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The House of Education Needs Overhaul

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Math and Politics

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EDITORS' NOTE: *This lesson plan is one of five projects created out of the Bard Early College Fellowship, which have been highlighted for the first issue of* Early College Folio. *Read Ligaya Franklin's "Early College Pedagogy: An Introduction to the Bard Early College Fellowship," for a comprehensive review of the fellows, their pedagogical approaches, and the broader goals of the fellowship.*

SUMMARY

In the introductory unit of my *Math and Politics* class, students develop their own voting systems and compare them to voting methods and other social choice procedures actually used throughout society, thus reinforcing the idea that many mathematical notions are human constructs, with strengths and weaknesses, rather than ideas that exist completely outside of human experience.

OVERVIEW

Near the beginning of my Year 2 *Math and Politics* class, students are asked to design their own voting systems (or "social choice procedures"). It's framed as a challenge; I give them a made-up list of voter preferences (and how many voters display each preference), and their challenge is devise five different reasonable-sounding systems, each of which results in a different winner at the end. Students are asked to be creative, but still defend their choices. We use the student-devised systems as a jumping off point for exploring various social choice procedures used in elections and other situations (e.g. reality television).

We work with this material over the course of five 50-minute periods (minus some time for start-of-year logistics), and thus I also use the work as the substrate on which I set the norms for how we do work in the course, including:

- A group work protocol that allows for individual think time.
- Visibly random grouping and vertical non-permanent surfaces.

I chose this sequence of lessons (and more generally, this course) to focus on because our students need more math experiences that help them to make

sense of the world. They need opportunities in math class to debate which of multiple ideas are more reasonable, and to connect their own ideas to the larger body of knowledge waiting for them. According to my students, this task been successful in jump-starting engagement with the material at the beginning of the course.

We then proceed to work on the first problem set of the course, where students both formalize their own voting system to handle special cases, and evaluate other voting systems with regards to whether or not they fulfill various desirable criteria for fairness and predictability. Students mostly work on large whiteboards (either loose or on wall) so that thinking is more visible to their teammates, and it is easier to transition to low-stakes presentations.

ISSUES ADDRESSED

- Student's view of mathematics as a fixed body of knowledge they either understand correctly or not ("no one best method").
- Moving back and forth between fuzzy notions ("what is fair?") and strict mathematical concepts ("does this system satisfy the Pareto condition"), and what is lost when translating back and forth.
- Difficulties in distinguishing between examples and proof, especially the idea that an example doesn't prove a statement, but a counterexample does disprove a statement.

STUDENT LEARNING OUTCOMES

- Learn all basic vocabulary around social choice procedures (i.e. voting methods).
- Learn several basic conditions which may or may not be satisfied by a given social choice procedure and how to determine if a given condition is met.
- Identify what social choice procedure is being used in a given situation.

SIGNATURE PEDAGOGIES EMPLOYED

- Visibly random grouping; use a deck of cards to group students in threes and fours every week. See this link for more information on the pedagogical benefits of grouping students randomly (and regrouping frequently) in the mathematics classroom.[1]
- Vertical Non-permanent surfaces; students use the boards in the room as work space, not just presentation space. See this link for more info on how this strengthens thinking in the math classroom.[2]
- Focused free-write (FFW) when writing up an "abstract" for the first problem set
- Process writing: students work to connect what math they have created

to the math I as teacher have introduced, and what challenges they still face.

TIMELINE

- **Class 1**: Introduction to task and work time
- **Class 2**: Presentations of systems and I connect them to the systems we'll study
- **Class 3**: Looking at whether or not systems satisfy given conditions
- **Class 4**: Checkpoint quiz and more work on conditions
- **Class 5**: Wrap up Problem Set 1 by doing FFW for abstract and process write

MATERIALS

- Name tent: students make them so I can learn quickly, and write daily feedback [page 14] on the back for the first four days of class (I respond in writing)
- Intro task [page 8]
- Problem Set 1 [page 9]
- Quiz 1 [page 12]
- Lots of whiteboards and markers

CLASS 1

Seat randomly and explain briefly why. Make a name tent. (5 minutes)

- Seat students randomly as follows: Give each student a playing card; they sit with their fellow numbers. Location doesn't matter where, but the cards neither allow friends to sit together nor specifically prevent the same. We will do this at the beginning of each week so you work with as many people as possible in this class, which is especially important in a non-mainstream math class like this one.
- Take the name tent feedback sheet, fold it into a triangle, and write your name on it. At the end of the class, you'll write a question, comment, something about yourself, or a piece of feedback. I'll respond in writing each day as I pass back the tents to learn your name.

Pass out intro task. (5 minutes)

- Have one student read aloud then read again silently.
- Clarifying questions?

Silent work/think time (5 minutes)

- If you are stuck, try to write down places other than elections where a

winner has to be determined. How do we decide who the winner is?

Set up board spaces for each group and have them share. (25 minutes:)

- Share whatever you figured out, one at a time. For each of the five different groups, assign them a candidate they want to win, and have them work to devise a system that appears as fair as possible where their candidate wins.

Extensions

- Can you devise other systems that make the other candidates win?
- Can you describe your system without alluding to this example?
- Does your system generate a predictable outcome or is there randomness/vagueness? Tighten those points up.

Logistics items for first day of the semester (5 minutes)

- Pass out syllabus and Google Classroom code.
- Name tent feedback: students write one question, comment, or concern on the back of their name tent and turn in.

CLASS 2

Return name tents to students to test myself, and let them read feedback for a moment. (5 minutes)

Prepare informal presentation of what your group did yesterday. (5 minutes)

- Presenters selected at random

Have each group present their system. (15 minutes)

- Central presenter per group, plus they can always "phone a friend" if they get stuck.

Anticipated Ideas:

- Plurality (but they'll call it majority)
- March Madness -style (i.e. bracket tournament)
- Some sort of points system (e.g. 5 points for first place in a list, 4 points for 2nd place, etc.) Watch out for non-linear point systems (e.g. 10, 5, 3, 2, 1).
- Elimination rounds (e.g. eliminate candidate that is most frequently last place, then repeat with shorter list)
- "Regular season" sport-style system (i.e. W-L record for every possible matchup).

Notes by me (25 minutes)

- Where possible, work to introduce the systems based as much as possible on the systems students have already devised. E.g. "In Scott's group,

their system is essentially a bracket tournament, which is a lot like sequential pairwise, but with the following differences..."

SYSTEMS

- *Plurality*: most top-place votes.
- *Borda count*: assign points, with 0 points for last place, highest total wins.
- *Condorcet*: round robin wins all matchups.
- *Sequential pairwise voting*: first two candidates face off, third candidate faces winner, repeat
- *Hare system* (instant runoff): delete candidate who is least common top choice and repeat (kind of like Survivor).
- *Dictatorship* (what it sounds like)

TERMS TO KNOW

- *People*: voters
- *Alternatives*: candidates
- *Individual preference list*: ballot
- *Profile*: sequence/group of ballots
- *Social choice procedure*: voting method, takes in profile and gives a winner or no winner
- *The social choice:* winner

Name tent feedback again (5 minutes)

CLASS 3

Pass out and introduce Problem Set 1. (5-10 minutes)

- Answer questions about formatting, grading, etc.

Rules for Problem Set work time:

1. Allow your teammates to think individually first.
2. No one gets left behind.
3. Never write down something you don't understand.

Silent work time on PS1.1 (5 minutes)

Share at the group.

- When your group feels good to go (i.e. any one member chosen at random could explain it to me), check in with me.

Goal:

- Everyone is done with PS1.1 by 25-minute mark and thus has time to work on PS1.2.

Finish with demonstration of how a counterexample can disprove a statement: D is the winner. This is not Pareto.

Profile 1		
2	1	1
A	B	C
B	C	D
C	A	B
D	D	A

Homework

- 30-45 minutes' work on PS1, wherever you are.
- When we are in problem set mode, I'll post on Google Classroom a suggested goal (e.g. "you should be done with problem x and at least try problem y before we start class tomorrow).

CLASS 4

Quiz 1 at the beginning on determining winners

- You have whatever time you need, and when you finish, you can work on PS1 quietly.
- Estimated last finisher: 25 minutes [was actually 28]
- When done with quiz, make sure your group understands Pareto condition by reviewing example from last time. If you're stuck, find me.

Work on PS1.3

- Same structure: individual (5 min), group (as needed), then check in
- Speedy groups can take an early look at PS 1.4.

Finish with name tent feedback (final time).

Homework

- 30-45 minutes work, should be done with PS1.3
- At least understand what 1.4 is asking.

CLASS 5

Read your name tent feedback and... (10 minutes)

- FFW: Summarize as best you can what we've been doing the past few days. If you had five minutes to explain to a friend not in this class what this class is about, what would you say?

Trade, read, partner feedback (5 minutes)

- What's missing? What could be clearer?

Rewrite. (10 minutes)

- As a challenge, don't mention the class or yourself, but instead refer only to the concepts. Some tips for this:
    - Using first person plural:
      ("We define a social choice procedure as…")
    - Passive voice:
      ("It can be determined whether or not each system fulfills various conditions.")

Last cycle: PS1.5, and if time, PS1.6 (20 minutes)

Process write (5 minutes)

1. What about this material did you like or dislike?
2. What questions do you still have?
3. Any connections to other classes?
4. What aspect of class time did or did not work well for you?

Homework

- Everything written up nicely and submitted via Google Classroom:
    1. Abstract
    2. Problem
    3. Process write

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Class 1

Name:_____

WELCOME TO MATH AND POLITICS!

You are running an election with 5 candidates and 100 voters. The exact preferences of all voters are known:

31: A > D > C > E > B
20: B > D > C > E > A
19: D > C > E > B > A
16: E > C > B > A > D
14: C > E > D > B > A

This means that there are 31 voters who have A as their top choice, D as their second choice, then C, then E, and finally B as their least favorite candidate. And so on.

Can you **come up with five voting systems**, each of which sounds like a reasonable way to count votes, such that each of the five candidates is a winner?

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Class 4

Name:_____

PROBLEM SET 1: FEATURES OF SIX VOTING SYSTEMS

INTRODUCTION

Here are five conditions that it would be nice for a voting system to have:

1. **Always a winner (AAW)**: Any sequence of preference lists produces (at least) one winner.
2. **Condorcet Winner Condition (CWC)**: if (IF) there is a *Condorcet winner* (it beats all other alternatives in 1-1 matchups), then that alternative is declared the winner.
3. **Pareto Condition**: If *everyone* prefers Alice over Bob, then Bob is not a winner.
4. **Monotonicity**: If Alice is already the winner and one or more voters decide to rank Alice higher than before, she stays a winner (new votes can't hurt).
5. **Independence of Irrelevant Alternatives (IIA)**: If Alice is a winner and Bob is not, and one or more voters change their preferences *without changing their Alice vs. Bob preferences*, then Bob does not magically become a winner.

Your goal is, in short, to determine which conditions are satisfied by which of the six systems from class: Condorcet, Plurality, Borda count, Hare, Sequential Pairs, and Dictator. Here's a chart to help you keep track:

	AAW	**CWC**	**Pareto**	**Mono.**	**IIA**
Condorcet					
Plurality					
Borda					
Hare					
Seq. Pairs					
Dictator					

Some answers will be explained in class, some you will have to figure out, and a few we will skip or will be stated in class without proof.

PROBLEMS

1. Figure out which one of the six methods does NOT satisfy the AAW condition. Do this by providing an example in which there is no winner.

2. Condorcet's method satisfies the CWC because they are defined the same way. Sequential pairwise voting also satisfies the CWC because any candidate who can win all 1-1 matchups will win the sequence of votes, no matter what order you put them in. Figure out which methods do NOT satisfy the CWC criterion. In other words, find examples where the Condorcet Winner doesn't match the winner according to the system you chose. These examples may help:

Voters 1-3	**Voters 4 and 5**	**Voters 1-4**	**Voters 5-7**	**Voters 8 and 9**
a	*b*	*a*	*b*	*c*
b	*c*	*b*	*c*	*b*
c	*a*	*c*	*a*	*a*

Voters 1-5	**Voters 6-9**	**Voters 10-12**	**Voters 13-15**	**Voters 16 and 17**
a	*e*	*d*	*c*	*b*
b	*b*	*b*	*b*	*c*
c	*c*	*c*	*d*	*d*
d	*d*	*e*	*e*	*e*
e	*a*	*a*	*a*	*a*

3. Checking the Pareto Condition

   - Explain why each system besides sequential pairwise voting does satisfy the Pareto condition.

   - Use this example to show that sequential pairwise voting does not satisfy the Pareto condition. You need to find one candidate which is always preferred to another candidate, but that other candidate wins.

Voter 1	**Voter 2**	**Voter 3**
a	*c*	*b*
b	*a*	*d*
d	*b*	*c*
c	*d*	*a*

- Determine one of the systems that satisfies monotonicity and explain why it does. For this, you will have to explain why, in the system you choose, moving Alice the winner up a list does not make her a loser.

- Determine the Hare procedure winner of both of these preference lists. Use this to explain which condition is not satisfied by the Hare procedure.

Voters 1-7	**Voters 8-12**	**Voters 13-16**	**Voter 17**
a	*c*	*b*	*b*
b	*a*	*c*	*a*
c	*b*	*a*	*c*

Voters 1-7	**Voters 8-12**	**Voters 13-16**	**Voter 17**
a	*c*	*b*	*a*
b	*a*	*c*	*b*
c	*b*	*a*	*c*

- Determine the Hare procedure winner of both of these lists. Use this to explain which condition is not satisfied by the Hare procedure.

Voter 1	**Voter 2**	**Voter 3**	**Voter 4**
a	*a*	*b*	*c*
b	*b*	*c*	*b*
c	*c*	*a*	*a*

Voter 1	**Voter 2**	**Voter 3**	**Voter 4**
a	*a*	*b*	*b*
b	*b*	*c*	*c*
c	*c*	*a*	*a*

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Class 4

Name:_____

QUIZ 1: VOTING SYSTEMS

Determine the winner using each of the six systems below:

- Condorcet: Head-to-head matchups, any undefeated alternative is the Condorcet winner.
- Plurality: Most common top choice wins.
- Borda Count: Assign points, with 0 points to the lowest person in each list.
- Hare (instant runoff): delete the least popular top choice and repeat.
- Sequential pairwise: set an agenda, and the winner of the first two competes against the third, etc. Here, use the agenda acdeb.
- Dictator: One person's vote decides the winner. Here, use last voter.

Here are multiple copies of the same profile:

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
a	b	c	c	b
b	c	b	d	d
e	a	e	a	c
d	d	d	e	a
c	e	a	b	e

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
a	b	c	c	b
b	c	b	d	d
e	a	e	a	c
d	d	d	e	a
c	e	a	b	e

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
a	b	c	c	b
b	c	b	d	d
e	a	e	a	c
d	d	d	e	a
c	e	a	b	e

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
a	b	c	c	b
b	c	b	d	d
e	a	e	a	c
d	d	d	e	a
c	e	a	b	e

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
a	b	c	c	b
b	c	b	d	d
e	a	e	a	c
d	d	d	e	a
c	e	a	b	e

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
a	b	c	c	b
b	c	b	d	d
e	a	e	a	c
d	d	d	e	a
c	e	a	b	e

FIRST WEEK FEEDBACK FORM

Name: _____

Use this form to communicate with me. Write a comment OR question OR make a suggestion about this class. (Ideally a couple of sentences). Each day I will respond to whatever you write. Thank you for your time. Leave this name tent on your table each day.

Possible questions to respond to: How do you feel about math? What would you like me to know about you outside of school? What hopes do you have for this class? What do you like about math or this class? What are you fearful of? What activities (sports/arts/jobs…) are you involved in after school? Draw a picture/sketch of how you are feeling. Draw a picture/sketch of something that represents you.

Day 1	**Day 2**	**Day 3**	**Day 4**
Comments: **(you)**	Comments:	Comments:	Comments:
Response: **(David)**	Response:	Response:	Response:

NOTES

1 Peter Liljedahl, "The Affordances of Using Visibly Random Groups in a Mathematics Classroom" in *Transforming Mathematics Instruction: Multiple Approaches and Practices,* eds. Patricio Felmer, Erkki Pehkonen, and Jeremy Kilpatrick (New York: Springer, 2014) 127-144, https://peterliljedahl.com/wp-content/uploads/Visibly-Random-Groups.pdf

2 Peter Liljedahl, "Building Thinking Classrooms: Conditions for Problem Solving," in *Posing and Solving Mathematical Problems: Advances and New Perspectives,* eds. Yeping Li, Edward A. Silver, and Shiqi Li (New York: Springer, 2014) 361-386, https://www.researchgate.net/publication/275953429_Building_Thinking_Classrooms_Conditions_for_Problem_Solving