv K[I-A]^{-1}$

(Quadratic or cubic approx?)

$p_i = w\lambda_i + r\frac{K_i^T(r)}{\lambda_i}$ $\qquad p_i = w\lambda_i + rpK + rpK^2 + \ldots$

$P_i = w\Lambda_i + r K_i^T(r)$

$P_i = w\Lambda_i \left[1 + r\frac{K_i^T(r)}{w\Lambda_i}\right]$

$$\frac{P_i}{P} = \frac{\Lambda_i}{\Lambda}\left[\frac{1 + r/w\, K_i^T(r)/\Lambda_i}{1 + r/w\, K^T/\Lambda}\right] = \frac{\Lambda_i}{\Lambda}[z_i]$$

$$\ln\left(\frac{P_i}{P}\right) = \ln \pi_i = \ln\left(\frac{\Lambda_i}{\Lambda_g}\right) + z_i$$

2. DATA NOT DUE TO HIGH OBSERVED WAGE SHARE

Critique of Neo Ricardians

I Materialist Analysis & Role of social labor time

→ Object of analysis

→ Look to Values as More Fundamental < Explain Inner Regulators

II Hence Classical & Marxian focus on Labor Time as Regulator of Value:

E.G. Ricardo: $\frac{p_i}{p_\gamma} = f\left(\frac{\lambda_i}{\lambda_\gamma}, z_{i\gamma}\right)$, $z_{i\gamma} = F(w, r)$

where λ_i/λ_γ = dominant component

Total Sales: $P_i = \Pi_i + W_i + M_i = (\Pi_i + W_i) + (\Pi_i^{(1)} + W_i^{(1)} + M_i^{(1)})$

= ...

Smith/Pasinetti decomposition: $P_i = W_i^T + \Pi_i^T = w\lambda_i + rK_i^{*T}$

$\rightarrow \frac{p_i}{p_\gamma} = \frac{\lambda_i}{\lambda_\gamma}\left[\frac{1 + r/w\, Q_i^*}{1 + r/w\, Q^T}\right] = \frac{\lambda_i}{\lambda_\gamma} \cdot z_{i\gamma}$

This is Ricardo's point → Two causes of rel. price variation, but $\frac{\lambda_i}{\lambda_\gamma}$ is dominant

→ Cross Sectional (Ricardian) Hypothesis $(p_i/p_\gamma) \cong (\lambda_i/\lambda_\gamma)$

→ Time Series (Ricardian & Marxian) $\left(\widetilde{p_i/p_\gamma}\right)_t = \left(\widetilde{\frac{\lambda_i}{\lambda_\gamma}}\right)_0$

→ w-r curves

Marx's

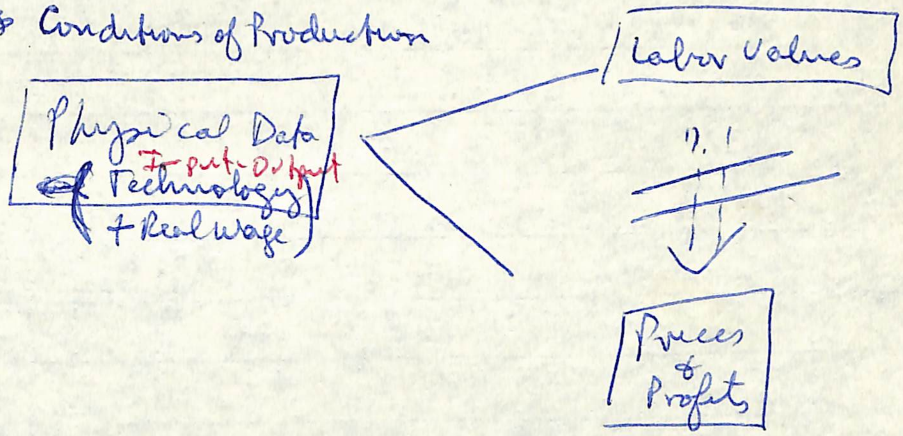
UCR 11/6/79

I ~~Cr~~ Critique of neo-Ricardians

1. NLR conference (see report on NLR conf.)
   Is LTV an impediment or not ← yes (Steedman, etc.) / no (Sweezy, etc.) / yes, in ~~practice~~, to both sides (Shaikh)

2. Basic argument of Neo-Ricardians (Steedman)

~~CP~~ Conditions of Production



Prod. as a labor process

(i) Conditions of production determine values and prices simultaneously. Thus values are an unnecessary ~~detour~~ since we can go straight to prices

Tendential Regulation π/S, transfers r, ρ etc. Abstraction as specification

(ii) ~~Moreover,~~ They give ~~different~~ different results

$r \neq \rho$, $\frac{\Pi}{W} \neq \frac{S}{V}$, etc.

~~P~~ Competition as war

See card [(iii) Tendential Regulation

Moreover, "prices" determine which technique is chosen, so they determine values [BUT THEN, SINCE CHOICE IS ALWAYS MADE AT MARKET PRICES, MARKET PRICES "DETERMINE" PRICES OF PROD. ALSO]

(iv) In J. P., ~~etc~~ etc., ~~negative profits~~ positive profits is possible with negative S.V. So ~~value~~ value categories are not merely redundant "approximations", they are directly contradictory.

(v) in any case, surplus product (or maybe surplus labor!) is sufficient; not need of surplus val...

VCR

3. Conceptual Differences

(i) Social Reprod < material basis ⟷ } class, division of labor, / Social Relations

(ii) Commod. Prod → forcible articulation of private labor in-and-through exchange < necessity of money / prices as immediate regulators / prices as dominated by social labor-time. / competition as trial through error [No stable equilibrium

(iii) Capitalist Commodity Prod < forcible generalization of commodity sphere / forcible alteration of labor-process (mechaniz.) / capitalist competition as a violent & ever present war [not a ballet]

(Immediate "choices" not made in terms of "price of prod.) [See card on choice of technique]

4. Implications

(i) Conception vs. Calculation — can differ in conceptions but not in (some) calculations (Astronomy / Astrology)

Difference between real determination and " our calculation

(ii) Concept of "physical data" of determination of value & price

a) confuses MOP + LP as C (commodity capital) with them as P (~~productive capital~~) and hence performance of labor. b) Confuses real process with our calculations labor process → appears as I-O data → calculation [definition of commodity roots in concept.]

(iii) Concept of "one-commodity" world of Sraffians → i.e. price / value deviations define commodities

(iv) ~~Prod / Unprod~~ Prod/Unprod Prod vs. Distribution. ~~external~~ displacement ~~effect~~ ⟷ essential unity

Limits of P-λ differences

a) Π/S b) v/p c) p_i/λ_i → Transfer within circuit of cap / Transfer Between Cap + Revenue

(v) Inability to face $p \cong \lambda$, for fear of neoclassicals ~~Σ deviations~~ $p_\gamma \cong \lambda_\gamma \Leftrightarrow \Pi_\gamma \cong S_\gamma$

(vi) Concept of Transfer of S.V < [why is rate of profit what it is for a particular capital < transfers due to formation of social value " " " " of price of prod ~~[illegible]~~)

(vii) Money & R[illegible] — gold as money has no price. How is $\bar{r}$ formed here? (AR?)

[illegible] "in plat" to reproduction,

118 Bldg

(v) Joint Prod: Real problem is market value, regulating value, individ. versus social value

— At this level, problem is not negative values but too many individ. values of ~~process of prod~~ → at this level of abstraction

— Marx ~~[illegible]~~ argues that competition within an industry produces common price of output but unequal rates of profit.

— Mkt value

— DR & AR,

— So by what process is this common price determined?

— Alternatively, what is process whereby alternative techniques can have same ROP?

$l_1 + x_{11} + x_{21} \rightarrow x_1^{(1)} + x_2^{(1)}$

$l_2 + x_{12} + x_{22} \rightarrow x_1^{(2)} + x_2^{(2)}$

— Marx defines value as value transferred, c, + value added by living labor = l. Thus

$(p_1 x_{11} + p_2 x_{22}) + wl_1 = \text{unit cost}_1 < p_1 x_1^{(1)} + p_2 x_2^{(1)}$

$p_1 x_{12} + p_2 x_{22} + wl_2 = k_2 < p_1 x_1^{(2)} + p_2 x_2^{(2)}$

→ what does

negative values ⇔ $c \leq 0$,

→ prior stage $l < 0$