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Writing as Active Learning in Gateway Undergraduate Mathematics

Amanda Landi
Bard Academy and Bard College at Simon's Rock

Kaethe Minden
Bard Academy and Bard College at Simon's Rock

Abstract

In this paper, we describe a writing and thinking intensive curriculum for gateway undergraduate mathematics. Furthermore, we report on our observations of this curriculum as implemented in our course called Elementary Functions. We found that using writing as active learning in the course increased transparency between students and faculty, improved students' metacognitive skills, and increased students' confidence in their mathematical abilities. In the conclusion, we describe several ways future research can contribute to these preliminary promising results.

1 Introduction

There exist many pedagogical theories for effective teaching in the mathematics classroom, such as problem-based learning, constructivism, inquiry-based learning, collaborative learning, and so on. In the United States, these theories of learning formed for many reasons. One such reason is to close the gap of women and people of color in mathematically related fields, i.e. the Science, Technology, Engineering and Math (STEM) fields. To address challenges, such as the gender and racial divides, there have been curricular "movements" to change mathematics classes. For several decades, there has been a strong push toward active learning within a mathematics classroom. Active learning requires the student to engage in and strengthen their skills, such as problem-solving and critical thinking, thus improving attitudes toward learning, as well as their overall capacity for thinking (Akınoğlu, Orhan and Tandoğan, R.Ö, 2007). Prince (2004) describes active learning as an umbrella term for any instructional method that engages students in the learning process (e.g., collaborative, problem-based, inquiry-based, constructivist).

A relatively new active learning model is the flipped classroom. That is, students learn the course concepts outside of the classroom while in-class time is reserved for more active problem-based learning. The purpose of the flipped classroom paradigm is to hold students more accountable for preparing properly for the course by reading, or watching a video, about the concept for the following class period. Students may be expected to bring questions or understandings to class discussion so that in-class time may be used more productively. Love et al. (2014) discuss the findings of a study on the flipped classroom in an undergraduate Linear Algebra course, which involved 55 students in 2 sections. One section used the traditional lecture format and another section used the flipped classroom format. Love et al. compared scores on exams for content knowledge, and found that the class with the flipped model had larger positive changes in scores between exams. Additionally, student attendance and in-class engagement were higher. Carter et al. (2018) describe a study they conducted in order to determine the effect of flipping the classroom in general education mathematics courses in a Liberal Arts college, and they state they believe there is potential for the flipped classroom pedagogy to reduce the gap in test scores of underrepresented students.

An active learning structure that has not been extensively studied enough as a method of increasing engagement in learning in STEM areas is writing. Although modern day theories of learning recommend environments in which students tackle mathematical problems and construct their own understanding of theory and how to problem solve, these approaches have gaps because they ignore the limits on working memory (Dubinsky, 2000; Kirschner et al., 2006). Writing as active learning is meant to foster critical thinking skills that current approaches do not quite do, using long-term memory in support of the working memory. Students will learn how to learn, reinforcing metacognition skills that will support them throughout their STEM education (McGuire and McGuire, 2015).

Writing in the mathematics classroom has been suggested and studied prior to this paper (Bicer et al., 2013; Connolly, 1989; Cross Francis, 2008; Doe et al., 2016; Hoffer, 2016; McCarty and Faulkner, 2020; Parsons, 2007; Seto and Meel, 2006). The setting in which each study is done varies, spanning from elementary school settings to first-year college courses and from private to public institutions. However, a common thread is the idea that in order to maximize a student's learning, both environment and individual learning factors must be considered.

Whitin and Whitin (2000) describe their study of the use of writing in a fourth grade math classroom. They express that, in mathematics, it is important for students to construct their own knowledge, and they encourage writing, discussion, drawing, and modeling as means for the creation of individual information representations. The purpose of allowing all these activities is so that students are empowered to think for themselves. They explain that writing and discussion allow students the opportunity to share their mathematical ideas to a real audience (not just themselves), and students respond to regular invitations to write and discuss in interesting ways: "the risks that learners are willing to venture in mathematics are a reflection of the kind of community in which they live. Creating a community that supports the expression of mathematical ideas in many different forms is an important dimension of this risk-taking stance." Whitin and Whitin recommend teachers highlight the process of problem-solving, recognize the difference in thinking of others, and invite the students to reflect on lessons with math journals.

While Whitin and Whitin highlight the benefits of writing in class for a student's learning, Urquhart (2009) states that writing in mathematics is also beneficial to the teacher. She points out that reading a student's writing gives the teacher a chance to better assess understanding and progress as time goes on, providing the teacher with information needed to adjust instruction. Not only does Urquhart say that it's beneficial, but she communicates that a teacher can easily implement writing in their class simply by taking their own lessons and add a writing activity that will enhance student engagement and heighten cognitive demands. Urquhart provides details for writing techniques an instructor might use in their class, mostly geared towards the high school mathematics classroom. One tool suggested is a math journal. The journal allows students to freely express their thoughts and questions on any given topic. Whitin and Whitin (2000), Park et al. (2014), and McCarty and Faulkner (2020) support the view that a math journal decreases math anxiety in students and increases positive student attitude toward math class.

There are fewer studies at the college level for a writing-based pedagogy in mathematics courses, though. Explanations for why vary, including time constraints, lack of professional training, or inability of faculty to see its usefulness (Doe et al., 2016; Parsons, 2007; Seto and Meel, 2006). In the Writing as Active Learning in Gateway Undergraduate Mathematics (WALGUM) research project, however, we have integrated a writing intensive pedagogy into an active learning environment. Moreover, this curricular design targets gateway undergraduate mathematics courses since they are often the gatekeepers for STEM studies and careers, and these courses influence a student's decision about whether to continue on a STEM-related path. Indeed, the importance of such courses is emphasized by Koch (2018); Love et al. (2014); Swan et al. (2018). Perhaps this new form of active learning can significantly impact the field of mathematics.

1.1 Writing and Thinking Pedagogy

The WALGUM model is inspired by the Writing and Thinking (W&T) pedagogy, based on work from Bard College's Institute for Writing and Thinking (IWT) and practiced by Bard College at Simon's Rock (SR). W&T is designed to emphasize that "writing is as much a means toward thinking as it is a result of it," and includes a robust set of techniques aimed at teaching students to explore ideas, share their thoughts, listen to others, and evaluate their own and others' work with thoughtfulness and respect (Weinstein, 2011). At SR, the workshop is a mandatory, five-day orientation to the liberal arts and "writing as a form of thinking" for first-year college students and is taught by full-time teaching staff and faculty at the college. The W&T culture of listening, sharing, and respect is then carried into the regular academic year in various other courses in slightly different ways, and students are expected to apply what they learn in the W&T workshop throughout their time at SR.

While W&T and its techniques are the basis for the WALGUM model, other similar, and more common, pedagogical research areas are "writing to learn" (WtL) and "writing across the curriculum" (WAC). Our referenced works tend to come from these realms (e.g., Hoffer, 2016; Urquhart, 2009; Whitin and Whitin, 2000). Although the names of W&T, WtL, and WAC may suggest there are significant differences between the pedagogies, they essentially have the same goals: (1) to cultivate a community in which all members bravely share their thoughts and actively listen to and respectfully respond to others' thoughts; and, (2) to encourage students to engage in their thinking process and the learning process of content-specific material. Because these are the shared pillars, and they are the foundation for writing and thinking in mathematics, it is important to understand better their significance. As the descriptions below demonstrate, the second pillar relies on a strongly formed first pillar and vice versa.

Pillar - Community of Practice: We can describe the first pillar of Writing and Thinking as a community of practice (CoP). That is, a CoP is formed by individuals who engage in collective learning, each person taking responsibility for creating and sharing knowledge (Wenger, 2011). The culture of W&T aims to foster in students the ability to hear thoughts different from their own, to critically think, and to respectfully respond to others. Each voice is heard and valued, and the instructor must highlight in each session that there is no right way to think or to write (Elbow, 1983). The writing allows each student to process their thoughts before speaking, and they read exactly what they have written.

While the goal is to de-colonize the learning space, the instructor does have power as the grader and as the engineer of the learning space. Therefore, their primary responsibility is to build trust so that students will feel safe enough to participate in materials and discussions that may challenge them to their very core, as bell hooks (2009) explains. This act of building trust will include transparency, not only about expectations and about the purpose of various activities, but an instructor must also make themselves vulnerable. The simple act of sharing thoughts and thought processes eliminates the "all-knowing" stereotype that students have about professors (bell hooks, 1994). All members of the CoP established become co-conspirators in the destruction of "the banking system of education"—the memorization and then regurgitating of knowledge being analogous to the deposit and storage of money in a bank (bell hooks, 1994).

Thus, members of the learning space should believe in the metacognitive teaching philosophy of W&T.

Pillar - Metacognition: The W&T pedagogy promotes the awareness and understanding of one's own thought process. This is the very definition of metacognition. The purpose of metacognitive training is to give students the ability to transfer their knowledge to new situations by training them to identify the essential elements that are similar or different from already completed and targeted tasks, which is documented as a limited skill among students (Kramarski and Mevarech, 2003; McGuire and McGuire, 2015 Schraw and Dennison, 1994). Moreover, learning to learn helps students change their language from blaming others to taking control of their own performance (bell hooks, 2009; McGuire and McGuire, 2015). Thus, this training will empower students as learners.

Writing and Thinking tools help students explore two types of processing. Type 1 thinking, or first order processing, is based on one's beliefs, thus it relies on immediate ideas, generating only representations of a concept. Type 2 thinking, or second order processing, is analytical and reflective, it's used to evaluate, so it should be more controlled and explicit (Elbow, 1983; Hand et al., 2014). A *one-minute paper* might be used after a concept is introduced as a way to engage type 1 thinking. A *read-around* can follow so that ideas are shared. To engage type 2 thinking, students might respond in writing to another's idea, or they might take the conversation home to explore in more depth what occurred in the class period. When students are participating in metacognition, the material matters more than how it is delivered (McGuire and McGuire, 2015). Therefore, the instructor is responsible for developing activities that helps to foster students as self-aware learners and critical thinkers. Students are responsible for seriously engaging in each activity with the knowledge that it is designed to benefit them.

Thus, the metacognitive training is only effective when a sound community of practice is established.

1.2 Target Population

This paper will focus on the implementation of WALGUM in the gateway course called Elementary Functions. Sometimes called "College Algebra" or "Pre-Calculus" at other institutions, Elementary Functions is the precursor to the Calculus sequence and explores properties of polynomial, exponential, logarithmic, and trigonometric functions. Students who typically take this course in college take it in the first two years of college. We use institution data, from 2008-2018, to describe the Elementary Functions population at Bard College at Simon's Rock[1]. As stated is typical of gateway courses by Swan et al. (2018), students who place into Elementary Functions have demonstrated inadequate backgrounds through our placement exam, on which they receive scores between 16.5 and 27 out of 44, which puts them in the 48th percentile. Approximately 23.6% of students who completed this course received a C or lower; 7.4% of students who enroll withdraw from the course; 23.3% of students who enroll drop the course. Of the students who enrolled and completed Elementary Functions, only 47% enrolled in Calculus I in any of the following semesters. Thus, a majority of the students who complete Elementary Functions do not continue in mathematics, barring them from pursuing 70% of the STEM programs at Simon's Rock. Of this 47%, only

26% of these students continued in a Simon's Rock STEM concentration. WALGUM aims to improve retention and progression in STEM courses, beginning with the Elementary Functions point in the core mathematics track (which after Elementary Functions goes in successive order into Calculus I, Calculus II, Linear Algebra, and Vector Calculus).

1.3 Outline

Section 2 will describe the most essential features of Elementary Functions WALGUM (EF-WALGUM) as well as the implementation details we have followed in the past. In Section 3, the successes and failures from the implementations of WALGUM, based on instructor observations, will be discussed. In Section 4, we highlight the strengths and weaknesses of this model in the gateway undergraduate mathematics classroom based on the evidence presented in Section 3. The manuscript concludes with Section 5 with a description of planned future work.

1.3.1 Contributions of WALGUM

Our two primary contributions are

1. The design of a Writing and Thinking curriculum for the undergraduate mathematics course, Elementary Functions.

2. A preliminary body of evidence for the math education community as to whether or not Writing and Thinking in mathematics improves students' experiences in and learning of mathematics.

2 The Curriculum

In the Elementary Functions course, before WALGUM, there were 110 learning objectives, and the goal was to teach them all over 35 class periods. This was usually done, by at least one of the authors, in a primarily lecture-based classroom with weekly homework assignments (from the textbook or an online homework system such as Mathmatize or WebWork) and four midterm exams, followed by a final exam. We note here that at Simon's Rock, every math class taught by a full time faculty member must have some writing component, which has typically been one manuscript. We speak more about manuscripts in Section 2.1.3.

In the design of WALGUM, we reduced the number of learning objectives to 80, and they are covered through readings, homework practice problems, W&T exercises, in-class problem sets, midterm exams, manuscripts, and a learning portfolio. The learning objectives for each lesson are covered in the WALGUM scripts, and a sample of the scripts are available in Appendix A. Moreover, students are only expected to demonstrate comprehension for 65 learning objectives as part of their final learning portfolio. See the full list of these objectives in Appendix B.

The WALGUM curriculum is designed to be W&T-intensive both inside and outside the classroom and requires the synthesis of engaging writing activities and mathematical exercises that support the most important concepts within a semester. To make sure that the optimal amount of content is covered with a manageable amount of writing and mathematical problem-solving, the course should have the flipped-classroom structure. Indeed, Doe et al. (2016) found that writing instruction fits more favorably with the flipped-classroom setting as well as provides variation and substance.

Beginning on day one, the instructor must make it clear the course will be writing-based. There should be a statement in the syllabus[2], and a discussion in class about the course design. Moreover, the first month of the course is WALGUM-intense so students understand that they should take the writing seriously. Because the math and the writing inform one another, a student can only find success in the course if they seriously engage in both components. Therefore, this first month is the most important. After the first month, there is a reduction in the amount of reading and writing prompts to lighten workloads.

2.1 The Most Essential Features

WALGUM aims to have more informal check-ins with students than formal check-ins, and it intends to have a mix of both formative and summative assessments for student learning, similar to Parsons (2007). There are five main components of the curriculum: homework assignments (math problems and math journal), class participation (written and discussion), manuscripts, exams, and a learning portfolio. We describe each one in more detail here, including recommended implementation design.

2.1.1 Homework

Each lesson is preceded with a "preparation" homework and ends with a "review" homework, and each assignment has at least one writing prompt and several math problems. A recommended time limit is given so a homework would not take longer than 60 to 90 minutes *and* students are encouraged to learn to use that time productively as described in the assignments. This cycle, and many of the activities, are inspired by the learning cycle described in McGuire and McGuire (2015).

The homework is due twice per week. One homework assignment is a collection of at least 5–10 math problems, and were assigned daily for each set of learning objectives. The other homework is a collection of the writing prompts plus responses, called the math journal. Note, one homework (containing prompts *and* problems) can be collected per week, but more frequent collections makes the feedback load manageable for the instructor!

Math Problems are an informal formative assessment. Students still require opportunities to practice/test their computational problem-solving skills and content knowledge in order to be fully prepared for exams as well as future mathematics courses. Students are requested to select 5–10 of the problems per week (the number varied weekly and depended on the number actually assigned) to turn in to the instructor, with a mix of problems they struggled with and wanted help completing and of problems they felt proud they completed. They have to identify what problems fall into these categories through a description that comes alongside each problem. The assignments are graded based on completion (one point for math, one point for reflection), and instructor feedback is provided.

Math Journal entries are another informal formative assessment. The journal is recommended by many sources as an outlet for students to reflect on how they are feeling about a particular concept, assignment, or about the course in general (Bicer et al., 2013; Park et al., 2014; Parsons, 2007; Urquhart, 2009). When collected once per week, the instructor can determine where students are in their learning and can more effectively plan for future classes. The students are asked to turn in 5, 7, or 10 prompt-responses once per week (the number depends on the week and the number of homework activities). The journals are graded based on completion, and the instructor leaves feedback for the student, as a way to open up thoughtful conversation.

Optional: As a way to encourage serious participation on in-class writing based activities and to encourage reflection on that work and to be considerate of students with significant responsibilities outside of this particular class, students can be allowed to use their prompts+responses from in-class WtL activities for math journal entries. In which case, the number collected can be increased. Because of this dynamic, math journals should be considered for both the final homework grade and the final class participation grade.

2.1.2 Class Participation

Each lesson is full of opportunities for students to work on verbal communication, team work, and argumentation, as well as their writing and thinking skills.

Discussion: The purpose of a student participating in discussion should be to practice formulating a thought and speaking it out loud, to a group. Also, a student needs to practice recognizing when they do or do not make sense through feedback, whether it be their own, through hearing themselves, or another's. A guide for mathematical discourse, based on Hoffer (2016), is provided on the course website, to provide support to students who are less comfortable with engaging with others in this way. Moreover, students can read directly from their own writing, and in some activities these are the exact instructions (the W&T way).

Writing Activities: The purpose of informal writing is to develop the ability to define, classify and summarize, to develop methods of close and reactive reading, of organizing and structuring data, to develop the knowledge base on concepts in a course as well as knowledge of one's own problem solving thinking and learning. The prompts are designed to have students experience the material in varied ways and make meaningful connections between thoughts and ideas about concepts through W&T inspired activities. We describe some of the different W&T tools used and highlight where in Appendix A the example scripts can be found.

Private Free Writes: The first five minutes is an important part of the class period, in which student attention could be lost for the entirety of the class. The private free write is an activity in which a student writes nonstop for the fixed amount of time about anything that is currently on their mind (if nothing is on their mind, then they might write about their journey to the classroom). The purpose of starting in this manner is to give students time to clear their minds and become more mentally prepared to learn, a kind of transition. Each class begins with three *flexible* minutes of private free writing; it is three minutes since the class periods are only 55 minutes long. However, it is recommended by W&T experts for private free writes to take between five and ten minutes (Vilardi and Chang, 2009).

Loop Write:

- A *focused free write* is a free write with a prompt designed to explore a particular term, issue, question, or problem for a fixed amount of time. The prompt is meant to get the writer started, but the writer is still free to continue exploring their thoughts and ideas beyond the prompt for the fixed amount of time.
- *Loop writing* is three or more back-to-back focused free writes. The purpose is to deepen the exploration through different prompts designed to inspect the topic at hand from other perspectives or angles. In the Day One script, we can see that a loop write is used to help students synthesize some of the information from the previous night's reading, bringing it to the forefront for the day's activities.

Process Writing: This writing can be a piece that requires students to be aware of their thoughts as they occur, reflect on their process-outcome afterward, or engage in learning activities/strategies. The process writing assignments are inspired by the realm of metacognitive skill building, teaching students how to learn by asking them to engage in thoughts about their process (Kramarski and Mevarech, 2003; McGuire and McGuire, 2015; Parsons and Taylor, 2011; Schraw and Dennison, 1994).

For example, in the script for Day One, the last activity for the day has students re-examine (maybe even examine for the first time!) the learning objectives for the day. Before leaving the classroom, they are able to determine which learning objectives they will need to work on, and they can create a plan for working on those objectives. This will help students to be more self-aware of their learning and learning needs.

Believing and Doubting: In this activity, students are asked to write for three to five minutes confirming an author's viewpoint. The students are then asked to write for three to five minutes disagreeing with the same author's opinion. The purpose of this writing activity is to put students outside their comfort zone, encouraging them to understand another's perspective (Weinstein, 2011). This technique can be used to have students critically examine multiple pathways to a solution for a mathematics problem.

In the Day 15 script, we see that students get to analyze a possible solution for a problem. Although students might find logical issues in the solution, they must develop an understanding for *why* someone might believe their steps have no fault. On the other hand, they must propose fixes to any issues that arise, practicing a different type of problem solving skill.

Writing from Images: An image is displayed, and students are asked to write about what they see. This is the practice of inferring a context from a single point in time. Depending on the image, this activity can inspire creativity or it can hone deductive reasoning skills. This is similar to the math classroom activity, *Notice and Wonder*, in which a graph may be posted on the board and students are asked questions such as "what do you notice?" and "what do you find interesting?" (Rumack and Huinker, 2019; Whitin and Whitin, 2000). If one posts one piece of a graph at a time on the board, then the activity can slow students down and get them to analyze what they see and to predict what they think might happen.

In the Day Eight script, we see that this activity (though listed as a loop write) can be used as a way to review material such as quadratic functions. It is also a unique way to introduce new material such as sine and cosine functions, as seen in the script for Day 23.

Dialectal Response Notebook: Students interact with different points of view with one another by passing a notebook between each other, writing as a mode of critical thinking, as described in Vilardi and Chang (2009).

In the script for Day Four, we have used a Dialectical Response Notebook as a way for students to problem solve together and even included a bit of acting!

2.1.3 Manuscripts

A formal formative assessment is provided to students through *manuscripts*. Manuscripts are long-term assignments in which students are asked to solve a complex problem and then write up their solution as if teaching another how to solve it. While research does not address *math* manuscripts explicitly, Urquhart (2009) recommends the layout of formal writing pieces to include a title, abstract, literature review, statement of the problem, body of report, ideas for future research, and a list of references. The purpose of this activity is to challenge the students outside the short writings such as free writes and journal logs. Moreover, manuscripts are well-established at the Simon's Rock institution. Most of our math courses assign at least one manuscript assignment.

In the EF-WALGUM model, there are two manuscripts assigned, one for each half of the term. There is an obligatory revision process; a rough draft is due before the final draft with three weeks between each draft. Students receive instructor feedback on the rough draft. The students are also encouraged to go through a peer review process, either with a classmate or with a class tutor. This mimics the academic peer review process, but also allows another pair of eyes to see the draft before the instructor sees it. Students are required to cite sources (even if it is just the course textbook, so they begin to learn to write like a professional in the field), and get a librarian's approval that their citations are correct. When students turn in their final draft, they are also required to turn in process notes. This way, students deeply reflect on their journey to the final product. There is a rubric to provide since this is typically the first assignment of its kind for students, and they don't know where to start or what to do. The rubric guides students, at least, on what the instructor will be looking for when they read the manuscripts.

All SR math faculty share material. We share the manuscript assignments, with only slight stylistic changes. We do not claim originality here. We chose to keep manuscripts as a central component to WALGUM because it is a writing-based activity with several benefits, such as the opportunity for students to write on a mathematical concept as though they are teaching it (much like a textbook) and to revise their writing. We also chose to keep manuscripts as a central component to WALGUM because they historically garner student appreciation, despite their difficulty.

2.1.4 Exams

There are three midterm exams, i.e., formal summative assessments. Students still require opportunities to test their problem-solving skills and content knowledge in order to be fully prepared for future mathematics courses. Exams help with this training; they encourage students to do a thorough summative review, students must apply their skills and knowledge on their own, and it provides students with feedback so they understand better what it is they are capable of doing without assistance. While the time-limit can cause an undue amount of anxiety and stress for some students, the benefits to the general population still justify using exams.

One midterm exam occurs at the end of "review" material, concepts usually covered in Algebra I, like linear functions and quadratic functions. The second exam occurs after polynomial and rational functions. The third happens at the end of the trigonometry unit. We have one more unit on exponential and logarithmic functions, and it is up to the individual instructor to have a fourth exam or not.

There are no practice exams provided, as recommended by McGuire and McGuire (2015). Students are informed that the list of practice problems for each class period is sufficient for their preparation. However, students are given the chance to earn up to 50% of points back using a make-up form like in Appendix C. This adds another revision process, further emphasizing that learning comes through practice and repetition.

2.1.5 Learning Portfolio

We replace the usual final, cumulative exam with a final, cumulative learning portfolio. A *learning portfolio* engages the student in a review process, and it is a formal summative assessment. According to Sole (2012), there are five types of portfolios:

1. an *ideal portfolio*, which contains all of the work a student completed,
2. a *showcase portfolio*, which consists of a selected subset of a student's best work,
3. a *documentation portfolio*, which contains work that the student has done over a period of time giving evidence of skills that have improved,
4. an *evaluation portfolio*, which includes some work that had not previously been graded, and
5. a *class portfolio*, which contains a student's grade on an evaluative assessment of the student by the teacher.

Although a portfolio can be more time-intensive to prepare for the student (as opposed to taking a two-three hour final exam), some portfolio types require less time, such as types 1, 2, 3, and 5, since materials have already been produced prior to any due date.

For the WALGUM portfolio, students are required to write a self-evaluation, using graded materials as evidence of learning, and must solve a never-before-seen problem set. They may choose to use problems from the final problem set as evidence of learning in the self-evaluation as well. The self-evaluation statement is scaffolded by math journal assignments throughout the semester. Additionally, there is a rubric for this portfolio project so students are provided guidance about what a reflection on learning might look like, with enough room for them to be creative stylistically. See Appendix B for the portfolio prompt and rubric.

The evaluation of the portfolio grade is determined by the student's self-reflection and their solution to the problem set, with the grade distribution up to the individual instructor. Though, we recommend leaning more heavily on the self-reflection since this emphasizes that the reflective and process skills learned throughout the semester, which are the most useful in the long run, are as valuable as the content knowledge. While content competency is important, our own experiences as students have taught us that the content mastery is lost shortly after the semester ends, regardless of whether or not we effectively learned a lot of the content. What we remember the most are the interactions with faculty, with peers, and the learning skills we gained. Focusing on the last item, we are able to relearn the content forgotten at any time. The question, then, is why test our own students for content mastery at all? Because, if the metacognitive training is effective, then students will have learned mathematics inside the short timeline of a three month semester too. Here, "mathematics" is not just content but also problem solving and communication abilities.

As described in this section, we have created a learning portfolio that combines types 1, 2, 3, and 5. The new problem set that is added as a way to help students test their own abilities would be akin to portfolio type 4.

2.2 Reflection on the Pilot Semesters

There were three pilot classes. Two sections were during the same semester and a third section was the following semester. One section was a remote class, taught on Zoom, and two sections were in-person. We will refer to these sections as SECTION A (remote; during semester one), SECTION B (in-person; during semester one), and SECTION C (in-person; during semester two). Please note: curricula are modified dynamically, and it may be true that, since the writing of this paper, EF-WALGUM may look very different.

There were a total of 29 students who completed the Elementary Functions course. For all three classes, the instructor kept a journal so as to keep track of the success of the script activities in terms of learning objectives, any timing adjustments that may be needed in the future, and to record their personal perception of how students received the curriculum.

In general, it was more difficult to read if students enjoyed class than if they disliked class. One challenging aspect of WALGUM, and we suspect any curriculum that aims to make students more autonomous learners, is the push-back from students. Some students were very vocal in their dislike (in particular, two male students who self-identified as good at math in their previous institutions), and this tainted the classroom atmosphere, especially in the remote setting, despite the instructor's efforts to encourage an open-mindedness in different techniques of learning. Others resisted this course approach more "quietly". For example, on Zoom, students could be seen laughing on screen at W&T prompts. Furthermore, to give an in-person example, when the instructor loosened the reigns on private free writing (PFW), i.e., "If you prefer to draw or meditate or review yesterday's material, do that! All of these methods will help clear your mind and help you focus on the day's lesson.", students stopped completely participating in PFW. This was observed when the instructor in the room looked up from their own

PFW. Finally, many students did not appear to be prepared for class, even with the recommended preparatory homework. They admitted this since many in-class activities depended on preparation assignments.

Despite this push-back, though, more students participated in the classroom than is usual. We don't mean only in number, but in the diversity of the participants. Many students who preferred to be quiet during class time participated thanks to techniques like *read arounds* or having a participator select the next participant. It should be noted that the instructor still primarily used a volunteer based system for sharing ideas and work, so an increase in participation is very exciting. Unfortunately, this still gave space to people who are overly confident (in the remote class, one student took up a significant amount of space and made others feel more afraid of sharing).

Another challenging aspect of this curriculum for the instructor is the practice of allowing students the space to struggle. A teacher's primary instinct is always to swoop in and "save" a student who is struggling with a problem. However, students must "get their hands dirty" with learning and with the mathematics before they can problem solve with confidence. WALGUM allowed students to so-to-speak play in the mud, if they chose to actively engage with the designed curriculum, and it did indeed increase their overall self-efficacy. This was made very clear through homework assignments.

2.2.1 Homework: Math problems

Requiring students to identify problems for feedback seemed to benefit metacognitive development. First, students had to connect with their work on a deeper level. For example, when determining whether or not they were proud of the solution, they had to take a step back and reconsider their solution: "am I proud of this? if so, why?" Furthermore, the task of marking problems they struggled with for feedback replaces the idea that homework is "busy work" with the fact that its purpose is to learn. Second, students had to explain their categorization, a form of process writing. This practice connected writing to doing mathematics.

Students who engaged in the identification and explanation process for practice problems demonstrated growth in a variety of ways. In Figure 1, we see two examples, one of a student who was proud of their work and another of a student who struggled with completing a challenging problem. The instructor was able to celebrate with the student who found a path to success, and even had fun along the way! And the instructor was able to give more specific help for the student who identified with what it was they struggled.

An additional benefit to this homework structure is that students can be invited to continue to work on problems rather than leaving them behind, further emphasizing that successfully understanding how to do something might take some individuals longer than the course pacing. For example, the student who struggled in Figure 1 could have redone the problem with the given feedback and turned it in with the next set of problems, and continue to do this until they felt proud of their work. Of course, it must be recognized that this is a privilege (the

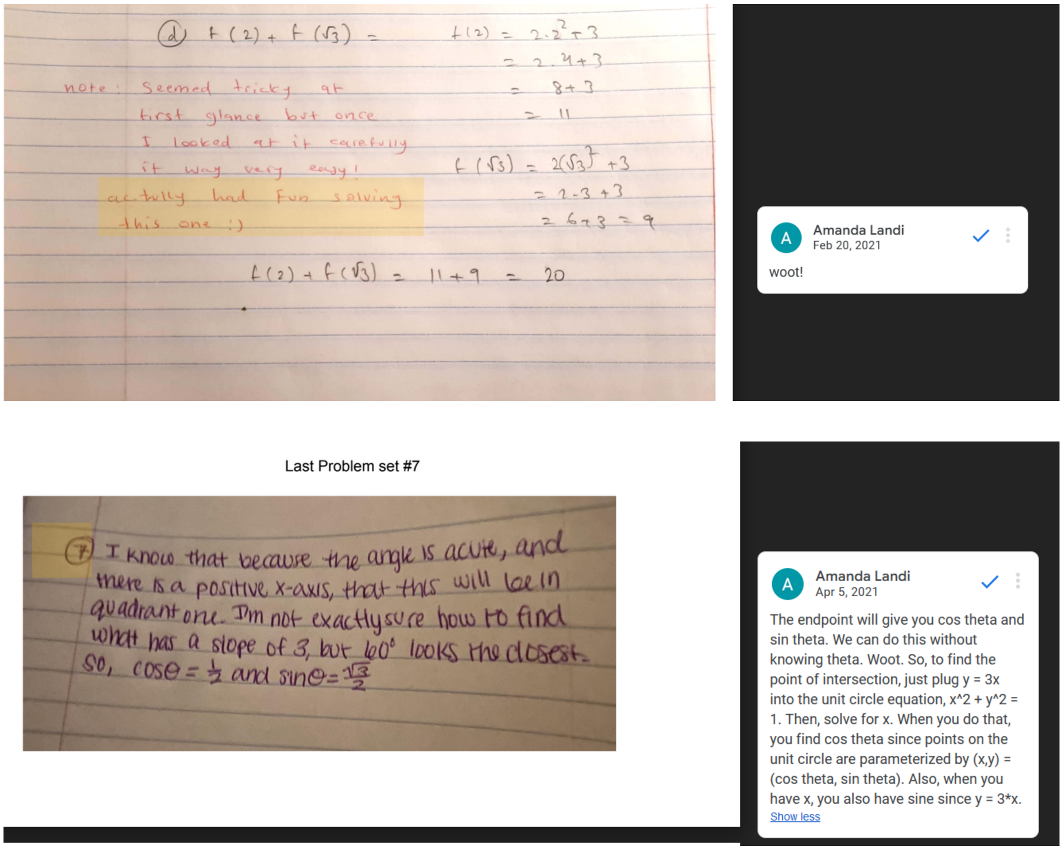


Figure 1: Process Write for Problems: on top, a student highlighted pride; on bottom, a student highlighted struggles

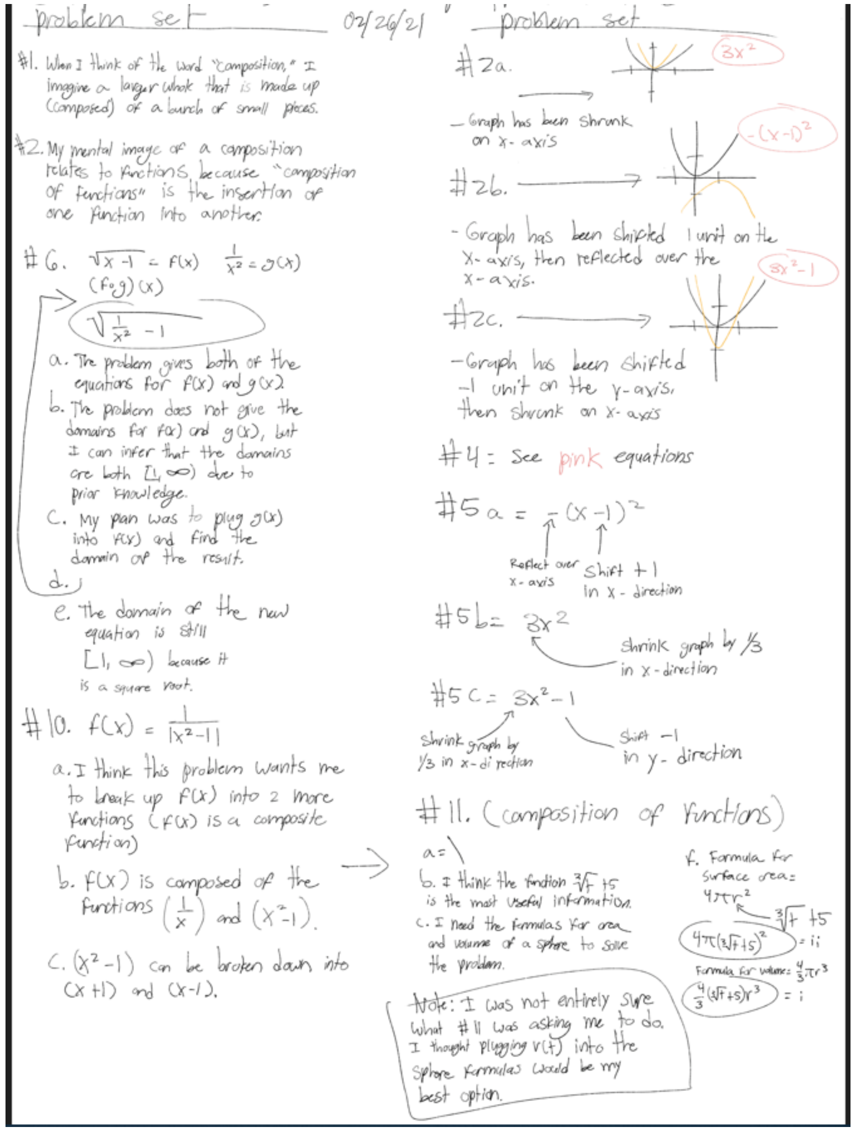
ability to continuously work on past problems while also keeping up with current work) that most college students do not have, especially those who have families to care for or those who have to work more than a few hours per week.

The work shown in Figure 1 is from two students who remained consistently engaged throughout the semester. They are examples of the best we could hope for on the math homework.

In general, it is not unusual for math instructors to ask students to write an explanation for their mathematical work, and Figure 2 would be an ideal assignment in other math classes. This student is showing their work and writing complete sentences when complete sentences are required. Moreover, the student is stating what it is they know from the problem and what they think they want to find based on that information. So, although this person isn't quite stating what they are proud of or what they struggled with, an instructor can still determine what it is the student has or has not learned and provide feedback.

2.2.2 Homework: Math journal

Although the identification process for practice problems was new for students, the mathematical content of the homework problems didn't require students to


problem set 02/26/21

#1. When I think of the word "composition," I imagine a larger whole that is made up (composed) of a bunch of small pieces.

#2. My mental image of a composition relates to functions, because "composition of functions" is the insertion of one function into another.

#6. $\sqrt{x-1} = f(x)$ $\frac{1}{x^2} = g(x)$
$(f \circ g)(x)$
$\sqrt{\frac{1}{x^2} - 1}$

a. The problem gives both of the equations for f(x) and g(x).
b. The problem does not give the domains for f(x) and g(x), but I can infer that the domains are both [1, ∞) due to prior knowledge.
c. My pan was to plug g(x) into f(x) and find the domain of the result.
d.
e. The domain of the new equation is still [1, ∞) because it is a square root.

#10. $f(x) = \frac{1}{|x^2-1|}$
a. I think this problem wants me to break up f(x) into 2 more functions (f(x) is a composite function)
b. f(x) is composed of the functions $\left(\frac{1}{x}\right)$ and (x^2-1).
c. (x^2-1) can be broken down into $(x+1)$ and $(x-1)$.

problem set

#2a. $3x^2$
– Graph has been shrunk on x-axis

#2b. $-(x-1)^2$
– Graph has been shifted 1 unit on the x-axis, then reflected over the x-axis.

#2c. $3x^2-1$
– Graph has been shifted –1 unit on the y-axis, then shrunk on x-axis

#4: See pink equations

#5a = $-(x-1)^2$
Reflect over x-axis
Shift +1 in x-direction

#5b = $3x^2$
shrink graph by 1/3 in x-direction

#5c = $3x^2-1$
Shrink graph by 1/3 in x-direction
Shift –1 in y-direction

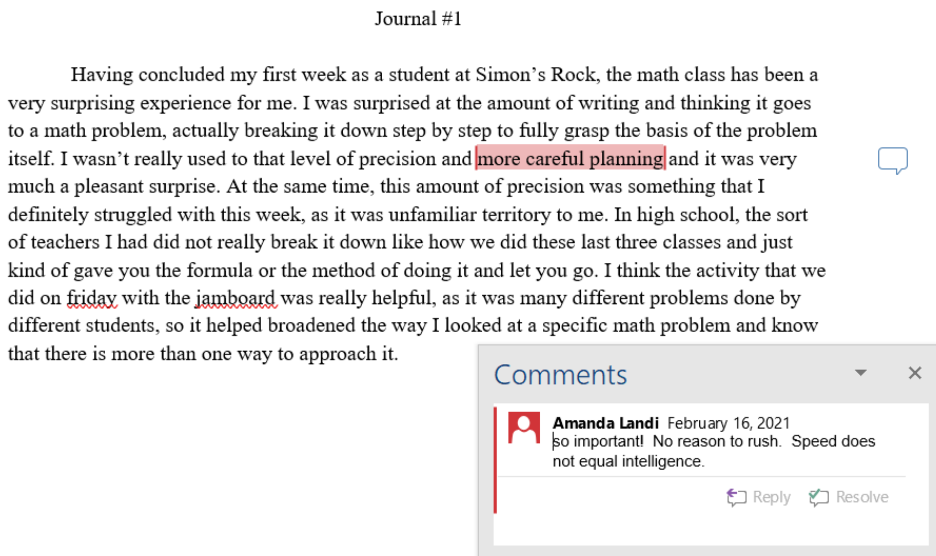
#11. (composition of functions)
a = \
b. I think the fnction $\sqrt[3]{t}+5$ is the most useful information.
c. I need the formulas for area and volume of a sphere to solve the problem.

f. Formula for surface area = $4\pi r^2$
$\sqrt[3]{t}+5$
$4\pi(\sqrt[3]{t}+5)^2$ = ¡¡
Formula for volume: $\frac{4}{3}\pi r^3$
$\frac{4}{3}(\sqrt[3]{t}+5)^3$ = ¡

Note: I was not entirely sure what #11 was asking me to do. I thought plugging v(t) into the sphere formulas would be my best option.

Figure 2: Process Write for Problems: a standard homework

think about mathematics uniquely. The writing prompts in the journal, however, were designed to engage students with learning and with mathematics in different ways than a standard math course might ask of them. In the Day Four script, students are asked to solve a math problem as if they are their academic advisor and they are asked to "act" out the solution at the board. This takes the pressure off students for getting the correct answer, but it asks students to think about how someone else might approach the problem. The presentations are always funny, even by students who don't know their advisors very well. This approach to the math classroom is not common in the undergraduate gateway math class. Besides this light-spirited activity, there are activities that are "more serious" but still have more creative approaches. For example, in the homework for Day 8, students are asked to state how they feel about quadratic functions. Students are also asked to tell the story of the first time they learned about the parent function x^2. While students might write about what class was like or what the homework was like in Algebra I, the prompt is purposely open-ended so that students can indeed tell a story, e.g., how their brain actually interpreted x^2.


Journal #1

Having concluded my first week as a student at Simon's Rock, the math class has been a very surprising experience for me. I was surprised at the amount of writing and thinking it goes to a math problem, actually breaking it down step by step to fully grasp the basis of the problem itself. I wasn't really used to that level of precision and more careful planning and it was very much a pleasant surprise. At the same time, this amount of precision was something that I definitely struggled with this week, as it was unfamiliar territory to me. In high school, the sort of teachers I had did not really break it down like how we did these last three classes and just kind of gave you the formula or the method of doing it and let you go. I think the activity that we did on friday with the jamboard was really helpful, as it was many different problems done by different students, so it helped broadened the way I looked at a specific math problem and know that there is more than one way to approach it.

(a)


I find it surprising that I was so frightened of domain and range. At this time I feel a lot more comfortable with them, thanks to your help, but I can still understand that they were so hard for me. Even now I find them to be annoying, but not as looming and terrible as before

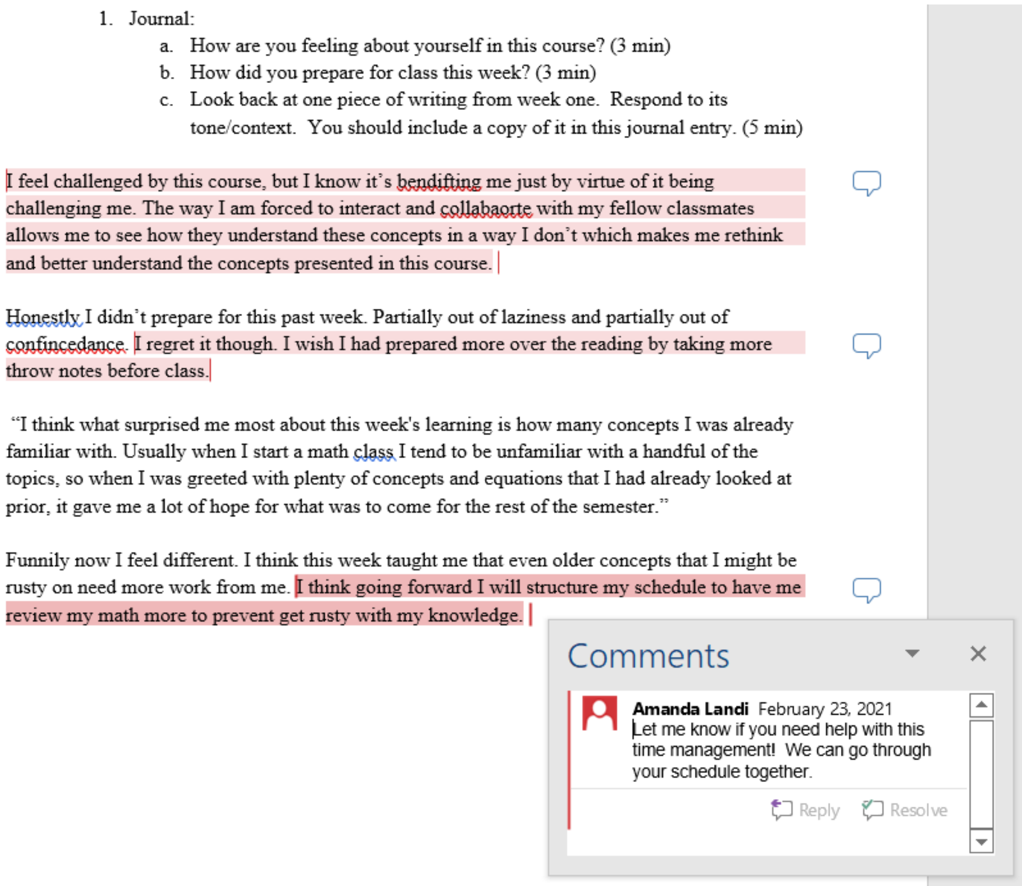
(b)

Figure 3: Reflective Responses

In the Day 10 script, to continue exposing students to the ways others think, including oneself, students are asked in one activity to discuss the pros and cons of all group partners' strategies and to consolidate strategies into one in which a majority of the group agrees on. Finally, for students who like to draw or visualize their learning, there is a prompt in the homework for Day 16 that asks students to make a diagram about all the functions they've learned about up to that point and illustrate how they relate to one another.

One of the biggest beneficial features of the writing assignments is that students respond in a variety of ways. That is, the intent behind a prompt may not always elicit a response the creator expected. This allowed the instructor to better understand how each student sees the world.

The other big beneficial feature about the writing assignments is the transparency it created between students, their thoughts, and the instructor. This allowed the instructor to better support students and to cater feedback for individuals that needed more specialized help.



1. Journal:
   a. How are you feeling about yourself in this course? (3 min)
   b. How did you prepare for class this week? (3 min)
   c. Look back at one piece of writing from week one. Respond to its tone/context. You should include a copy of it in this journal entry. (5 min)

I feel challenged by this course, but I know it's bendifting me just by virtue of it being challenging me. The way I am forced to interact and collabaorte with my fellow classmates allows me to see how they understand these concepts in a way I don't which makes me rethink and better understand the concepts presented in this course.

Honestly I didn't prepare for this past week. Partially out of laziness and partially out of confincedance. I regret it though. I wish I had prepared more over the reading by taking more throw notes before class.

"I think what surprised me most about this week's learning is how many concepts I was already familiar with. Usually when I start a math class I tend to be unfamiliar with a handful of the topics, so when I was greeted with plenty of concepts and equations that I had already looked at prior, it gave me a lot of hope for what was to come for the rest of the semester."

Funnily now I feel different. I think this week taught me that even older concepts that I might be rusty on need more work from me. I think going forward I will structure my schedule to have me review my math more to prevent get rusty with my knowledge.

Comments

Amanda Landi February 23, 2021
Let me know if you need help with this time management! We can go through your schedule together.

Reply Resolve

Figure 4: Reflective Responses Continued

In Figure 3(a), the student describes, in a fair amount of detail, their learning process during the first week or so of classes. It's clear the student is reflecting upon their past learning experiences and attempting to determine why this new semester feels more difficult for them. Meanwhile, in Figure 3(b), this particular student describes negative feelings associated with a mathematics concept. Although both students are highlighting struggles, they also seem to have a positive attitude about the challenges with which they are currently tackling.

The student's response in Figure 3(a) was written during the first week of classes when students haven't learned very many of the techniques to become more autonomous learners. However, it's a good example of a student who is engaged in the writing activities that take place during the first weeks of the WALGUM curriculum. There is clearly a lot of thought behind the response.

The student's response in Figure 3(b), although short, demonstrates the level of engagement this student has with their feelings about the concepts of the course. This involves deeper reflection as opposed to simplifying how one feels about the course or the instructor. It is fully possible for a student to enjoy a course or like an instructor while disliking material, or dislike an instructor while being agnostic about the material. Often, however, students tend to conflate all three and do not process what they feel, allowing the negativity to hinder successful learning.

In Figure 4, the student responds to a writing prompt assigned approximately one month into the course. We can see that by this point of the semester, although this particular student recognizes the benefit of being challenged with and by classmates, they admit they did not prepare effectively for the class this week. Part of the challenge for their group mates is likely that this student does not "pull their own weight" during group work. This can often be a sufficient motivator to come to class prepared, but maybe not as often as a teacher would like. In the reflection of a past entry, however, the student highlights they also need to be reviewing past concepts as part of their preparation for class time.

The metacognitive benefits go beyond simple reflection. In Figure 5, we can see that students are honing skills in making connections, analyzing diagrams, synthesizing clues to form a clear understanding, noticing a difference in their own work, and taking responsibility for their lack of growth because of choices they make.

Although many times it felt like students did not engage in the writing activities with as much diligence as we would have liked (as measured by length of responses, which usually indicates the amount of time spent on a response in the W&T pedagogy), we see how students that did engage in some manner received benefits to their learning.

The evidence that the math homework and the math journal entries benefited the students is qualitative. There was not a rubric followed for the grades students received on these assignments, so they received 1 point for each of the requested number of practice problems and prompts+responses turned in. Since the purpose of these assignments was metacognitive and mathematical training, we feel there is no need for a different grading system. However, this can be up to an individual instructor.

2.2.3 Manuscripts

We look at Figure 6 for the distribution of manuscript final averages among the students for all three sections (the averages are the arithmetic mean of the grades earned by students on their two manuscripts according to a rubric). Based on the graphs, we see that although the distributions have different shapes (left skewed vs. right skewed), all classes are centered in the B+ to A- range by the end of the semester. These centers are typical since all final manuscript drafts are

3. Writing prompt (5 min): Examine Figure 4 on page 476. Describe the concepts you think the figure is illustrating.

One thing I notice is that the figure shows which functions are even and odd. Cosine and Secant are shown to be positive in the 1st quadrant, but not positive in the second quadrant, where *x* becomes negative. Sin t and cos t are also shown to be in the 4th quadrant, making it so that in the right half of the unit circle they are always positive. This hints at symmetry about the origin. Since cos t and sec t are not positive in the 2nd quadrant, they are negative, meaning that cos t and sec t are odd functions. However, sin t and csc t are positive in both the 1st and 2nd quadrants, meaning sin t and csc t are even functions. They are symmetric about the *y*-axis.

Overall the image is illustrating where each function; sin t, cos t, tan t, sec t, csc t, cot t, are even or odd.
tan(t) is odd because tan(t)=sin t/ cos t and sin t is odd and cos t is even. The quotient of an even and odd function is an odd function.

(a)

I looked at some of the earlier responses from September and early October. I think that the way I write about math has definitely gotten better. I used to just kinda write what I thought about math, and now I feel that I am more analytical and precise. I can also see that I care more now about the writing prompts than I did in the past. I don't necessarily think my ideas have changed much because the ones I choose are more opinions. This being said however, I think that my understanding of the math we are currently working on and the math we went over prior has gotten better. I think my success is due to taking notes. At first, I didn't, and then I did poorly on an assessment and that changed quickly.

(b)

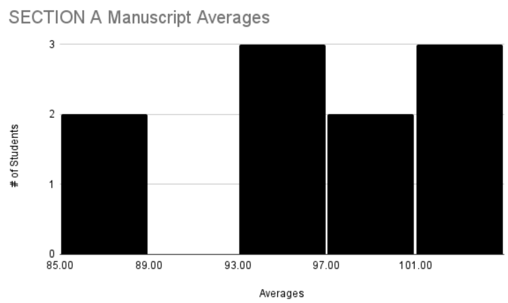
Figure 5: Sample Math Journal Entries

preceded by a rough draft with feedback *and* a second manuscript is preceded by a first manuscript[3]. This is in addition to the resources we provide students, such as "Writing in Mathematics" by Dr. Annalisa Crannell, and some students taking advantage of the peer review process.

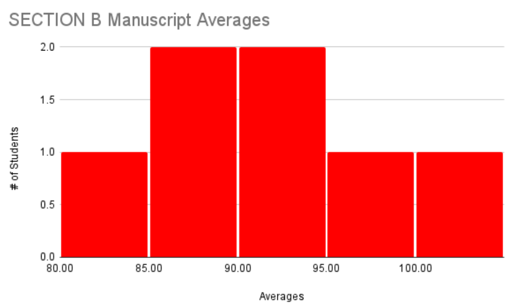
2.2.4 Exams

While homework assignments and manuscripts provided opportunities for metacognitive development and mathematical growth, the struggles students experienced on exams can sometimes bring doubt about whether the course design was successful. Each semester had a different exam arrangement; the second semester arrangement was a way to address the struggle observed by the instructor during the first semester.

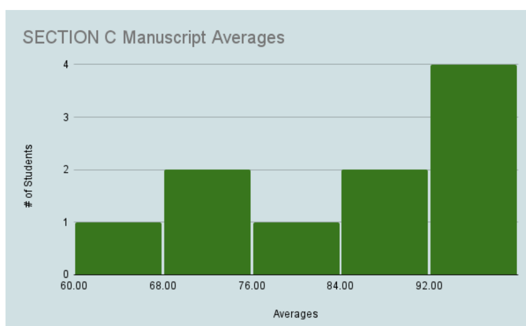
For the first semester of the pilot, there were three midterm exams, with practice exams released one week prior to each exam. The exams came at the end of three of four main units (the review unit, the polynomial and rational functions unit, and the trigonometry unit). The performance for both SECTION A and B was not atypical for the first exam, with class averages of 77.31 and 83, respectively.



(a)



(b)



(c)

Figure 6: Distribution of Manuscript Averages

The second exam saw significant improvement for both sections, with class averages of 81.95 for SECTION A and 91.64 for SECTION B. Perhaps this improvement could be thanks to students getting acclimated to the new curriculum they were experiencing! However, the third exam, which came at least one month later *and* after the semester's workload began to take a clear toll on students, saw performance even lower than the first exam. SECTION A had a 67.4 class average and SECTION B had a 69.57 class average. At least the difference in averages had gotten smaller!

To us, there were several possible reasons for the scores from the first semester. First, the students may have assumed the exams would be similar to the practice exams. If they believed the problems were cookie-cutter problems, then their study strategy could have been to memorize solutions to the practice problems. Second, the time limit for the exams caused so much emotional distress for students that they blanked during the exam and could not use the critical thinking skills they were cultivating in the coursework. Third, students may have relived trauma from previous math experiences, which prevented them from using new skills like writing process notes as a way to address a problem (a perfectly acceptable response for an exam in EF-WALGUM). Lastly, the students were not practicing problem sets within a simulated exam environment frequently enough outside of class time. That is, they were not working on problems, on their own without resources, in isolation without distraction, within the timeframe of 55 minutes.

To hopefully improve this aspect of the WALGUM curriculum, in the second semester, there were four midterm exams, no practice exams, and a revisionary process for an opportunity to earn points back. The first two exams came at the same points in the semester (the review unit and the polynomial and rational functions unit). The third exam came halfway through the trigonometry unit so that the instructor and the students could assess how well they understood the material much sooner than the end of the unit. The fourth exam was at the end of the trigonometry unit. SECTION C's class exam averages were 72, 79, 84, and 68. In general, this class did a lot worse on exams if we analyze the hard numbers.

The grades for exams here are listed because they were the motivation for the changes we made between semesters. And, because of these results, it may be easy to say WALGUM is not effective. However, grades on exams are not good indicators of critical thinking nor a student's ability to learn. They are a projection of a multi-dimensional human being onto the range [0,100] for a performance during three to four single hours of their semester. Unfortunately, students continuing in STEM must learn how to prepare more effectively and perform better for exams since they are the standard manner of assessment throughout undergraduate and graduate schools. In the future, the WALGUM model will maintain three midterm exams, no practice exams, and the revisionary process. However, elements from "ungrading" pedagogy, like is described in Blum (2020), may be used to improve this portion of the class even further.

Despite exam grades disappointing students, it was surprising to learn through their portfolios how positive the experience was for them.

2.2.5 Learning portfolio

While not all students received a high grade[4] on their portfolios, many wrote positive self-evaluations. We were surprised to learn how much students felt they learned from the course! Although it may not have been all the mathematics, as the exam grades seem to indicate, they definitely learned *some* mathematics and *a lot* about themselves as learners. We consider this a success! In Table 1, we've listed some direct quotes and we've coded the students' remarks to provide a set of the insights by students.

Although the portfolio required more time to prepare for students and for the instructor to grade, the benefits outweigh this cost. A majority of students per-

Type of Comment	**Student Comment**
Highlighting where the most learning happened:	*The two manuscripts were incredibly enjoyable to write. Their thoroughness of investigation into one particular topic made me think into great detail and intensity, and felt much needed to go along with the rest of the class work.*
Appreciation for being challenged:	*This semester has been very tough. It's insincere to say that past math classes have been easy, but this has been the first where I repeatedly struggled to understand material and was forced to adapt and learn and grow lest I'd risk getting swept away by the tide. It was stressful, but definitely a growing pain, and I'm glad I was able to experience it in a relatively tame environment rather than being thrust with expectations in a setting with higher stakes.*
Reflecting on one's attitude and approach:	*One of my main objectives, apart from the ones tied to this course, was to be as a spike. What I mean by that is to be as nimble as possible – uncomfortably nimble. To think about things in ways that might not work fluidly at first, to try the strategies of others that worked well, and those of the textbook, and hybridize them into my own ... Up until around two weeks into this class, I looked at math as flatly as doorways look at the sides of people.*
Commenting on strengths and struggles:	*When I really think about it, there is nothing learnt this semester that I could not understand with some small textbook reading or research ... my single greatest challenge this semester has been my time-management skills; even though I have had plenty of it, I lacked the discipline to do things when it was best to do them.*

Table 1: Words from student's self-evaluations

formed at C-level or higher. This grade is made by grades for the self-evaluation, graded by the rubric, and for the problem set, twelve problems each worth 10 points. If we examine the problem set separately in order to determine how "successfully" students "learned" the mathematical content, we see that students also seemed to perform relatively well on the corresponding problem set. SECTION A had a 90 class average on the problem set, SECTION B received an 85 class average, and SECTION C had an average of 74.44. It could be said the students successfully received the bare minimum knowledge[5] to move forward into Calculus. It should be noted that almost all students who went on to Calculus I during a following semester received an A on their problem set knowledge. One student received a C. One student received a D.

2.2.6 An Aside on Remote vs In-Person

The scripts are designed for the in-person classroom. However, in the end, many students in the remote section learned to collaborate with peers more effectively and became more independent from the instructor than those in the in-person section, perhaps by virtue of "break-out rooms" allowing instructors to have less of a presence. When the instructor was physically present in the classroom, students appeared to rely on each other to figure out answers less often (as measured by the number of hand raises received each class period for questions and reassurance) *and* they were significantly more uncomfortable with responding freely to writing prompts (as measured by the number of pens that stilled when the instructor walked nearby for in-person students).

3 Overall Reflection on WALGUM

While some of the implementation details differed between semesters, and may still change in the future, the WALGUM curriculum and its goals remained the same. Recall, the objectives for this paper were to report on the design of a W&T curriculum for Elementary Functions and to present and describe its effectiveness based on student work.

To determine whether WALGUM has long-lasting effects, we collected information on (a) how many students from SECTIONS A, B, and C were retained in mathematics, and (b) did they continue to use WALGUM techniques for their learning in those following mathematics courses. This data is only on students who remained at Simon's Rock campus while furthering their mathematical studies.

After their first semester, four students went on to take Calculus I in the immediate following semester. One student took a five week summer course, and three took a full semester Calculus I course in the following fall. Amanda reached out to the Fall semester students, using the email in Table 2, toward the end of their Calculus I semester. Only two students responded to the email. Their responses are shared in Table 2.

It's clear that these students were impacted by aspects of the WALGUM curriculum. Although they may not have been able to use W&T learning techniques as rigorously as they did in EF-WALGUM, the students understood that these tools helped them learn Elementary Functions so effectively that they found success in their Calculus I studies.

This was only those in the most immediate semester. Two other students, from SECTION A and B, state they planned to continue on to Calculus I later, when we last checked in. For the second semester, SECTION C, two students did continue with Calculus I in the following term. Though, we did not track completion or use of WALGUM techniques. While association does not imply causation, at least one of these students discussed above told me directly they did not originally have plans to continue with mathematics. What a success!

Email:	“Hello friends, I hope your semester of Calculus was full of growth! I am writing to wish you luck on your final, but I also have an ulterior motive... * I am wondering whether the concepts from El Func helped you in Calculus? If so, which ones helped you the most? Detail would be welcome! * I am wondering whether you used any of the learning skills you developed (math journal-like explorations) in our El Func course together in order to learn Calculus? If you did, could you tell me a bit more about which techniques you used. * If nothing we did in El Func helped you, maybe you could speak to why. Thank you so much for the time it’ll take you to answer these questions.”
Response 1:	*Hi Amanda!* *... skills I utilized during Calculus, I tried to squeeze in a 5-minute free-write prior to class when I could, though most of the techniques I picked up from last year I used mentally rather than with a pen and paper. ... much of my learning was done outside of the classroom. It was during these periods that I would work on any homework and attempt to recall and apply any and all strategies I had developed during El Func to learn the new concepts. While still rather grueling, I found that these strategies for tackling Calculus certainly helped quite a bit alongside reaching out to others in the class when I needed someone to explain new material to me.* *Best, STUDENT LL*
Response 2:	*Hello Amanda,* *... First of all, I don’t think I could have continued Calculus 1 if I hadn’t taken Elementary Functions... The concepts that helped me the most were: the right triangles (soh-cah-toa, the unit circle, and so on), even and odd functions, logarithmic and exponential functions, and inverse functions. A skill that I also learned in El Func is graphing, which has helped me both in calc and in general and has made math far more beautiful than it used to be (for me). One of the skills that I learned in El Func and have always used is writing in math, which I learned through writing journals and manuscripts. What I learned from my journal writing was how to be mindful and observe myself when I am solving a problem. It helped me be patient with myself and my learning process. And from writing our manuscripts, I learned how to understand math as a language.* *Sincerely, STUDENT HM*

Table 2: Email Responses from Students who Continued to Calculus

4 Observations on the Strengths and Weaknesses of W&T in Math

The **strengths** of WALGUM in our experience are:

- Increased transparency for instructor.
- Increased awareness for students about themselves and their learning.
- No one did worse than they would in a traditional math classroom (self-proclaimed math wizzes still performed at B or higher level by end of semester); students who would perform poorly (based on their own impressions) improved in some way.
- More diversity, than is usual, in students who participated.
- It could be argued that students became better problem solvers or writers depending on the student (in general, we cannot see significant improvement in the course of one semester).
- Students who actively engaged in the W&T activities continued to use the tools learned in Calculus the following semester.
- W&T prompts do not have to involve writing only, there is flexibility so that students can act , draw, and express themselves musically.

The **weaknesses** of WALGUM in our experience are:

- Instructor struggles with letting go of the reins, or, finding a balance between demonstrating concepts and allowing students to struggle.
- On the one hand, drills are necessary for learning since learning comes from repetition and practice. A lot of this burden is placed on the students outside of class time.
- Students may go into STEM because they want to write less.
- Some students experience writing anxiety. Although W&T does not judge a student based on their rhetorical writing, this curriculum may be exclusive to them.
- The push-back from students increases the emotional labor of the instructor.
- There is a lack of empirical evidence that there is a benefit over other forms of active learning.
- A curriculum study is only as good as its replicability, and the WALGUM evidence in this manuscript is based on a very specific population. See Section 1.2 for more details on our population.

5 Conclusion and Future Work

Our experiences with the pilot semesters have helped us to:

1. finalize the list of most essential concepts for the course,
2. determine how many of each writing activity type is appropriate,
3. figure out how much time should be allotted per activity, and
4. determine that the flipped classroom best supports the writing and thinking model.

Some WALGUM considerations for future investigations are to

- provide a list of the in-class prompts on course website (after class) so students can include them when they turn in their journals (often, it was unclear what students were responding to because they did not fully engage with the prompt).
- redesign exams so they better test students on their explanations and reflective abilities and process writing; or, continue to have a make-up process.

5.1 Remaining Questions for EF-WALGUM

The following are remaining research questions for future WALGUM work. They are questions that have not been fully answered through this manuscript.

- Was the push-back because students don't want to write?
  - Do they not want to write because they have writing anxiety?
  - Are they taking STEM courses to avoid writing?
- Was the push-back because of inherent biases students are unaware of (we are younger and female-presenting faculty)?
- Which students benefit by participating in a WALGUM curriculum? (*i.e.* examine their demographics, course history, timing in academic career)
- Does participation in WALGUM reduce math anxiety?
- Does participation in WALGUM increase math ability (performance and cognitive ability)?

There are several long-term goals for the curriculum: (1) extend WALGUM to other undergraduate gateway mathematics courses, (2) conduct a study to gather empirical evidence that writing-to-learn pedagogy benefits the fields of STEM as well as the larger community, (3) have other instructors (not the authors of this manuscript) implement the curriculum in order to remove instructor bias from the data, and (4) expand WALGUM to other liberal arts institutions in order to remove institution bias from the data.

Acknowledgements

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Amanda Landi (she/her/hers). Amanda is currently, and has been since 2015, a faculty of Mathematics at Bard College and Bard Academy at Simon's Rock. She received a PhD in Applied Mathematics from North Carolina State University in 2015, and she received a BA in English and a BA in Mathematics from North Carolina Wesleyan College in 2009. Amanda appreciates a good cup of coffee, a very soft blanket, and dustless chalk.

KAETHE MINDEN started teaching mathematics at Bard College at Simon's Rock as an assistant professor in 2019. Before Simon's Rock, Kaethe taught at Marlboro College as a mathematics fellow and also taught mathematics courses while a graduate student at The Graduate Center of the City University of New York (CUNY). She received her PhD in Mathematics from CUNY in 2017 and her BS in Mathematics from the University of California, Los Angeles in 2010. Her research lies in the fields of logic and set theory—particularly large cardinals and forcing. Some recent work with undergraduate students connects to combinatorial group theory, involving infinite latin squares.

Notes

1. We should also note here for replicability purposes that Bard College at Simon's Rock is a small, liberal arts early college. Its student body's average age is 16. All students are required to complete a Writing and Thinking workshop in order to obtain their AA degree. The pedagogy of this workshop emphasizes writing as a method of thinking and to build a community of young people who share fearlessly and who listen/respond respectfully to others. Although the Writing and Thinking workshop materials are humanities- and social science-based, the WALGUM curriculum is an adaptation of the techniques rooted in mathematics.

2. Example statement for syllabus: *This course is designed to use Writing and Thinking (WT) inspired activities. To find success in mathematics, we need to read texts and write notes and express our ideas in an accessible manner. We must engage with the math, not only by calculating this and solving that, but by mulling over concepts and ideas. Writing provides a medium for us to do that. While we'll still have practice problems, there will be writing and discourse activities in the same vein as writing and thinking workshop integrated into the curriculum. The purpose of using WT pedagogical practices is to train your metacognitive skills in mathematics so that you may successfully learn to learn math. This will not only benefit you in the Elementary Functions course, but it will also benefit you in future [math or otherwise] courses because you will be building a toolbox of learning*

techniques you can use in the future! WT does this by providing you a place to reflect on what you know and what you need to know as well as allows you to connect your thoughts and ideas in a more purposeful way. I ask you to keep an open mind to new ways of learning, especially new ways of learning math! It may feel like the workload is higher, because it is. Learning the concepts (well) in this course requires significant time and effort than when there is no writing, but your learning with writing will lead to better content knowledge and you will be a more confident learner overall.

3. It is the instructor feedback on rough drafts and a second manuscript that sets this part of the Writing as Active Learning model apart from other Simon's Rock mathematics courses.

4. At least one student turned in their portfolio and received an F and two students did not turn it in and received a 0 on the assignment.

5. This is a relative threshold, and for WALGUM this threshold is determined by the percentage of learning objectives the student demonstrates mastery over. Since 12 out of 15 problems is 80% of the 65 learning objectives already, a C on the problem set demonstrates a D mastery over the total number of learning objectives I hope students will learn. D is passing at SR, and so I accept that as the bare minimum level of knowledge too.

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A Sample of Instructor Scripts

Script for Day 1, Review:

Learning Objectives:

1. To find a value on the real line
2. To determine the set of values that satisfy an inequality
3. To use and interpret set notation
4. To use and interpret interval notation
5. To identify x-axis, y-axis, and the origin
6. To identify the quadrants
7. To plot points $\{(x,y)\}$
8. To determine the x- and y-coordinates from a point (x,y)
9. To find the distance between 2 points
10. To describe the set of points that are equidistant from a point C

You will not cover all these learning objectives, because it is assumed the students understood Algebra I enough to get into Elementary Functions. Thus, they should be comfortable enough with the 10 learning objectives to review some of them on their own, especially 1, and 5–8. Rather, we continue to concentrate on establishing the WT culture as well as building metacognitive skills. Thus, going through the review may feel very slow.

- Private Free Write (3 min)
- Loop Writes
  1. What connections can you make between the real line and the Euclidean plane? (2 min)
  2. What skills/strategies used to analyze sets on the real number line do you use to analyze sets of points on the Euclidean plane? (2 min)
  3. What new skills/strategies did you learn from the reading? (3 min) (maybe skip)
- Put example on board:

  Use set and interval notation to list value of x that satisfy $-3x+4<5$.
  - FFW. What are your 1st thoughts? (30 sec)
  - FFW. Recall your earlier list of skills/strategies. What are your 2nd thoughts? (1 min)
- Students turn to a neighbor and share both sets of thoughts (if remote, use 5 min breakout rooms). Then, respond to: What do you know about the problem from what was shared? (3 min)

Highlight here that many mathematical problems are solved by teams of mathematicians sharing ideas, often those ideas are different and the mathematicians must work together to knit them together to form one cohesive strategy/solution.

- Collaborative solving: go around in a circle and students give next step until problem is "solved" (if remote, could use Zoom whiteboard for this), and the instructor will justify each step to model appropriate justification. (15 min)

  Remind students they must explain why for each step.

- Collaborative Q&A: Students will write any lingering questions w.r.t. inequalities on an index card. Pass to instructor. Mix the cards up. Pass back to students randomly. Go around and discuss/answer each question. (if remote, have students write questions on Google doc and respond to each other there) (10 min)

- Collect remaining questions, and they will be posted and everyone must participate in a discussion about one of the questions. Any questions students cannot answer before class, post a video with an explanation an hour before next class. (3 min)

- Reflect. Look at the 10 learning objectives for today's class. Which do you feel most confident doing right now? What do you still need to work on? What is your study plan for them? (5 min)

Script for Day 4, Functions:

Learning Objectives:

1. To determine whether a relation between two variables is a function
2. To use vertical line test to show a graph is a function
3. To identify the independent variable
4. To identify the dependent variable

- PFW. (3 min)

- FFW. Where have you seen the concepts on functions before? Do they remind you of anything (outside of math class)? (5 min)

- Break students into groups of 3. Have one problem ready for each group. Each problem will be a function (either a graphical representation or algebraic expression).

  The students will discuss whether it is a function or not through a dialectical notebook (students are asked to compose a dialogue of two or more voices that interact for a few lines or a few pages; writing in dialogue allows students to enrich an analytical piece of writing by comparing two view points). They will role play as their academic advisor. (15 min)

- Put yourself into the mind of your academic advisor. What kind of mathematician do you think your advisor is? Based on that, what stance would they take? What language would they use to argue? Who are they more likely to side with? How would they support that person?
- If you and one or more of your partners have the same advisor, how do you think your advisor might argue with themselves?

- Each group will "act" out their discussion. Basically, they will read their discussion in the "voice" of their character. (10 min)
- Problem set, see the next page. Students will choose 1 problem and solve it on their own. They must justify their steps in writing. These solutions will be taped around the room. Make sure they don't put their name and that they use a separate piece of paper. (8 min)
- Everyone will walk around the room. This is the gallery walk activity. Each person will give feedback to at least 3 others. The feedback will be written on a post-it so it can be attached to the individual's work. They must use "respectful discourse", such as "Explain this differently" or "I never thought of it like that." or "This is a good point." (8 min)

Homework for Day 8, Quadratic Functions:

1. Review (11 min):
   (a) Writing prompt: How do you feel about quadratic functions? (3 min)
   (b) Writing prompt: Tell the story of the first time you learned about $f(x) = x^2$. (5 min)
   (c) Writing prompt: Describe in what ways linear functions and quadratic functions differ and in what ways they are the same. (3 min)
   (d) Practice Problems
      i. sec 1.3 # 5, 7, 11, 13, 16, 19, 21, 22, 23
      ii. sec 1.5 # 6, 8, 18, 20, 26, 34, 36, 41, 44, 53, 57, 61, 62, 67, 68

Script for Day 8, Quadratic Functions:
Learning Objectives:

1. To find the x- and y-intercepts of a quadratic function
2. To find the vertex of a quadratic function
3. To describe end-behavior of the quadratic function

- PFW. (3 min)
- Place a cup-like figure on the board, with no labels/axes on the board.

- Loop Writes:
  - Describe what you see. (3 min) Instructor will add the x-axis.
  - What has changed in the image? How does this modify your description of what you see? (3 min) Instructor will add the arrows at the endpoints of the cup-like image.
  - Reflect on your description. Did you notice the missing arrows? Why do you think it's important to make note of them? (3 min) Instructor will unveil the y-axis.
- Pair students up. Discuss the remaining features of this function that you can now find. Make a list or label the parts on the function. (5 min)
- Have each student go up and write a feature on the board or label it on the graph. (5 min)
- Together, compile a list. Think about sections 1.1-1.3, in which we learned about functions, in general. What important features did we learn? Did you consider them here? (3 min)
- Using transformations, determine the $f(x) = ax^2 + bx + c$ for this function. (3 min)
- Turn to a partner. See what they came up with. Teach the other person what you did, out loud, step-by-step. Explain why you did what you did. Does it make sense? (5 min)
- As a group, go over the problem. (5 min)
- Address left over questions/concerns/comments. (4 min)

Script for Day 10, Inverse Functions:
Learning Objectives:

1. To determine if a function is invertible
2. To find the inverse of an invertible function
3. To determine whether $g(x)$ is the inverse of $f(x)$
4. To graph the inverse from the graph of $f(x)$

- PFW. (3 min)
- Work on review problem with students.

  Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x} + 1$.
  (a) What is the domain and range of each?
  (b) Find $(f \circ g)(x)$.

(c) What is the domain and range of $(f \circ g)(x)$?

- Problem posted on board:

  Let $f(x) = x^2 - 2x$.

  (a) Show that the given function is not one-to-one.

  (b) Determine a subset of the domain of the function on which it is one-to-one.

  (c) Find the inverse on the restricted domain.

  FFW. What is the problem on the board asking of you, in your own words? (3 min)

- A sample of students will share their response. (1-3 min)
- FFW. What strategies have you used in the past while solving problems that may be useful now? (3 min)
- In groups of 3, consolidate your strategies. Then, discuss the pros and cons of them as they pertain to the problem. (10 min)
- Select 3 of your strategies. Each person use one, and attempt to solve the problem. (8 min)
- Go over problem together as group.

Script for Day 15, Rational Functions:

Learning Objectives:

1. To state the range of a rational function
2. To describe end-behavior of a rational function
3. To sketch the graph of a rational function based on its features

- PFW. (3 min)
- Together as class, determine the asymptotes for $f(x) = \dfrac{x^2-4}{x(x+2)(x+1)}$.
- In groups of 3 (10 min): With a problem plus “solution" on the board:
  - Believe this solution and justify each step.
  - Doubt the solution and propose a new approach at each step.

  Sketch the graph of the function $h(x) = \dfrac{x^2+x-2}{x+1}$ after finding the axis intercepts and the asymptotes.

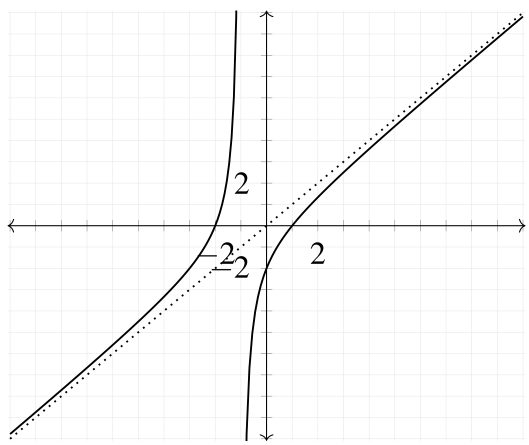
First we begin by finding the axis intercepts. Since to find the x-intercepts we need $h(x) = 0$, and since a fraction is only equal to 0 when its numerator is equal to 0, we set $x^2 + x - 2 = 0$. We can factor the LHS into $(x-2)(x+1)$ and we see that the x-intercepts, roots, and zeroes of $h(x)$ are $x = 2$ and $x = -1$.

To find the y-intercepts, we want $x = 0$. So, we plug 0 into each x. $h(0) = \frac{-2}{1} = -2$.

Woot. We're off to a great start. We'll find the vertical asymptotes first since they're "easier". We check out what makes the denominator 0. That's $x = -1$.

To find horizontal asymptotes, we look at the dominating term in the numerator and in the denominator and see what happens as x goes to positive or negative infinity: $\frac{x^2}{x} = x \implies$ in the long-term, the function $h(x)$ behaves like the line $y = x$.

Therefore, the function looks like



- If there is time, problem set for remainder of it.

Homework for Day 16, Rational Functions:

1. Review (approx 10 min):

   Writing prompt: Based on what you've learned so far, attempt to make a diagram of the types of functions (constant, linear, quadratic, power, rational, ...) and how they relate to one another.

Script for Day 23, Periodic Functions:

Learning Objectives:

1. To graph $\sin x$ and $\cos x$
2. To graph $A\sin\theta + D$ and $A\cos\theta + D$, where $\theta = Bx + C$
3. To identify the amplitude and midline of $A\sin\theta + D$ and $A\cos\theta + D$

4. To find the period and frequency of $A\sin\theta + D$ and $A\cos\theta + D$
5. To determine the phase shift of $A\sin\theta + D$ and $A\cos\theta + D$

- PFW (3 min).
- Put a graph of $\cos(x)$ on the board, but without axes or labels.
  - (2 min) Have students respond to: what do you see? then add one axis:
  - (2 min) what changes about your previous observations? then add the last axis:
  - (2 min) explain how this adds/removes anything to your description.
- Go over together their descriptions and make sure they are using the appropriate mathematical vocab!
- Inform the students the graph is of $\cos(x)$. In pairs, students should work together to relate the graph to what we know about $\cos(x)$ from the unit circle. (8 min)
- Place the graph of $\sin(x)$ over the graph of $\cos(x)$. List all the similarities and differences you observe about the two graphs. They can do this out loud. (5 min)
- FFW (3 min) Recall as much as possible about function transformations. In particular, the algebraic expression for the transformation of $f(x)$.
- Together, relate what they know about transformations to $A\sin(Bx+C)+D$ and $A\cos(Bx+C)+D$
- (15 min) Go over meaning of function and the graph together using the example $2\cos(3x+\pi)$.

B Learning Portfolio Prompt

Bard College at Simon's Rock
MATH 109 Learning Portfolio
Due December 17th by 9 am

Learning is a journey. You are supposed to struggle, and you are meant to grow from it. This portfolio serves as a way for you to demonstrate your growth over the course of a semester during Elementary Functions at Simon's Rock. Use your own voice! The self-evaluation is not meant to be an academic work, and it should be honest and insightful. Read all sections below and come to me with questions. Do NOT wait until the last minute to put everything together. The time and effort you put into your thoughtfulness will be reflected in these materials and there are no redos.

INSTRUCTIONS:

- This portfolio is due by **December 17th at 9 am**. Please include a README file (call it README_YOURLASTNAME) that directs me on the appropriate order of documents to read. The format preferred, as was for all semester, is PDF file format. Always use underscores when naming files, do not use spaces, periods, or dashes. It's bad practice.
- Include (a) self-evaluation of your learning, (b) at least 8 artifacts as evidence of your learning, and (c) the final problem set demonstrating your mastery of the course learning objectives.
- Please see the rubric on page 3 on how the portfolio will be graded. Guidelines for the different sections of this portfolio are directly following these instructions. On the last page, there is a checklist to help you determine when your portfolio is ready for submission.

GUIDELINES on Self-Evaluation:

Your self-evaluation is a personalized reflection on your learning experience. You want to illustrate your growth over the semester as a student of Elementary Functions. Given this is the first time you may be writing a self-evaluation, I've included some ideas for what you might put into your self-evaluation. This is not an exhaustive list.

- What did you discover about yourself as a mathematician this semester?

- With respect to written/verbal communication of mathematics: what struggles did you encounter? why do you think you had those struggles? explain how you attempted to overcome them. do you think you were successful?
- With respect to thinking about mathematics: prior to this course, how did you think about math? how do you think about math now? if there was an evolution in your thinking about math, discuss that evolution and how it impacted your performance on materials with time. if there were no changes in your thinking about math, discuss why do you think that is.
- With respect to individual units (basics of functions from algebra 1 (ch 1 and 2), rational functions (ch 3), trigonometric functions (ch 5–7), exponential and logarithmic functions (ch 4)) — see the learning objectives to help you with details
  - What concepts/objectives did you struggle with at first?
  - Have you overcome those struggles? why or why not?
  - Tell the story of how you learned to master the concepts/objectives you did

The self-evaluation should be 3 pages to 8 pages double-spaced. Anything in between is acceptable. You want to use your selected artifacts as evidence of your learning within your self-evaluation (see the next section for guidelines). That is, reference the artifacts during your discussion. Let the rubric below help guide your writing. The self-evaluation should both describe what it is you learned as well as reflect on the learning experience.

GUIDELINES on Artifacts:

Artifacts are pieces of graded material from the class that support claims of learning. Artifacts can include any exams, math homework problems, in-class math problems, or rough-final drafts of manuscripts. You should have at least 8 artifacts as evidence, but these can be paired artifacts. For example, a problem you submitted for homework and that same problem reworked after digesting feedback I provided — this would count as two separate artifacts but it makes for a more meaningful analysis. For the sake of time, you may only want to use things that you have already spent time working on. Hopefully, you already went back and redid every problem you “got wrong", as suggested. What a time saver that is now!

The Final Problem Set:

There are 4 units from this course, listed previously as basics of functions from algebra 1 (ch 1 and 2), rational functions (ch 3), trigonometric functions (ch 5-7), and exponential and logarithmic functions (ch 4). The problem set covers 65 learning objectives (listed toward the end of this prompt, just before the convenient checklist). You may use the solutions you create to these problems as part of documenting your learning this term. You should demonstrate your mastery of the learning objectives by working out the problems and justifying your steps. If there is no justification, I will not consider it.

Clarifying the Rubric:

You should review the rubric before reading this section.

1. *Reflection on Learning*: You should demonstrate that you thought deeply about, what produced your learning, and what actions generated growth.

2. *Demonstration of Learning*: Your portfolio should demonstrate that you have met the learning outcomes. Your portfolio must include documentation or artifacts that support the knowledge and skills you have acquired. So, you want to include graded class materials (e.g., manuscripts, exams, or math problems you have worked on) that support your claim that you have met course learning outcomes. Revisions of these materials are welcomed as part of the artifact submission. You want to clearly describe and document your learning experience and show that you have met the course learning objectives (listed on the pages following the rubric).

3. *Mastering Knowledge and Skills*: You need to demonstrate that you have mastered the knowledge and skills reflected in the course learning objectives and that you can apply the learning to math problems. So, provide examples!

4. *Presentation*: Make sure your portfolio includes all elements described in these guidelines, organize the presentation of your portfolio, and strive to be error free (though, our differences in writing styles will be valued! and for those whose first language is not English, any instance of your first language will be celebrated! Neither of these things will be used as tools to hurt your grade).

	0	1	2	3
Reflection on Learning (15%)	The portfolio provides little or no evidence of reflection to increase learning aligned with the course learning outcomes for which credit is being sought.	The portfolio provides inadequate evidence of reflection to increase learning aligned with the course learning outcomes for which credit is being sought.	The portfolio provides evidence of reflection to increase learning aligned with the course learning outcomes for which credit is being sought.	The portfolio shows that the student has reflected with substantial depth upon how the semester's learning experience is aligned to the course learning outcomes for which credit is being sought.
Demonstration of Learning (25%)	The portfolio's materials and artifacts are not appropriate and/or adequate to demonstrate learning of course learning outcomes; That is, there is little or no evidence of learning.	The portfolio's materials and artifacts are not fully supported by or connected to the course's learning outcomes; That is, there is some documentation of learning, but insufficient.	The portfolio includes appropriate artifacts that support the demonstration of learning outcomes; That is, the learning of course objectives are effectively presented.	The presentation of artifacts is convincing, with strong support for the course's learning outcomes. The portfolio provides clear evidence of learning.
Mastering Knowledge & Skills (50%)	The portfolio provides little evidence of student's ability to use knowledge and skills for the course's learning outcomes in practice.	The portfolio's demonstration of student's ability to use the knowledge and skills for the course learning outcomes in practice is limited.	The portfolio documents the acquisition of knowledge and skills for the course learning outcomes, with some ability to apply them in practice.	The portfolio demonstrates student has mastered the knowledge and skills for the course learning outcomes and can apply them in practice.
Presentation (10%)	Assembly instructions have not been followed with critical portfolio elements not included; the quality of presentation does not meet standards.	Most of the expected elements are included; the quality of presentation does not meet standards, with too many errors in spelling, grammar, and punctuation.	The portfolio is well-organized with all critical elements included; the quality of presentation is competent with minor errors in spelling, grammar, and punctuation.	The portfolio is well-organized with all critical elements included; learning is well-documented with writing and production skills that exceed expectations for average college student.

C Exam Make-Up Form

Instructions:

You need to fill out the form for *each problem* on the assessment for which you are attempting to earn points back. You can earn up to half the points back for each problem. You must revise your work with a class tutor, a Think Tank tutor, or with the instructor during office hours. When you turn in this form, include a copy of your original, graded work from the assessment. A picture is sufficient since the make-up assessment should be turned in via Google Classroom.

Assessment # ____
Problem # ____
Helping Tutor or Instructor: _____

What is the original problem asking in your own words?

What mistakes did you make during the assessment? Why do you think you made those mistakes?

What skills are required to complete this problem that we covered in class?

What is your new solution to this problem? Explain your reasoning.

What do you now know that you didn't know before?

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