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run determinant of changes in the latter. This is likewise an empirically testable hypothesis.

III. PRICE SYSTEMS

We will present labor-values and three different types of price systems for the U.S. economy for the years 1947, 1958, 1961, 1963, 1967, 1968, 1969, 1970 and 1972.

1. Labor-Values

Labor-values are here assumed to be identical to total (direct and indirect) labor requirements for unit of output. The approach followed in dealing with the problem of heterogeneous labor is discussed in appendix C. In addition, we are ignoring dynamic effects such as rapidly changing technology and changing patterns of demand. The former effect would make the socially-necessary labor-time as determined in the sphere of production by the state-of-the-art technology different from the (average) total-labor requirements for the industry as a whole. The latter effect would imply a disjunction between the two senses of socially-necessary labor-time (the second sense being the amount of social labor-time which "society" is willing to allocate to the production of a given good, as evidenced by the level of demand). Assuming these effects away is an abstraction comparable to assuming that the rate of profit is uniform across industries. Moreover, it should be possible to infer the real importance of these effects from the empirical results.

The system of 71 linear equations which define labor-values is:

(i) $$v = a_o + v(A + D)$$

where a_o is the homogeneous-labor-coefficients row vector, A is the matrix of input coefficients whose elements a_{ij} represent the amount of good i used by industry j, and D is the physical capital depreciation matrix (see appendix C for a discussion of data sources and methods). The solution to (i) is given by

$$\text{(ii)} \qquad v = a_o(I - A - D)^{-1}$$

Equation (ii) defines the row vector v as the amount of labor required directly and indirectly to produce one unit of sectoral output. That physical unit is a "market-dollar's worth," given the way a_o, A and D are defined. When we use their deflated versions, the unit is a constant (1972) market-dollar's worth, which is then a constant measure of physical output. The vector v thus has the units of labor-time per physical unit of output.

2. Direct prices

We define the row vector of direct prices d, following Shaikh (1977), as the set of prices directly proportional to labor-values, the constant of proportionality relating the money unit to the unit of labor-time. We define the money unit by requiring that the sum of sectoral outputs at direct prices equal the sum of sectoral output at market prices. In vector notation,

$$\text{(iii)} \qquad dq = mq$$

where the vector of market prices m is the unit vector, since q is measured in market prices. To convert values to direct prices, we must find the proportionality constant μ, the value of money.

$$\text{(iv)} \qquad v_j = \mu d_j \quad , \text{ for all } j.$$

Combining (iii) and (iv), we get

$$\text{(v)} \qquad vq = \mu mq$$

or

(vi) $$\mu = \frac{vq}{mq}$$

Hence, the final expression for d_j is:

(vii) $$d = (1/\mu)v = \frac{mq}{vq} v$$

3. Marxian Prices of Production

Marx defines prices of production as the sum of costs plus an intersectorally uniform rate of profit on capital advanced. These magnitudes are seen to be the centers of gravity of the continually fluctuating market prices. By capital advanced, we mean the capital invested in plant and equipment (fixed capital), plus the accumulated investment in inventories of materials, plus the stock of money necessary to pay out wages. The level of material inventories and wages-fund is related to their flows by the turnover time of circulating capital t_j in each industry. If we have the flows per year, and the turnover time in (fractions of) years, then the necessary stocks are simply (annual flows x turnover time). This would give us the stock levels necessary to produce one year's output. Dividing through by the output level, we obtain the stocks required per unit of output.

In order to specify the wage, we must include the real counterpart of the value of labor-power. This is given by the column vector b', which is the real wage basket per unit of homogeneous labor-time. The expression for p, the Marxian prices of production vector, is then as given below:

(viii) $$p = p(b'a_o + A + D + \langle g\rangle) + \rho p\,[\,K + (A + b'a_o)\langle t\rangle\,]$$

where $b'a_o$ is the matrix of wage-good inputs, $\langle g \rangle$ is the diagonal matrix of indirect tax coefficients (see appendix C), and ρ is the uniform rate of profit. Let $A^+ = (b'a_o + A + D + \langle g \rangle)$ (i.e., total costs) and $K^+ = [\,K + (A + b'a_o)\langle t \rangle\,]$ (total capital advanced). Then (viii) reads

$$p = pA^+ + \rho pK^+$$

or

$$\text{(ix)} \qquad (1/\rho)p = pK^+(I - A^+)^{-1}$$

Equation (ix) is the eigenvalue problem for the matrix

$$K^+(I - A^+)^{-1}.$$

The economically meaningful solution requires p to be a strictly positive, real vector. If we assume that

$$K^+(I - A^+)^{-1}$$

is an indecomposable matrix--and we know it is nonnegative--then the Perron-Frobenius theorem insures that the only such eigenvector is the one associated with the largest eigenvalue $(1/\rho)_{max}$ (to which corresponds the lowest ρ).

Since p is an eigenvector, it is defined up to a constant. In other words, (ix) only defines a set of relative prices. To set the price level, we need a normalization condition similar to (iii):

$$\text{(x)} \qquad pq = mq$$

Let p^* and p be the unnormalized and normalized eigenvectors, respectively. Then we define the normalization constant β such that $p = \beta p^*$. We can then rewrite (x) as

$$\text{(xi)} \qquad \beta p^* q = mq$$

which implies

$$\text{(xii)} \qquad \beta = \frac{mq}{p^* q}$$

Therefore,

$$\text{(xiii)} \qquad p = \frac{mq}{p^* q} p^*$$

Summing up, Marxian prices of production are given by solving for the eigenvector associated with the maximum eigenvalue of the system of equations (ix), and then normalizing this eigenvector according to (xiii).

4. Sraffian Prices of Production

The generalized price system developed by Sraffa in part II of his book (Sraffa, 1960) treats fixed capital as a joint product. Consider the following price system with joint products:

$$\text{(xiv)} \qquad (1+\rho)sK + sA + a_o\omega = sB$$

K is the capital stock (which becomes a flow in a joint-product model) and A is the flow of circulating capital (materials), B is the matrix of joint products, ω is the scalar wage, and s is the price vector.

Define $\hat{B} = B - I$. Then $B = \hat{B} + I$, so that the joint-product matrix B is the sum of the identity matrix (i.e., the unit output of each industry) and the matrix $\hat{B}$, now interpreted to be the used-machinery coefficients (i.e., capital as a joint product). Then the above equation can be rewritten as follows:

$$(1+\rho)sK + sA + a_o\omega = s\hat{B} + s$$

or

$$sK + \rho sK + sA + a_o\omega = s\hat{B} + s$$

or

(xv) $$s(K - \hat{B}) + \rho sK + sA + a_o\omega = s$$

The matrix $(K - \hat{B})$ represents the difference between the stock of capital going into the production process and the stock of capital which emerges out of it; in other words, the scrappage matrix. This is precisely the series which we use to construct the matrix D in our price systems, so the above equation is none other than

(xvi) $$s = s(A + D) + \rho sK + a_o\omega$$

Sraffa's model is still different from Marx's, since he computes the profit rate on fixed capital advanced only. He also does not have a concept of the value of labor-power, so the distribution of the surplus product between wages and profits is left open as a degree of freedom of the system. We can solve for the resultant system as follows:

(xvii) $$s = \omega a_o + s(A + D) + \rho sK$$

or

$$s(I - A - D - \rho K) = \omega a_o$$

or

(xviii) $$s = \omega a_o(I - A - D - \rho K)^{-1}$$

This is a system of n linear equations and n+2 unknowns: s, ω, and ρ. We also have our usual normalization condition

(xix) $$sq = mq$$

Combining (xviii) and (xix) and specifying the level of ρ, we get:

$$\omega a_o(I - A - D - \rho K)^{-1}q = mq$$

or

(xx) $$\omega = \frac{mq}{a_o(I - A - D - \rho K)^{-1}q}$$

Solving (xx), we obtain the money wage, which we can use in

(xviii) to obtain the price vector s. This can be done for a number of values of ρ in the range (0,R), where R is the inverse of the maximal eigenvalue of the system given below, which is (xvii) with $\omega = 0$:

(xxi) $$s_R = s_R(A + D) + Rs_RK$$

V. SECTORAL PRICE-VALUE DEVIATIONS

Appendix A gives the sectoral outputs q, labor-values v, direct prices d, and Marxian production prices p for the 9 years studied. This section of the paper will present results based on those basic computations.

In order to measure the extent of the cross-sectional deviation between direct prices and Marxian prices of production in a real economy, we developed the following statistics (here illustrated for the production price-direct price case of Table 1):

(xxii) Mean Absolute Deviation [MAD(p,d)]

$$= (1/n) \sum_i \frac{|d_i - p_i|}{d_i}$$

(xxiii) Mean Absolute Weighted Deviation [MAWD(p,d)]

$$= \sum_i \frac{|d_i - p_i|}{d_i} \cdot \frac{q_i}{\sum_j q_j}$$

(xxiv) Normalized Vector Distance [NVD(p,d)]

$$= \frac{[\, \Sigma(p_j q_j - d_j q_j)^2 \,]^{1/2}}{[\, \sum_j (d_j q_j)^2 \,]^{1/2}}$$

We also computed the cross-sectional correlation coefficient, which we report squared as R^2. In order to avoid spurious correlation--since prices and values must be correlated as p_iq_i and v_iq_i to have any variation in the market-price data--we computed R^2 on the logged data points. This admittedly arbitrary measure prevents us from taking credit for that large portion of covariance between prices and values which is entirely due to cross-sectional variation in real output levels.

And yet we cannot eliminate this source of variation entirely: to do so would eliminate all variation from prices (market prices). In fact, the R^2 statistic becomes 0 by definition when one of the variables is constant. If we had true physical units for quantity, it might be argued that we could factor the latter out entirely and still have variation in all price series. But this only highlights the conventional nature of the solution. Suppose we chose ounces as a unit of measure in one sector, and tons in another. This could lead to large covariations in all price series which are entirely due to the choice of physical units.

What all this shows is that measures of covariance such as R^2 are not the proper statistics to assess the relation between alternative price systems. Rather, the measures of deviation presented above are the proper statistics to focus on.

To measure the extent to which individual values determine the behavior of production prices over time, we also performed 71 time-series linear regressions of values on Marxian production prices. The associated correlation coefficients were averaged and squared to obtain a general measure of explained variation which is dimensionally comparable to the cross-sectional R^2. (The reason we squared after averaging was that we did not want to take credit for negative correlations over time. Using this procedure, the latter actually