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# Analogy: A Decomposition of Space and Time

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# A Decomposition of Space and Time

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by

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**Abstract**

The written part of this project is divided into two sections, with the first section focusing on the human eye as a biological tool for gathering and processing physical information. The second section strives to provide a model of human vision by utilizing Fourier Analysis. Out of this model came a focus on Fourier Analysis as not only a model, but as a methodology that can be applied in a variety of ways. The Fourier methodology provided a conceptual bridge that allowed me to more thoroughly explore art, physics, and how these two fields can contribute to each other.

Table of Contents

## Introduction

Throughout one's life skills are acquired through practice and an accompanying desire to learn. After a multitude of skills have been learned they become the foundation for one's life, most likely determining what one will do both for work and pleasure. Within the skill set that one develops there are individual tools that can be used for a variety of needs. From each area of one's life, academic and otherwise, one's own perception of the world is limited to one's unique experiences and the skills one was able to master. Too often these experiences are limited by the need to focus one's attention on a specific field of study. Young adults find they have been deprived of certain opportunities because they chose their field too soon. Children are told they could grow up to be a doctor or a writer, but never both at the same time. The idea that science and art are entirely separate entities permeates our society. Almost every young person interested in several different fields is forced to make a choice between what have been deemed creative or non-creative, art or science while pursuing an education. My desire to study Physics and Art simultaneously is an attempt to, at least for myself, bring together the skills from each of these two fields and explore a territory that is often not taught in schools.

The most common trait between art and physics emerges from the idea that there are still aspects of the world that need to be explored. These separate fields attempt to supply their practitioners with the tools necessary to expand on what has happened in the past. There is an understanding within each community that anything done today is linked to a foundation that spans millennia. I'm using generalizations, but these are necessary to ease one's way into this topic from any direction. The current state of these two fields, at least in popular opinion, are completely disconnected from each other.

To see how widespread this opinion really is, I might suggest mentioning to someone that you chose to major in both physics and art, and then explain how you are actually trying to bring these two majors into one. What you will see is a rapid succession of emotions that go from surprise, to confusion, to an impressed sigh of encouragement. After the tenth time receiving this reaction I began to wonder why people were so flabbergasted in the first place. As a society we have been taught that subjects can be split into 45-minute time slots and remain in these separate slots forever. What I am proposing is that these overlaps do happen and these moments are where humans can push forward.

From a knowledge of the past comes an understanding of how a variety of problems have been solved. Most physicists will adhere to the tried and true scientific method. The scientific method begins with a question about a specific physical phenomena or experience. From the question phase one begins to construct a hypothesis based off of some preliminary exploration. After a hypothesis has been established one is able to begin designing experiments that could test one's original ideas. If the experiment supports one's original hypothesis the experiment can be performed multiple times to catch any discrepancies. The way many artists work is very similar to that of a scientist. The original concept that an artist wants to explore manifests as the original problem or question. Before an artist can begin the artist has to decide which medium would be most efficient in conveying their idea. Most similarly to the scientist, the artist must experiment with different iterations of the same idea. Each of these people are attempting to answer a question or solve a problem and then show the results of their efforts whether they were successful or not.

One must think of artists and physicists as explorers looking out into the world, trying to discover something that has yet to be found. After this thing, whatever it may be, is found it has to be communicated to an audience. An artist utilizes color, sound, light, or any other material to synthesize their complex message into something a viewer can access. A physicist creates models, equations, and language to report their findings. Both the artist and the physicist communicate their messages with the hope that they have contributed to the furthering of their field. All of these methods of communication are not the "real thing," but instead an attempt to capture and convey the real thing as effectively as possible. Even Further, the goal of an artist and a physicist is to help their audience gain an understanding of something that was, for this audience, previously missing.

To even begin on the path to understanding one must observe one's surroundings and gather data. From the earliest attempts to those of a seasoned observer, one is entirely dependent on the natural faculties of perception. Touching, hearing, seeing, smelling, and even tasting are the ways through which we, as humans, experience our environments. The senses provide a gateway for the mind and help one in one's quest for knowledge. After data is gathered, it is broken down into more fathomable and manageable pieces. This project as a whole is an attempt to not only explore the process of decomposition, but consciously apply this method to the conveyance of my work.

The following section was born out of an interest in the human body and a lack of understanding in how humans do what we do. My interest in the human body made a gradual shift from the more macroscopic scale of biomechanics, such as walking or climbing stairs, to the smaller tools humans use to gather and process physical data. Being fortunate enough to have the ability of vision, I attribute the vast majority of what I know to be derived from the things I observed through my eyes. Vision became a compelling subject due to its reliance on multiple parts of human biological innovation, with the eye being the main subject.

In approaching the eye, it became important to separate its functions into two groups. One group includes the parts of the eye that work to gather data from the physical environment. The second group includes the tools we use to understand the data our eyes worked so hard to obtain. These two groups began to play a larger role in this project as a whole, with the idea that one needs to not only have access to raw data, but one must also have a way to understand this data. This train of thought led me to Fourier Analysis, which has become a kind of mathematical and physical model for the visual processes humans experience naturally.

Admittedly, I knew very little about Fourier Analysis before attempting to write about it at length. The importance and usefulness of the general idea made itself known through the manner in which my advisors spoke about Fourier's contributions to the field of physics. Being a person less inclined to learn from numbers and equations, new explanations were needed to grasp the subject at hand. This need for different types of explanations eventually became the uniting theme for this project as a whole.

I began with an interest in a general topic that we will label as human perception. Basically, how are humans able to experience the world around us? Obviously that question has a number of answers, many of which are outside of my expertise . Eventually, I came to focus on vision because of its most obvious connections to the physical environment. As I delved into my research  on the human eye, new points of inquiry began to appear. I soon began making connections between the biological processes of humans and the methods we have developed through the results of these processes. I provide this context not as a simple synopsis of events, but instead to highlight the fact that these thoughts did not arrive all at once and came about through months of thinking about the same things. Only after this foundation was cultivated could I find the most fruitful and worthwhile links between vision and physics. But, as I have already alluded to, the artist and the physicist need to quickly become adept at helping their audience and themselves understand their work in greater depth. One can do so many ways, but I found analogies to be especially helpful.

Before there was the scientific method there was the desire to understand something, anything really, better than it had been before that point. When one knows a little about one thing, one will probably try to compare it to something else that one knows really well. Analogies serve as a useful tool to convey complicated ideas. When one says, “It’s like finding a needle in a haystack,” another person understands that one is not actually talking about a needle or a haystack. Instead one imagines the difficulty of finding such a small object in such a large, complicated shape and then applies that feeling to the situation at hand. While this analogy is rather simple, analogies can be used in the most complex scenarios.

In Daniel Kennefick’s “Traveling at the Speed of Thought” the analogy behind the research and eventual discovery of gravitational waves is explained. In general, the analogy was made between gravitational waves, which lacked experimental data, and electromagnetic waves, which has a very large base of knowledge and data surrounding it. In the first chapter of this book, Kennefick focuses on two types of analogies, one being a heuristic type and the other being an analogy where the two things being compared actually share innate similarities (Kennefick 7-8). For the sake of my project, the line between these two types of analogies is less clear. When I first began, I was using the heuristic type of analogy, with the sole desire to understand Fourier Analysis on a deeper level. As I kept working with this analogy something actually quite common occured, which is that some innate similarities between the two things I was comparing started to appear on an abstract mathematical level. I mention the distinction between these two types of analogies because it is important to be aware that just because an analogy works does not mean the two things being compared function in the same exact way.

An analogy becomes just another tool one can employ to understand even the most intricate of concepts. In my case, the most difficult concepts to understand revolved around the concept of Fourier Analysis. My research led to equations and formulas, which are explanations in their own right, but I needed to gain a more conceptual understanding of the main point. To gain this understanding I had to turn back to something that was stressed in my first physics class, and probably even back to high school. As you will come to see, this concept that I call upon to ground Fourier Analysis in more basic ideas is called vector decomposition.

As I mentioned, on a more abstract mathematical level these two things do in fact share innate similarities. And just to be clear, when I say "innate similarities" I am speaking to the idea that these two methodologies exist to solve similar types of problems and do so in the same way. But, in this project, I used the analogy between Fourier Analysis and vector decomposition to discover and learn about something new to me. For right now it is not important exactly what vector decomposition is, but instead how I was able to use it. Vector decomposition became a bridge between the more solid parts of my knowledge and Fourier Analysis, which is what I was attempting to learn about. In understanding this subject on a deeper conceptual level, I was able to integrate the Fourier methodology into my scientific work as well as my artistic work.

The main ideas behind Fourier Analysis served as a driving force in developing the final concept of the artistic side of my senior project. In the most general of terms, the Fourier methodology rests on the notion that complicated things can be broken down into pieces. These pieces are meant to help one understand that complicated thing even better than before. This method is analogous to the process I go through in life on a daily basis. Most of my work has centered around my own past, approaching it as this “thing” that can be picked apart and analyzed piece by piece. I have chosen to believe that I am not dwelling on the past, as some people may say. Instead, I am trying to gain an understanding of who I am, knowing that I am the product of those pieces being added together. In an attempt to avoid confusion due to a lack of vocabulary and understanding of the underlying concepts, my art installation will be addressed more thoroughly at the end of this project.

One thing this project is attempting to do is give the reader a stronger understanding of human vision, fourier analysis, and how these two things relate. Vision is an integral tool for humans in their endeavors to gather and process data from the physical world. Fourier Analysis can be viewed as a mathematical model of vision and contribute to a better understanding of this process. This project is also meant to showcase the learning and understanding won through analogy in art and science. In showing my work, I hope to shed light on how the fields of art and physics can contribute to each other in a meaningful way. Within this project I am exposing the methods I have used to grapple with information that I could not immediately comprehend. I have found that the easiest thing to do when one is faced with something unknown is choose to keep it that way. But, the better thing to do is think about what one does know and use that knowledge to know about something else.

## Introduction to Human Vision

Eyesight, perhaps one of the most important senses in regards to human survival, serves as a key to gathering the physical data of our surroundings. Eyes are able to provide the information to answer some of the most basic questions one might have: who, what, where, when, and how? On an instinctual level, early man used this information to decide if their environment posed any immediate threat to their survival. In today's world, where survival is somewhat easier to achieve, we are able to use our ability to see for pleasure. In an attempt to provide an understanding of how the eye is able to see anything at all, I am trying to shed light on the complexities and wonder that normally go unnoticed.

## Structure of the Eye

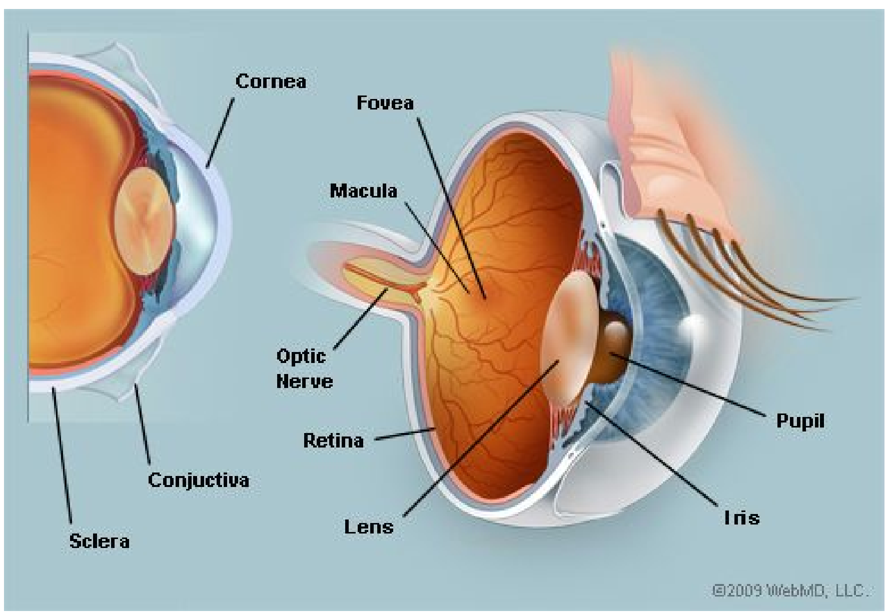



Image depicts one half section of the human eye in profile and three-quarters view (Web MD).

The eye can be split into two main sections: the front and the back. These two different sections work together, with the front half funneling light in and the back half beginning to process that information. The frontal section of the human eye includes the pupil, iris, cornea, lens, ciliary muscle, and aqueous humor. Each one of these parts play a specific role that helps humans collect accurate visual information. Basic structures and their functions include:

**Cornea:** The cornea is the outermost layer of the eye. The cornea is tightly packed with transparent fibers that allow light to easily move through its surface and into the eye.

**Aqueous Humor:** The aqueous humor is a transparent liquid located between the cornea and the lens. This substance, which is similar to blood plasma, is necessary for the continued health of the eye. Its main function is to carry nutrients, like oxygen, to the lens and cornea as well as removing waste. The aqueous humor also helps maintain the correct curvature of the eye. When this substance is unable to be adequately removed, pressure can build which can lead to vision-threatening conditions such as glaucoma (Wolfe 31, Encyclopædia Britannica).

**Pupil:** The pupil is a hole located in the center of the iris that allows light to enter the main body of the eye. Its size adjusts in accordance with the amount of light present in the environment, getting smaller with more light and larger with less light.

**Iris:** The iris is a pigmented ring around the center of the eye surrounding the pupil. The iris controls the size of the pupil, dictating the amount of light able to pass through. This function allows the eye to effectively adapt to varying light intensities. When one is outside on a sunny day the iris will constrict to make the pupil smaller which will protect the eye from unnecessary strain or damage by getting too much light.

**Ciliary Muscles:** The ciliary muscles function similarly to the iris in that they control another structure within the eye. They are located on the top and bottom of the lens, and their primary function is to help with focusing the eye at near and far distances. They focus the eye by contracting to change the actual shape of the lens.

**Lens:** The lens is perhaps the most important part of the eye. Its main function is to focus an image on the back of the eye. The lens is relatively flexible in that its shape can be changed using the ciliary muscles. A relaxed lens is rather flat, allowing it to focus on objects at farther distances. When the lens is contracted, becoming more tightly curved in the process, it changes how the light is refracted and then focused on the retina.

## Light and Refraction

The most pertinent physical function of the front half of the eye revolves around the transmission of light, so it's necessary to understand the concept of refraction. In general, this phenomenon occurs when any wave is moving from one medium into another medium with different physical properties. A common example for the refraction of light involves a pencil in a glass of water.

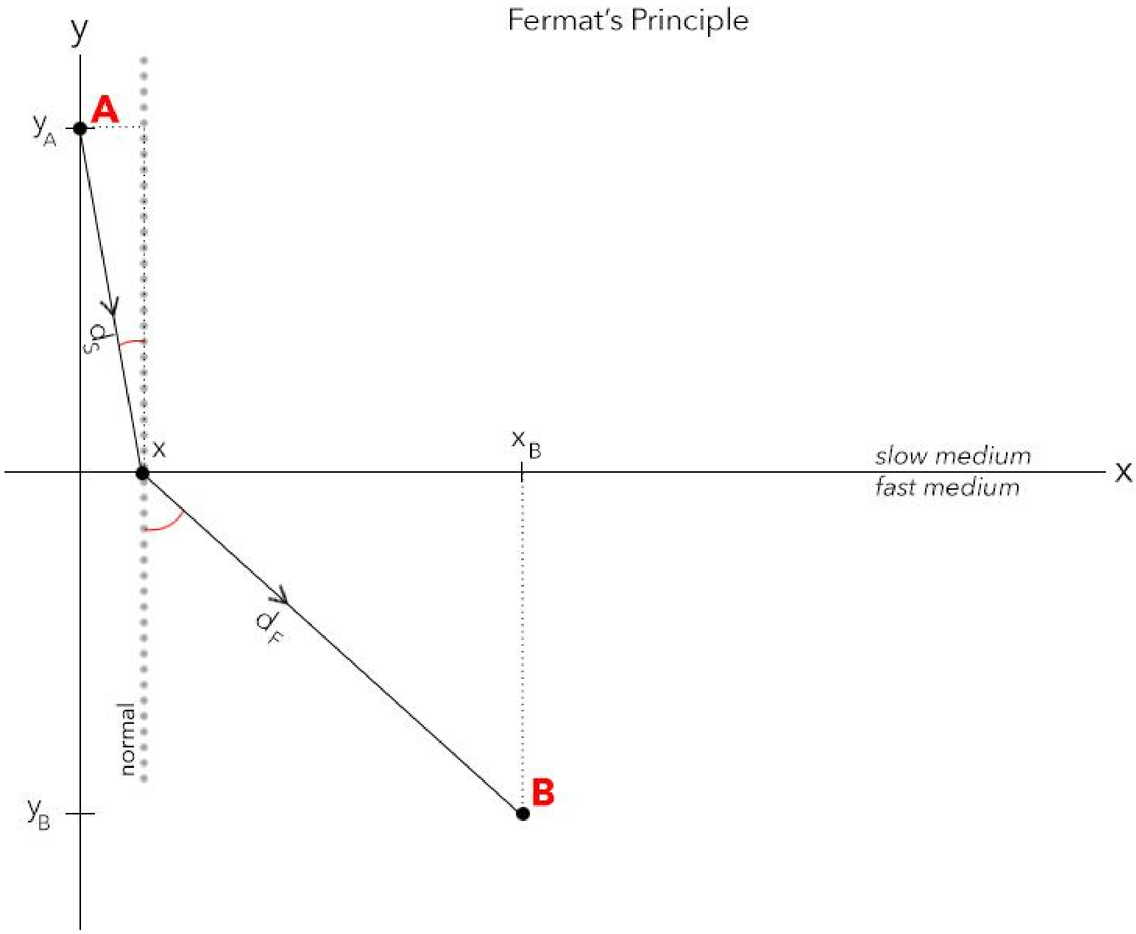
Depiction of pencil and water example of refraction taken by Dr. Roy Winkelman

One will notice that the segment of the pencil that is submerged in the water seems to have been shifted or bent. This discontinuity occurs because the light has traveled from within the water back into the air. The index of refraction for these two mediums are different, so light will travel at different speeds. It is difficult to imagine light changing speed, but it may be easier to think of the reasons light may change speeds in the first place. Light is able to move from a rather "light" medium like air, where few molecules are present, to a more dense medium like water, where molecules are more tightly packed together. When an incident like this occurs light encounters a kind of "friction," and I use this term very loosely here. As light enters the denser medium it encounters more molecules that are absorbing and then emitting it which in turn affects light's speed.

The "index of refraction" is what refers to how fast light is able to travel within a given medium. For example, the speed of light was measured in a vacuum to be approximately $3 \times 10^{8}$ meters/second while the difference when the speed of light is measured in air is negligible. The relatively simple equation $n = \frac{c}{v}$, with n being the index of refraction, c the speed of light in a vacuum, and v the speed of light in a given medium, supplies enough information to determine the n-value for any medium if one knows how fast light is moving through that medium. For air the speed of light in the medium is extremely close to the speed of light in a vacuum, so the index of refraction for air is quite nearly 1. It is known that n=1.33 for water, which means light travels $\frac{3}{4}$ the speed in water that it does in a vacuum. But how was this determined before anyone was able to measure how much light slows as it enters this medium? The answer to this question lies in another equation called Snell's Law.

Snell's Law is written as $n_1 \sin\theta_1 = n_2 \sin\theta_2$ with the left and right sides of the equation referring to two separate mediums. The $\sin\theta$ portions of Snell's Law correspond to the angles at which light enters (angle of incidence) the second medium and the angle at which it leaves (angle of refraction) in relation to the normal to the interface. These angles of incidence are highlighted in red in the diagram. A simple algebraic manipulation reveals an equality between the ratio of the sine components and the reciprocal of the ratio of indices of refraction. There is a certain beauty to Snell's Law and refraction that can be understood through a mathematical proof and that can be illustrated through real world experiences.

Fermat's Principle states that a light ray will take the path that will require the least amount of time. By using Fermat's Principle, one can derive Snell's Law. The general setup includes a ray of light going from point A to point B on a simple, two dimensional coordinate system. I will use this setup to prove Snell's Law.



The total time that it takes the light to travel from A to B is

$$f(x) = t_{SLOW} + t_{FAST},$$

and this can be expressed in terms of the speeds and distances as

$$f(x) = (d_{SLOW} / v_{SLOW}) + (d_{FAST}/v_{FAST}).$$

The speeds can in turn be expressed in terms of the indices of refraction in the two media ($n_{SLOW}$ and $n_{FAST}$) and this yields

$$f(x) = ((n_{SLOW}*d_{SLOW})/c) + ((n_{FAST}*d_{FAST})/c).$$

Now the distances can be expressed in terms of x and y, giving

$$f(x) = \frac{1}{c}[(n_S \sqrt{}(x^2 + y_A{}^2)) + (n_B \sqrt{}((x_B-x)^2 + y_B{}^2))].$$

After this substitution, to find the minimum time it takes the light to travel, one can take the derivative with respect to x and set the equation equal to 0, which leaves

$$0 = (n_S x/\sqrt{}(x^2 + y_A{}^2)) + (n_B(x_B-x)/\sqrt{}((x_B-x)^2 + y_B{}^2)).$$

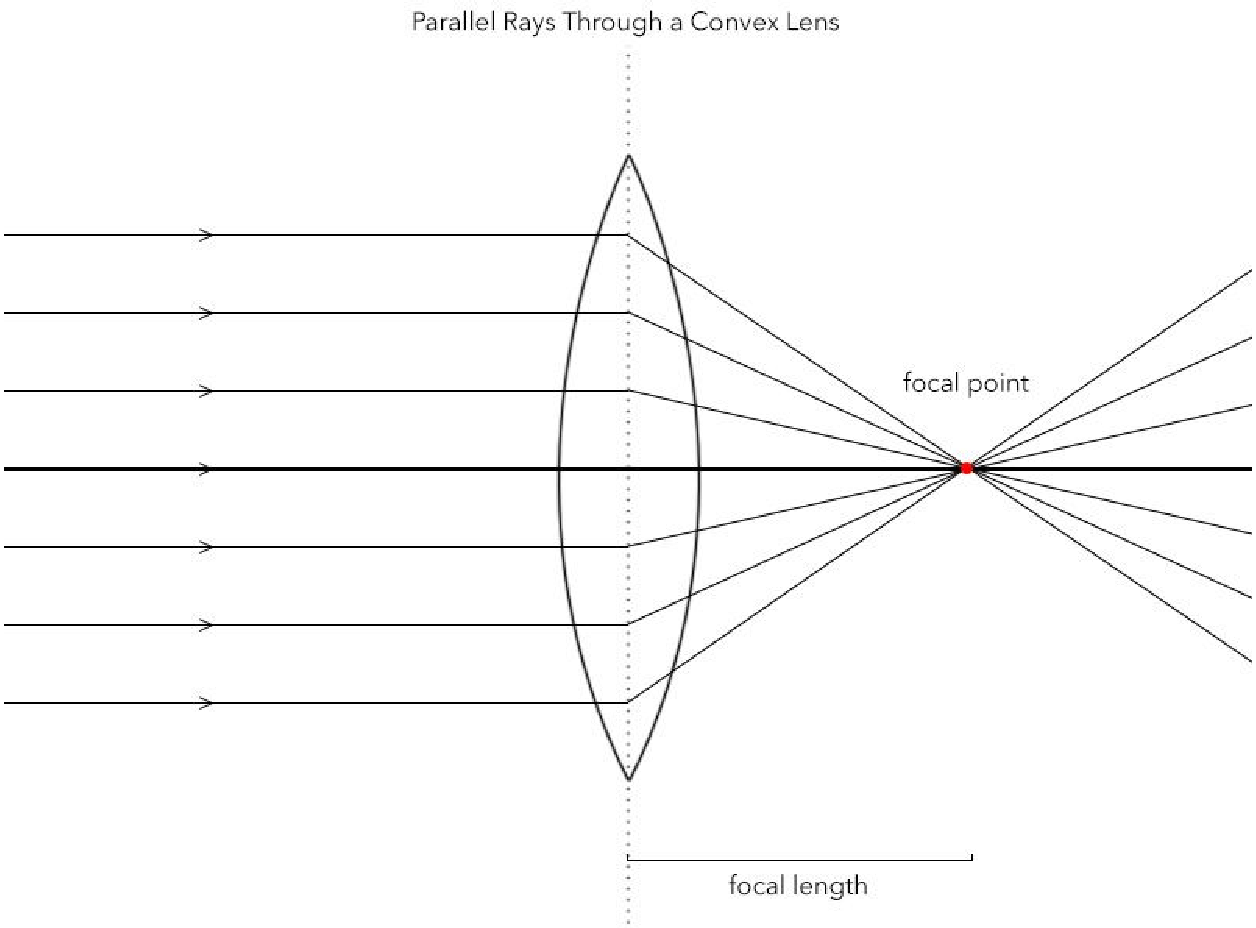
Finally, one can rewrite x and y in terms of θ, leaving one with Snell's Law in the form

$$0 = n_S \sin\theta_S - n_F \sin\theta_F.$$

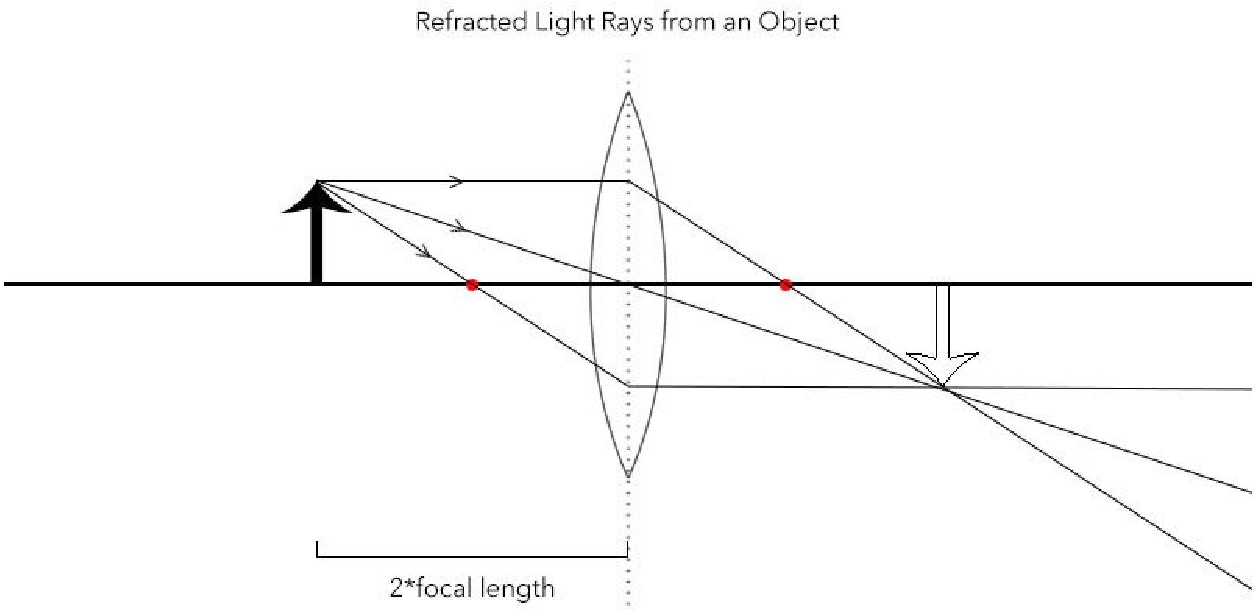
The fact that Snell's Law can be directly related to Fermat's principle helps shed light on the simplicity of nature. From the idea that light will always take the path of least time one has explained why light bends as it moves from a slow medium to a fast medium and vice versa. To use a real world situation one can think of a lifeguard at a beach. The lifeguard has made their way out to someone drowning in the water at point A, but now the lifeguard must carry them in to point B. The water is much harder to move through, so the lifeguard knows to find the fastest way out of the water while still making their way to point B. This means that the lifeguard will want to spend the least amount of time possible in the water and cover more distance on land where it is easier to move. This example is meant to allude not only to the efficiency of nature, but also show how simple intuition can be applied to a physical problem. A greater understanding of Snell's Law and refraction led to one of the most useful technological innovations to ever exist: eyeglasses.

Eyeglasses, as well as anything involving lenses, depend on refraction to alter the path of light rays. The specific attributes of a given lens, as well as its position, dictate what will happen to light as it is transmitted. The light that has passed through the lens will form an image. Eyeglasses were invented to correct any deficiency in our biological lens by adjusting where the final focused image appears. There are several types of lenses, but looking forward to the lens of a human eye I will focus on convex lenses. A convex lens can be defined as having two faces with positive curvatures around a center point. To begin addressing any type of lens in more detail I will need to define some key vocabulary.

First, there is an object, which can be thought of as where the light originates. This object is what one is trying to "see" or focus through a lens. The image, as I have already alluded to, is where the light that has come from the object ends up. There are two types of images that can be formed after light has passed through a convex lens. The first type, a virtual image, which can be observed but there is no actual light interaction where that image is formed. The easiest example to comprehend this idea is a simple plane mirror. Looking into a mirror in one's bathroom shows you an image that appears to have dimension, but one knows no light has penetrated the mirror. On the other hand there are real images. In this case the image is formed by light rays converging after they have been focused by a lens. Our biological lenses form real images on our retina. The focal point is the place at which parallel light rays that have passed through a lens intersect. Lastly the focal length of a lens describes where a given lens is able to bring light rays together. This quantity helps you to understand where an image will form. In the following diagram I show how parallel light rays travel through a convex lens to meet at the focal point.




From the simpler diagram of parallel light rays being refracted by a convex lens one can move to a slightly more complicated setup that includes an object.




This diagram depicts light rays from an arrow's tip being refracted through a convex lens which in turn creates a real, inverted image on the other side. To keep this diagram as simple as possible I have placed the object at a distance equal to two times the focal length to avoid any sort of magnification. Using three key light rays alongside some basic rules of optics one is able to pinpoint where an image will appear. The first ray is exactly like the parallel rays from the previous diagram, it goes through the lens and is refracted through the focal point. The second ray leaves the arrow's point and goes through the center of the lens. This ray will avoid any refraction allowing it to travel in a straight line. One can think of the third ray as an inversion of the first. It leaves the arrow to intersect the focal point before it has passed through the lens. Once it does pass through the lens it is refracted to be parallel to the x-axis. As seen in the diagram, the point where these three rays intersect is where the image of the arrow's point appears.

This diagram is a precursor to the slightly more complicated human eye. The first important idea to understand is that the human eye is made of many different types of organic material. Unlike a glass lens one may encounter in a lab, the lens found in a human eye is pliable and able to change shape. As I have explained while defining the human lens, its flexibility allows one to change what is in or out of focus. Working alongside the cornea, which accounts for about seventy percent of the refraction that occurs in the eye, the lens is an active component in producing a focused image. Nonetheless, a raw image is useless if there is no way to understand it, which is where the back of the eye becomes important.

## The First Steps of Visual Processing

After light has been focused through the cornea and the lens it finds itself on a surface rich with light-sensitive cells, which are a part of the retina. The front half of the eye worked to gather an image, but now the retina is able to make that data into something one can *think* about. The most interesting, as well as the most important, parts of the retina are those that begin to transduce light into signals that the brain can understand. These parts, called photoreceptors, are responsible for tracking any changes in light intensity in the physical environment.

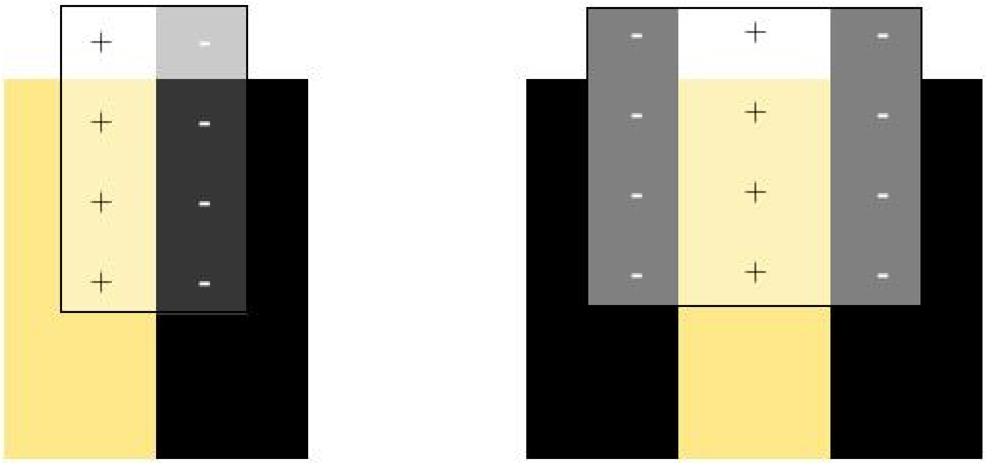
The two most relevant types of photoreceptors in the human eye are referred to as “rods” and “cones,” whose names are derived from nothing more than their shape. Rods and cones have different strengths when analyzing the light data they encounter. Cones operate most efficiently in environments that have a relatively high amount of light. The largest concentration of cones in the retina can be found near the fovea located in the center of the visual field, which is where light is most strongly focused after it passes through the lens (Wolfe 37). This location allows cones to pick up smaller, or more subtle variations in an image. Additionally, cones also allow humans to see in color due to the presence of three types of photopigment in three different types of cones. Having two types of photoreceptors allows rods to make up for any inherent weaknesses found in cones.

Rods function best in situations where there is a low amount of light. Unlike cones, rods do not contribute to the perception of color, but they make up for this weakness with their strong sensitivity to light. There are an astounding 90 million rods with only 4-5 million cones, so the importance of rods should not be underestimated (Wolfe 37). The majority of rods can be found away from the fovea. The location of rods makes them more sensitive to changes in intensity on the periphery of our vision. The distribution of these two different photoreceptors makes sense, when one is looking forward one will most likely need to gather data of the most detailed parts of an image. Changes in light around one's periphery require less detailed information, but communicate necessary information about objects that may be coming into view.

After light has triggered the rods and the cones, information in the form of chemical and electrical signals continue to be processed through the retina. These signals continue on a path through cells that continue to process this data, until they finally arrive at the ganglion cells. This is where all of the data collected and processed thus far is prepared to be sent to the brain. This is also the moment where variations in light, also known as contrast, begins to play a larger role in how one understands one's surroundings. After the ganglion cells, the signals leave the immediate vicinity of the actual eye to begin their journey to the brain.

After the signals have made their way through the photoreceptors and ganglion cells, their pilgrimage ends in the visual cortex. In the visual cortex, "simple cells" exist, which can be defined as neurons that have "clearly defined excitatory and inhibitory regions" (Wolfe 64).

Included below is a reproduction of a figure featured alongside Wolfe's definition of a simple cell (Wolfe 64).




(a) edge detector (b) stripe detector

The rectangles including the plus signs and minus signs represent two versions of simple cells. The sections including plus signs can be thought of as the "excitatory regions" and the minus signs the "inhibitory regions" of the cells. The black and yellow parts of the diagram represent clear variations in light. When the cells line up with stimuli that correspond to these regions, as depicted in the diagram, the cells will send a signal to the brain. This signal will correspond to the type of simple cell that has been triggered, with a cell like example (a) signalling the presence of an edge or example (b) a stripe.

As I alluded to in the discussion on the importance of rods in the human eye, the ability to detect changes in light intensity is integral to constructing an image of our environment. The two examples included, (a) and (b), are only a couple of the different types of simple cells in the visual cortex. It may be obvious, but the ability to detect edges and stripes, where stripes can be thought of as spatial patterns, is the foundation of vision. Simple cells are an important tool for the detection of spatial variation in the physical environment. Humans depend on the detection of these variations, including edges and stripes, to look into the world and separate one object from another. Once a single object is identified, smaller edges appear to reveal intricate details and supply not only objects, but other people with unique traits and physical identities.

Human vision relies on a team of biological tools that developed throughout years of evolution. These tools have been crafted by our needs to gather information from the world around us and then interpret that information. Our brains supply us with the natural ability to take in raw data, but after that our minds must take over to truly observe. In the next section, one will find how the mind itself must craft its own tools to understand ideas beyond the physical environment, but you should see for yourself.

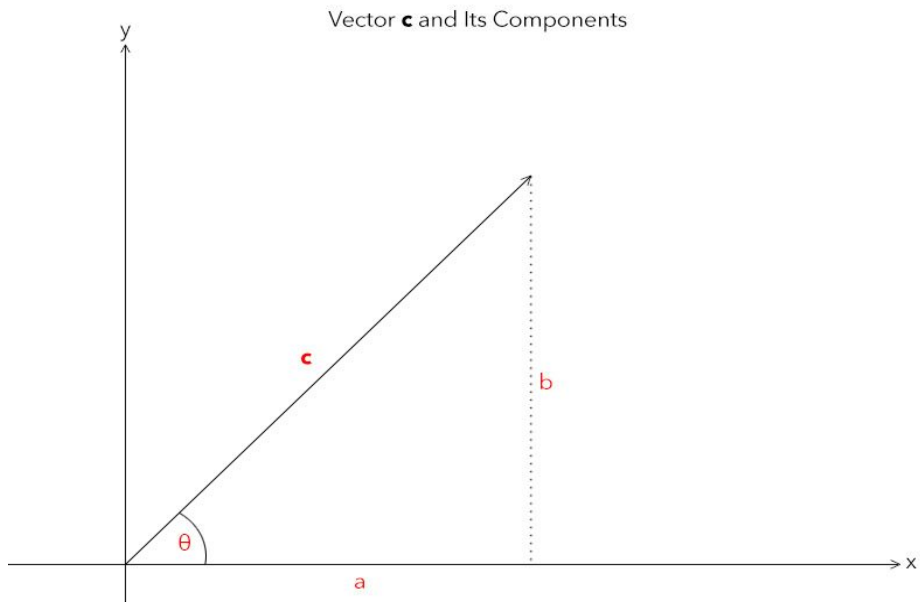
## Introduction to Fourier Analysis

When I first happened upon the concept of Fourier Transforms it seemed clear that they are used to break something up into smaller, more useful pieces. My initial introduction to the topic of Fourier Analysis occurred within conversations pertaining to sound and hearing. This may be a surprise for some, but the sounds humans hear throughout our everyday lives are incredibly robust and consist of their own set of unique parts. Continuing in regards to sound, one can think of a simple chord as it might be played on a piano. The C major chord is one of the most popular chords found in Western music. The chord itself, when played on a piano, appears to sound like one note. In the simplest of terms, performing a Fourier Transform on this C major chord would reveal that it is comprised of three totally distinct notes, C, E and G. Of course in the real world one could go even further to look at each of those notes on their own. In doing so, one would discover a new set of frequencies that come together to achieve what humans perceive as a pure tone from the piano. I will show how the main principle of Fourier Analysis can be expanded to address a multitude of phenomena and used as a tool to understand and manipulate spatial information.

## Fourier Analysis Through the Lens of Vector Decomposition

To understand Fourier Transforms one can compare them to the process of vector decomposition. While “vector decomposition” sounds even more daunting, the process is relatively simple. One can think of a vector as an object, picturing a line in space, that contains information pertaining to its magnitude and direction. To analyze vectors more accurately, physicists and mathematicians create a coordinate system to provide context and the means to apply numerical measurements. Decomposing a vector requires the consideration of its separate parts, with these parts being the x and y components as represented in the chosen coordinate system.

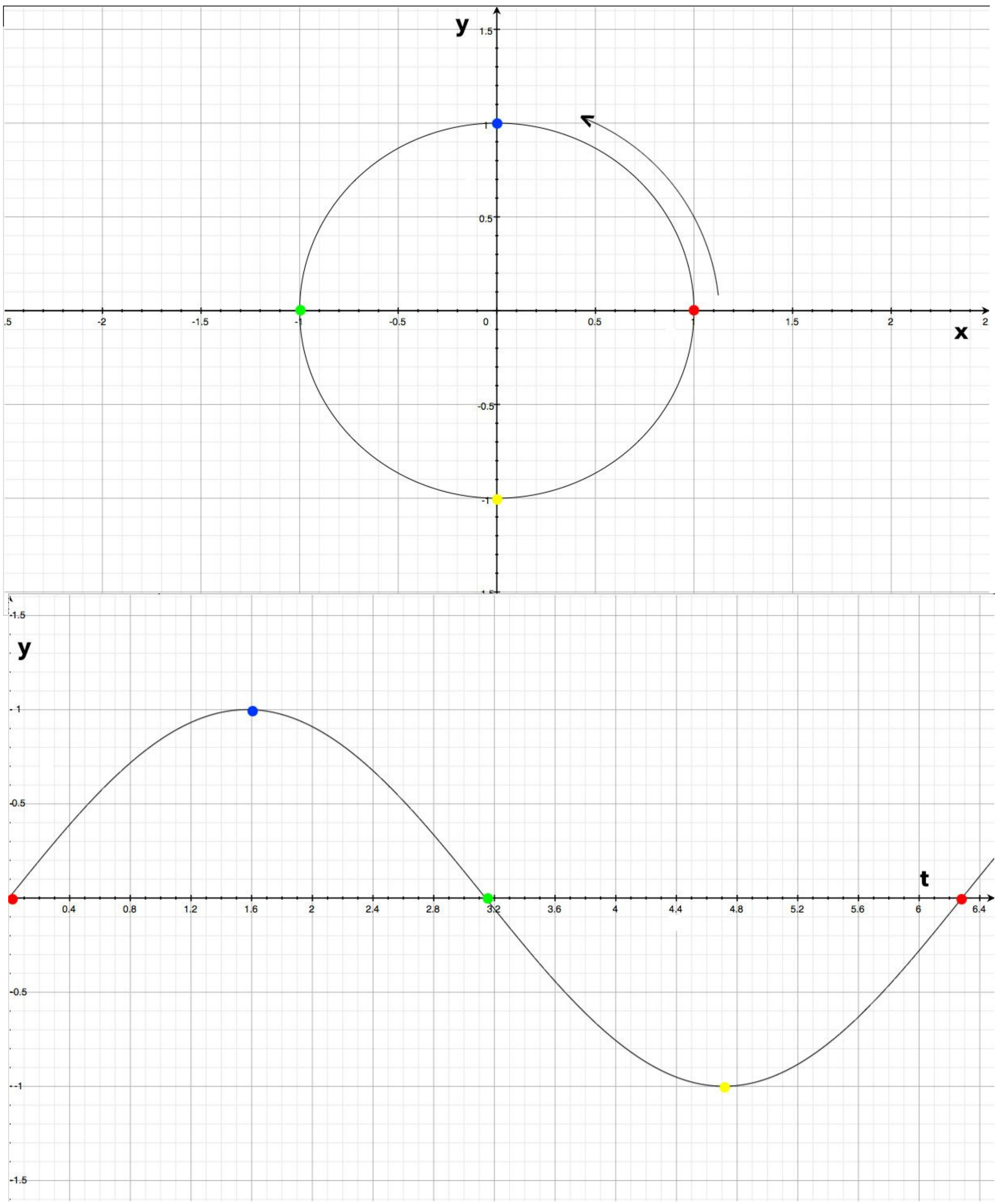
The components of a vector are merely tools for describing it in more useful terms and providing information in a more digestible form. To explain further I will use an example involving a vector, which I will call **c**, that starts at the origin and ends at a point in the first quadrant of our two-dimensional cartesian coordinate system. One can draw a line straight down from the end of the vector to the x-axis. The length from the origin to where that line intersects the x-axis will be defined as a. Going through the same process for the y-axis gives a magnitude which will be defined as b. These two numbers, a and b, are the x and y components of the vector **c**. Components a and b show us "how much" x and y are hidden within the slightly more complicated vector. The last variable that will soon be important is the angle between the vector and the x-axis, which one can define as θ. Working off of this example, as depicted below, one can begin to explore the relationships between each of these variables and how they relate to each other.



Understanding the most basic way a and b relate to the vector **c** allows us to move on to θ and how all of these variables are inherently linked. Looking at the diagram above, it is clear that the vector c can be drawn as the hypotenuse of a triangle. To bring a, b, and **c** together one can use the Pythagorean theorem: $a^2 + b^2 = c^2$. This is the Pythagorean theorem we all know and undoubtedly love, but there is another representation often overlooked. This same theorem can be written as $sin^2\theta + cos^2\theta = 1$.

The jump between a, b, and **c** to their trigonometric counterparts may be confusing for some and that is understandable. One can make the magnitude of vector **c** equal to 1, with 1 being a single unit, and maintain the vector's direction. This can be done to any vector regardless of its original magnitude and, just to be clear, making a vector a unit vector is just a mathematical process to make calculations easier. From basic trigonometry, one knows $sin\theta = \frac{b}{c}$ and $cos\theta = \frac{a}{c}$, and since c=1 the denominator goes away and one is left with the simpler equations: $sin\theta = b$ and $cos\theta = a$. Finally, one can think of the x and y-components in terms of $cos\theta$ and $sin\theta$. This new way of representing these components provides one with the means to start relating vectors to real world physical phenomena.

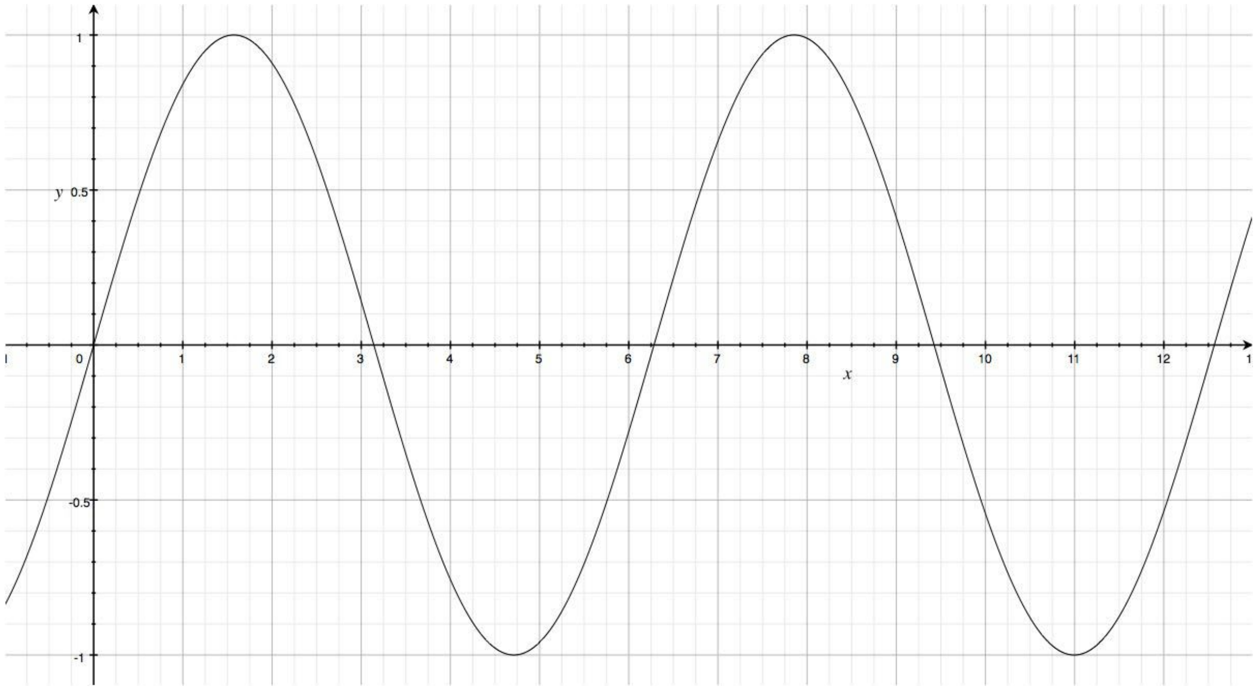
The introduction of $sin\theta$ and $cos\theta$ and the new representation of the Pythagorean theorem encourages the transition from triangles to circles. Graphing the equation $x^2 + y^2 = c^2$ with $c = 1$ reveals a circle with a radius of one, which is also known as the unit circle. For any point P on the unit circle, there is a corresponding, unique value for theta in the equation $sin^2\theta + cos^2\theta = 1$. Imagining point P as an object that can move around the circle, the four different colored dots mark four "snapshots" of its journey over time. Continuing to think of point P as an object allows one to think of different ways to analyze its motion around the circle. As point P moves around the circle in a counterclockwise motion, it ascends until it reaches its maximum at the y-axis to begin its descent crossing zero at the x-axis. The opposite sequence occurs until it reaches its original position. Graphing the object's vertical position along the y-axis reveals a sinusoidal waveform with its corresponding maximum, zero, and minimum points.

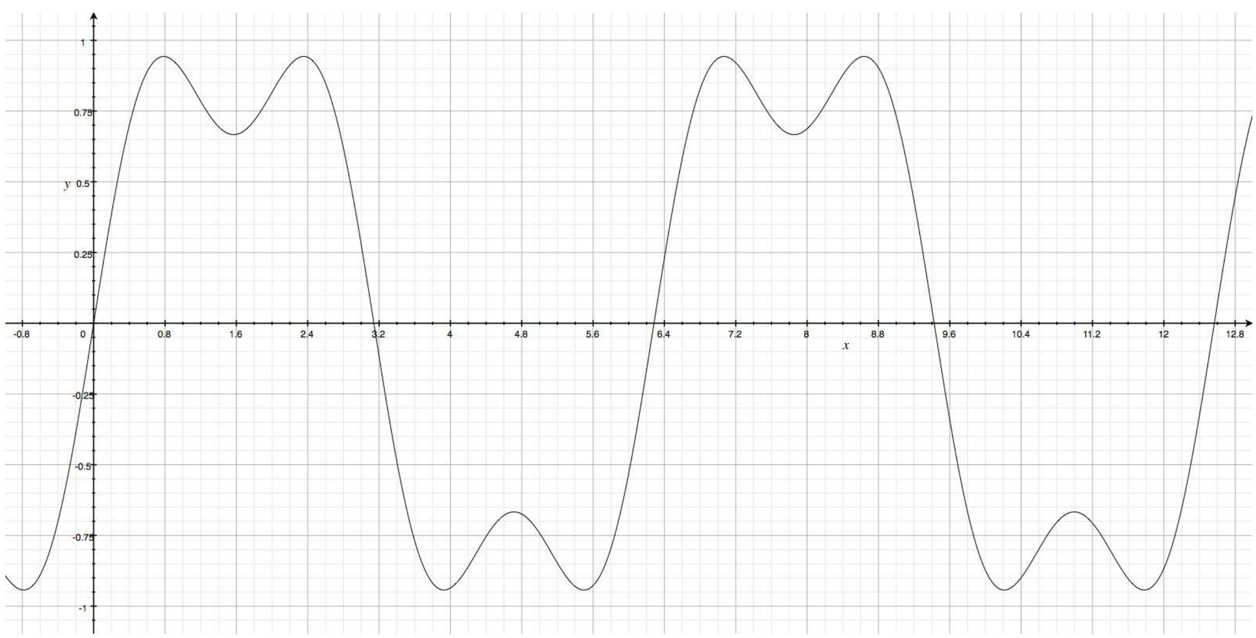


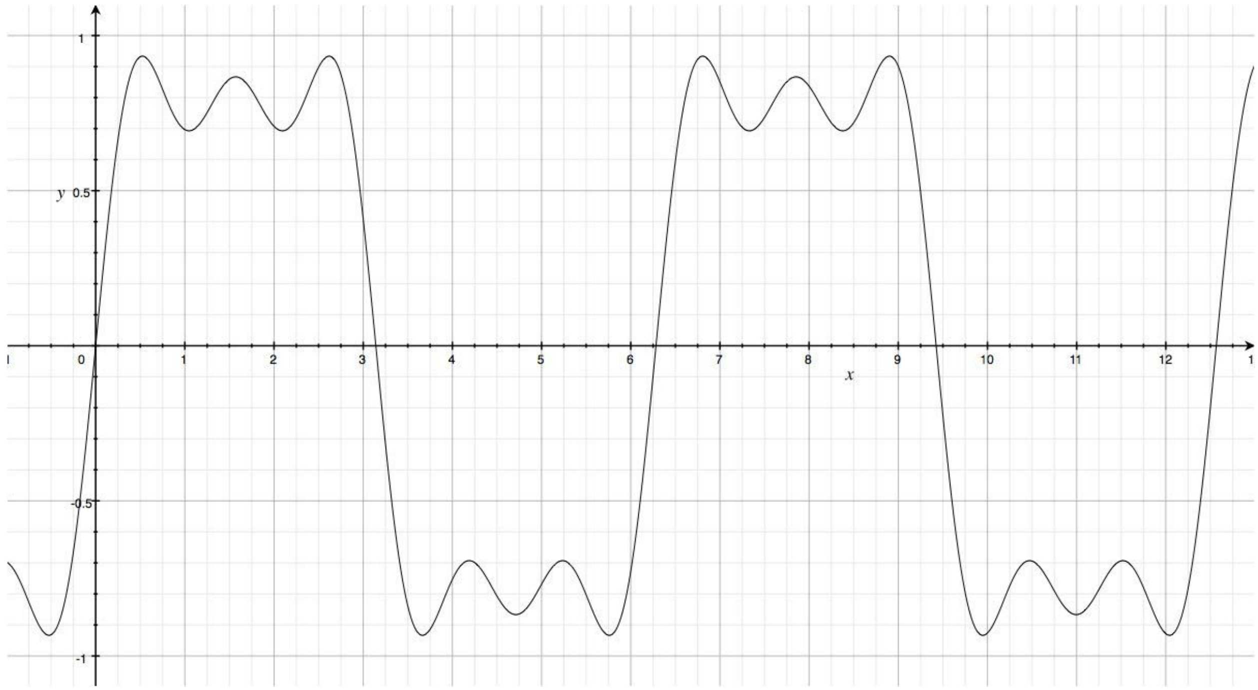
The graph of this point's vertical or horizontal position alone is what physicists call simple harmonic motion and comes up quite often. In basic terms, simple harmonic motion is characterized by cyclical variations around an equilibrium. When one is observing an object moving around the unit circle, one knows that the object will travel the same path over and over, passing over its original position every time. The different representation of this object's movement with a sine curve only further reinforces this idea with an important addition: time.

The idea of an object rotating around the unit circle should be clear by this point, but there are still factors to explain. Perhaps the most important of these factors is the speed at which the object is traveling its path. Assuming this object is traveling at a constant angular speed $\omega$ around the circle, then $\theta$ is equal to $\omega t$. This new relationship allows one to transition from thinking about $sin\theta$ to $sin(\omega t)$. The introduction of $\omega$ gives one a new degree of variability within the sine function itself allowing for the production of an infinite number of unique sine (and cosine) functions. These functions are the aforementioned building blocks that constitute even the most complex "real world" phenomena.

One of the most common periodic functions we generate in the lab is referred to as a "square wave." An ideal square wave is characterized by instantaneous shifts between the function's minimum and maximum values, maintaining each for equal intervals. It probably comes as no surprise that this type of function appears most frequently when dealing with electronics and the binary information these electronics produce. Included below is an example of a square waveform built from the combination of five different sinusoidal functions. The first image depicts a single sine function, the second shows the next sine function added, and so on. Adding the overtones to the first function one by one emphasizes their individual contribution to the final waveform, where none of these unique parts are discernable.

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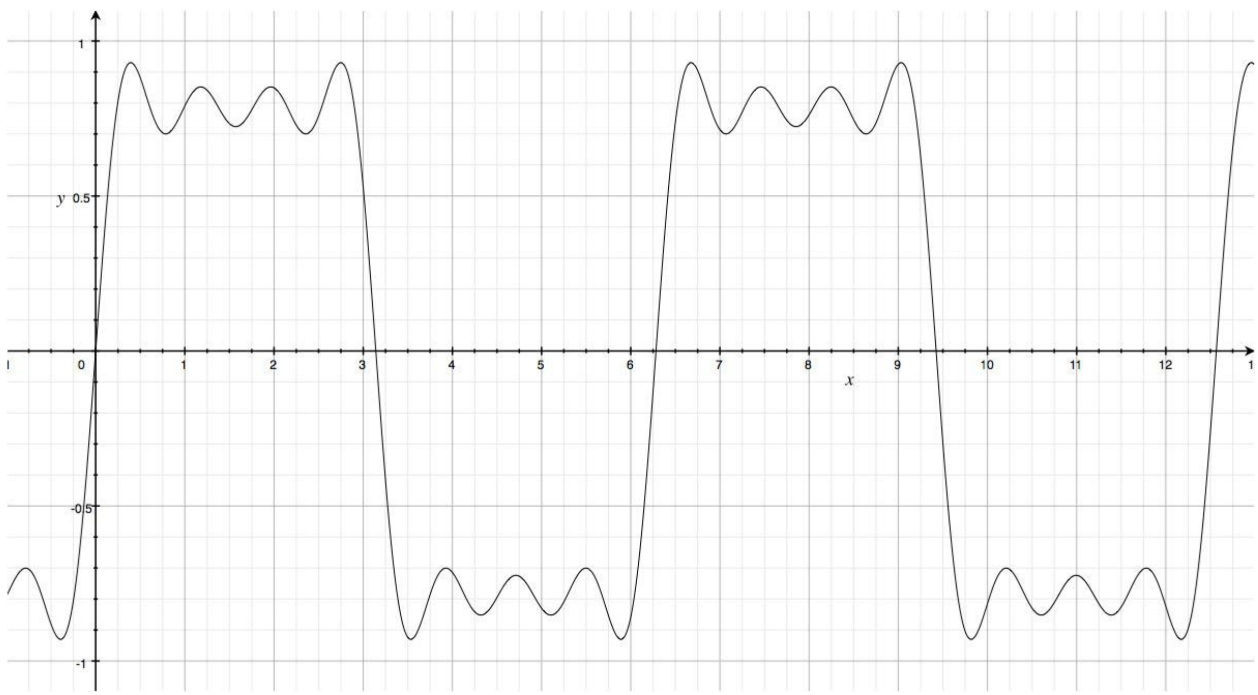
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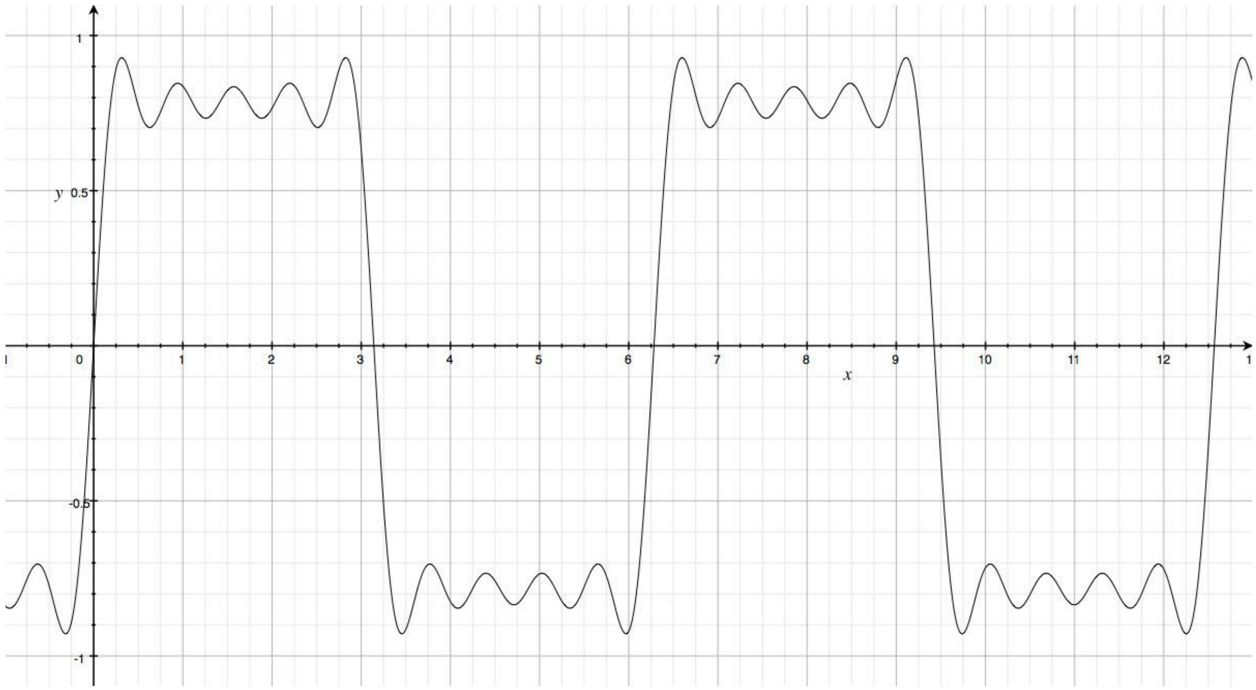



Again, a square wave is just one example of a waveform that can be decomposed into different sine functions. These sine functions are in the form $Asin(\omega t)$ with A and $\omega$ being the only parameters that determine the physical aspects of their motion. The addition of A, which is the amplitude, can be thought of as a normal coefficient that dictates the function's maximum and minimum magnitudes. The angular frequency describes how fast it takes to complete one cycle, in other words, $\omega$ is describing the number of variations in one second. This means that an increase of the angular frequency will cause a cycle to be completed in less time. By adjusting these two parameters alone, one is able to approximate a more complex function that approaches a more accurate representation of what may be found in reality.

**Fourier Analysis and Spatial Information**

As I have pointed out, sound is an especially useful physical phenomena when thinking about Fourier analysis. Most people understand that sounds change based on their frequency (pitch) and amplitude (loudness). From there it is quite easy to think of pure tones coming together to make more complex sounds that are found in music or even our own voices. The same core ideas that have been discussed up to this point carry over to a discussion of Fourier Analysis and images.

To begin, one will have to shift from thinking about time, as one did with sound, to space. Perhaps the best way to illustrate this shift is through naturally occurring waveforms in nature. Imagining the landscape of a desert will almost immediately bring the image of large sand dunes for as long as the eye can see. Instead of a desert, one can imagine ripples on the surface of a lake deprived of its placidity. While time certainly played a role in creating these patterns, the final product is a representation of variations in space.

Images depicting examples of spatial variation. Ripple image by Sergiu Bacioiu and sand dune image by Sherbaz Jamaldini.

The main goal in applying a Fourier Transform to an image is the same as before: decompose the image into its more easily manageable parts. But, before one can address an actual image, it is important to revisit the concept of vectors. An image that one views on a computer screen is two-dimensional and most people understand that they exist in a three-dimensional world (if one forgets about time), but what does that actually mean? One can imagine a person in an empty room, but how would one describe this person's position within that room? At any given moment the person's position can be described by where they are in relation to each wall and the floor, resulting in a total of three numbers totally independent from one another. Since only three values are needed for a complete description of the person's position, the person is in a three-dimensional space.

An image, more specifically a digital image, consists of little dots of color called pixels. These pixels are similar to the person in a room except one only needs two values to describe their position. This is just like finding a point on a Cartesian plane, each point has an x-component and a y-component. Picturing a grayscale image, void of any color, aids one in realizing that each pixel has its own value or intensity between white and black. This gray value can be thought of as the pixel's magnitude. So, to have the full description of an image, one needs to know the grayscale value for each pixel inside of it. If one were to move from pixel to pixel in an image, one would most likely encounter variation in magnitudes. Similar to tracking the object's vertical motion around the unit circle, one is able to track the variations between each pixel in an image.

To more clearly illustrate this point, one can move from a two-dimensional image to a one-dimensional image. Included below is a row of four pixels with the possibility of four different magnitudes.

Row of four pixels:

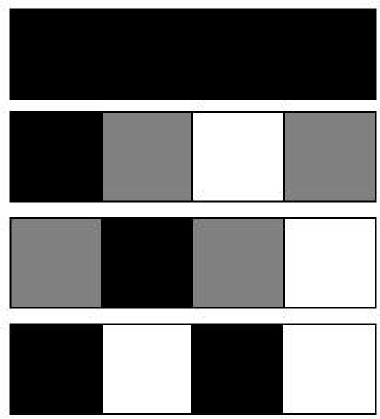
| | | | |
|---|---|---|---|
| a | b | c | d |

One needs to construct a set of "basis vectors." These basis vectors are similar to the five different sine functions that came together to form the square waveform. It is important that each vector is "linearly independent" of one another. Linear independence assures us that none of the basis vectors are replicas of one that has already been constructed. Again, think in dimensions: If one has already defined an x-axis, one would not want to include another x-axis to avoid redundance.

The values of a, b, c, and d can be -1, 0, or 1. To help visualize this information each number will correspond to a gray value, with -1 as white, 0 as gray, and 1 as black.

Basis Vectors:

| | | | | |
|---|---|---|---|---|
| $v_1$ | 1 | 1 | 1 | 1 |
| $v_2$ | 1 | 0 | -1 | 0 |
| $v_3$ | 0 | 1 | 0 | -1 |
| $v_4$ | 1 | -1 | 1 | -1 |



Remember, the basis vectors can be thought of as the individual components that already exist within the row of four pixels. Through the grayscale representation of the basis vectors one can see that one is now tracking the variation in magnitude across the row. One is still trying to see the extent to which each basis vector contributes the original form. Now one is able to find the coefficients that show how much each basis vector appears in the row of four pixels.

| | a ↓ | b ↓ | c ↓ | d ↓ |
|---|---|---|---|---|
| $v_1$ | 1 | 1 | 1 | 1 |
| $v_2$ | 1 | 0 | -1 | 0 |
| $v_3$ | 0 | 1 | 0 | -1 |
| $v_4$ | 1 | -1 | 1 | -1 |

$\alpha v_1 + \beta v_2 + \gamma v_3 + \delta v_4 =$

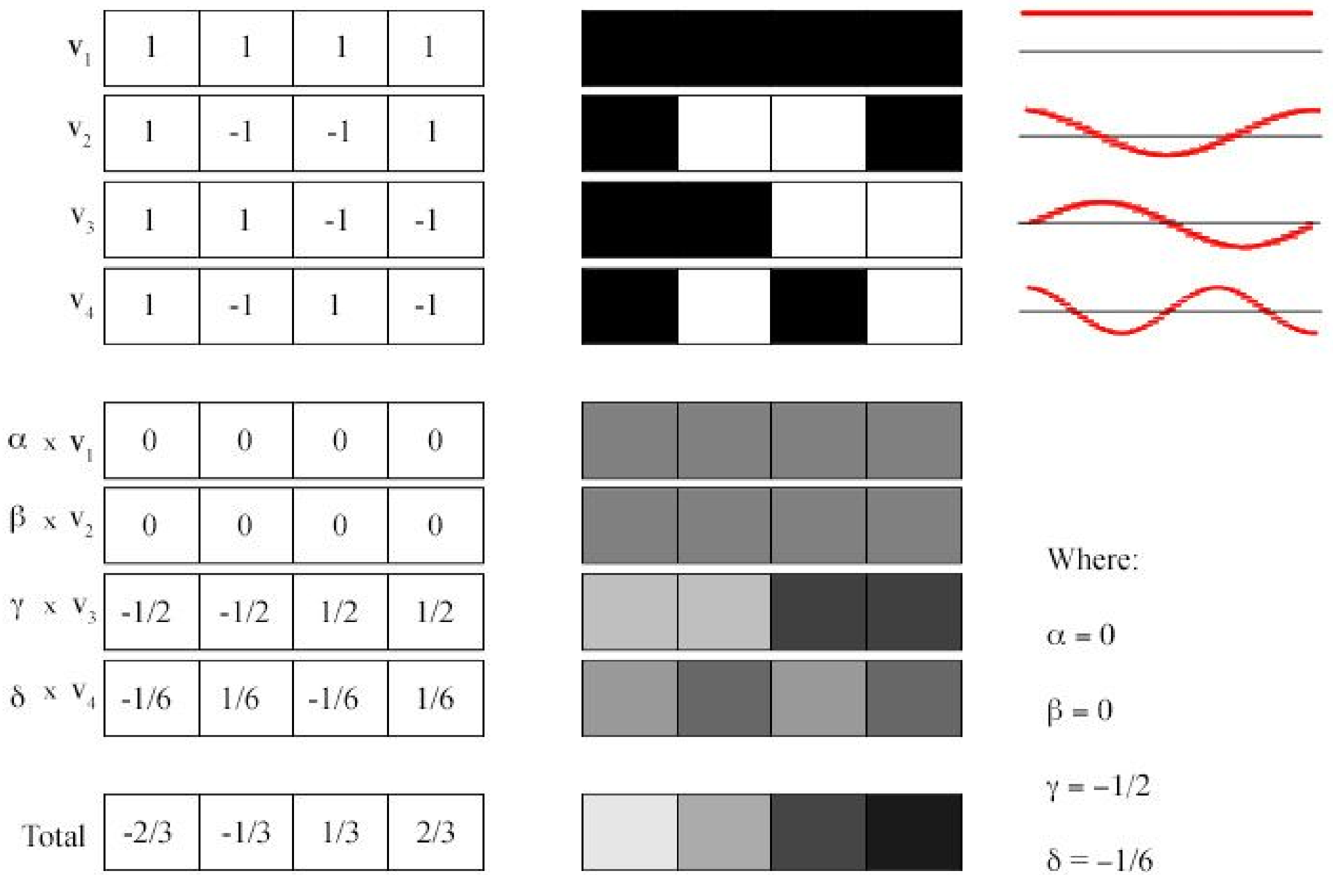
| | | | |
|---|---|---|---|
| a | b | c | d |

So, $\alpha + \beta + \delta = a$

$\alpha + \gamma - \delta = b$

$\alpha - \beta + \delta = c$

$\alpha - \gamma - \delta = d$




Finding the solutions for each of these different variables supplies one with a new understanding of the different variations being tracked or isolated. The first solution, which also might be the most obvious, is $\alpha = \frac{(a+b+c+d)}{4}$ . So $\alpha$ is nothing more than the four pixels averaged together. The solutions to $\beta$ and $\gamma$ are very similar, with $\beta = \frac{(a-c)}{2}$ and $\gamma = \frac{(b-d)}{2}$. These two variables are sensitive to two-cell asymmetries. Finally, one will find that $\delta = \frac{(a-b+c-d)}{4}$ . As the visual representation of $v_4$ may suggest, $\delta$ is sensitive to 1-cell asymmetries across the whole row. To help understand this concept even more, one can look at an example where both the numerical and visual values of a, b, c, and d correspond to their basis vectors and coefficients.

This example implements a different set of basis vectors, but the idea remains the same. It is important to note that $v_2$ and $v_3$ are now sensitive to different variations across the row of four pixels. Now $v_2$ is sensitive to two cell variations between the outer edges and middle section and $v_3$ is comparing the left section to the right. This new set of basis vectors, which is one combination out of many, is slightly more intuitive than the set in the previous example. These basis vectors are directly related to the sinusoidal waveforms of different frequencies and magnitudes that make up something like a square wave. The waveform equivalents for these spatial basis vectors are included in red to help understand the parallels between the two.

There are multiple ways to understand this information, but perhaps the simplest way is to imagine that one starts with finite values for a, b, c, and d which can be found in the "total" section above. After this point it is necessary to find a set of basis vectors to provide the means necessary to fully describe the magnitudes of the pixels. After the basis vectors are found, the next step is to solve for the coefficients for each of the basis vectors, that is, "how much" of each of these basis vectors is needed in order to add up to the final row of pixels. Pairing the coefficients with their corresponding basis vector reveals the proportion of that unique type of variation present in the original four pixels. In this example, it becomes clear that a gradient from light gray to dark gray holds variations from left to right and from pixel to pixel.

This type of analysis can be expanded to a set of any finite pixels, regardless of how many rows or columns it may hold. For some context, a picture taken with a typical eight megapixel camera will yield an image with approximately eight million pixels. Fourier analysis allows incredibly difficult data, like those eight million pixels, to be redefined and decomposed into a more manageable form. Included below are photos with their corresponding Fourier Transformed image.

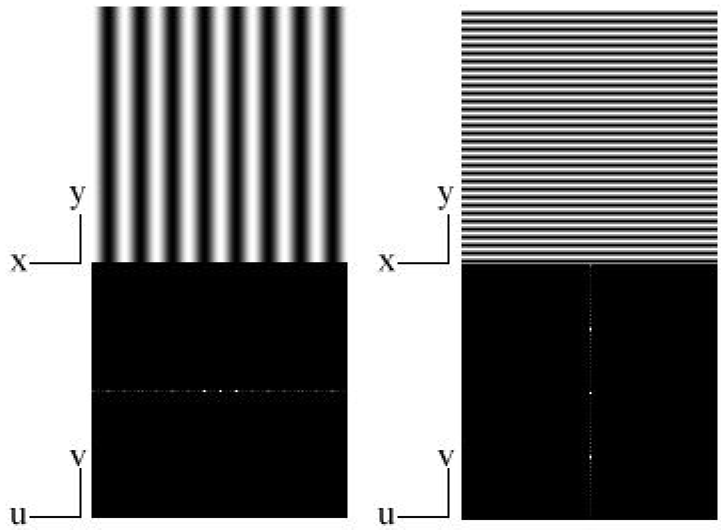



Image created by John Brayer

Starting with this image will hopefully allow one to more adequately understand what the Fourier Transform is analyzing in the original image. As I have been saying, Fourier Analysis in regards to images is tracking the variations between pixels. The previous example with the row of four pixels only had a horizontal component. But, with the two images above, it is clear that there is now a vertical and horizontal component. These image's transforms appear directly below them and illustrate the u-axis and v-axis.

These images are transformed from their original "spatial domain" to their new "frequency domain," which is why one needs to switch from x and y to u and v. The u-axis measures the variation along the horizontal direction and the v-axis measures variation along the vertical direction. From the first image one is able to see that the clear variation across the horizontal axis translates to a highlighted u-axis. The second image shows how a larger amount of variation, or a higher frequency, across the vertical axis leads to brighter regions that move away from the origin. One may notice that the bright dots in the left image are closer together than the transform of the second image. In the right image this difference is present because there are more variations between black and white that take place in the same amount of space as the image on the left. More variations between black and white translates to a larger frequency overall.

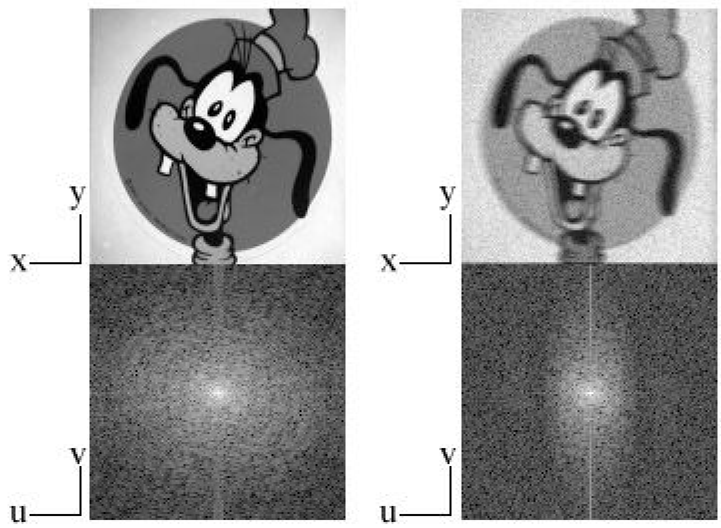



Image created by John Brayer

These two images show the Fourier Transforms of more complicated and more realistic images that one may encounter. The transform on the right shows one that the image is relatively similar with a strong brightness at the origin, or in other words, the majority of the pixels within the image are not too far off from the average value. Outside of the bright region in the center, the main brightness one will see is along the v-axis. This means that the strongest variation in the image is along the vertical axis from top to bottom. The second image is the same as the first with a blur added to it. This blur was only applied in the horizontal direction which, admittedly, I only knew because the creator of this image said so. But, upon further inspection, one can see in the transform below it that this greatly reduces the range of frequencies across the u-axis. This helps confirm that a Fourier Transform highlights aspects of spatial variation in an image that would be difficult to see or analyze without it. This reduction in the range of frequencies makes sense because high frequencies in an image are related to more distinct variations, like the edges that have been blurred. The low frequencies are left relatively unaffected because they correspond to the overall brightness or intensity of the image as a whole. In the next section, one will find that using Fourier Transforms for visual data facilitates the implementation of image processing techniques by effectively eliminating the need for complicated mathematics.

## Fourier Analysis and the Convolution Theorem

Convolution, in the simplest of terms, is a mathematical operation not unlike the more basic operations such as adding, subtracting, multiplying, or dividing. Similar to its more basic counterparts, convolution takes two inputs and gives one output. But, unlike arithmetic, convolution deals with functions instead of numbers. To illustrate this idea I will use an example of a common application of convolution in which one applies a filter to an already existing image.

Applying a filter to a digital image will result in some change in that image, whether it be something like a blurring or sharpening effect. One can think of this filter as an object on its own or a function that will act on an image. The filter, which I will refer to as a "kernel," will take the form of a matrix. A matrix is really nothing more than a collection of numbers in a form that allows one to easily perform normal arithmetic with other matrices. In this case, one will be multiplying the kernel with a set of numbers that correspond to the intensities of individual pixels of an image.

The way this process has been explained purposefully ignores the more complicated aspects of convolution. To bypass a difficult integral one must implement the convolution theorem. The convolution theorem states that convolution in the spatial domain is equivalent to multiplication in the frequency domain. So, all one must do to avoid convolution is perform a Fourier Transform on both the kernel and the image. This allows one to simply multiply (or matrix multiply) these two terms together. Included below is an example of this process where each matrix has yet to be transformed.

$$\begin{pmatrix} .25 & .5 & .25 & 0 & 0 & 0 \\ 0 & .25 & .5 & .25 & 0 & 0 \\ 0 & 0 & .25 & .5 & .25 & 0 \\ 0 & 0 & 0 & .25 & .5 & .25 \end{pmatrix} * \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} .25a_1 + .5a_2 + .25a_3 \\ .25a_2 + .5a_3 + .25a_4 \\ .25a_3 + .5a_4 + .25a_5 \end{pmatrix}$$

Kernel * Original Pixels = Filtered Image

The convolution theorem allows one to transform both of the matrices on the left, and doing so will make the kernel into a diagonal matrix. Instead of dealing with three terms being multiplied and added together, the terms from the kernel matrix and the original pixels can be multiplied term by term. All one must do after the final result is determined, is transform back to the spatial domain. This change may not seem drastic, but as soon as this method is implemented in computing the amount of time saved is exponential because many fewer multiplications are required. In the end, the convolution theorem allows computers to convolve the blur kernel with an image more efficiently. More efficient convolution in computing allowed scientists and engineers to move from computers that occupied entire rooms to the computers we carry in our pockets.

This kernel in particular is causing the original image to be blurred. The final matrix is a column of new values which have been calculated using one pixel for each new value and its two closest neighbors. Instead of each pixel maintaining its original intensity it has been averaged to some extent with the intensities of the pixels surrounding it. Imagine dots of wet paint being smudged into each other effectively obscuring where one ends and the other begins. This specific type of blurring, which is known as a gaussian blur, targets the high frequency components of an image. Looking back to the image of Goofy and its corresponding transform this idea makes more sense. Blurring the image, or smudging each pixel into its neighbors, will reduce the amount of sudden variation in intensity. Reducing the amount of sudden variation in an image is quite obvious to the human eye. To put it quite simply, once a sharp edge has been blurred, it will appear out of focus.

An alternative to blurring, which diminishes high frequencies, is to filter out low frequencies to highlight the high frequencies. This new type of filter, called a high-pass filter, will amplify the parts of an image where variations are the most distinct, which most commonly occurs at the edge of an object within an image. An example of a filter of this nature is a Gabor filter, which is used within the field of image processing to extract the most useful aspects of an image. Focusing on the high contrast parts of an image reveals the unique qualities of the objects present in an image. The Gabor filter can be used to identify recurring and unique attributes in an image. If one were to think back to the first section on human vision, one would probably see the clear similarity between what the Gabor filter does and what a simple cell does in the visual cortex. In fact, the Gabor filter is one of the best models for human vision with the main purpose to identify complex edges and patterns.

## Conclusion

One of the most basic needs of any human is the need to gather information. As I have shown, the eye serves as a gateway into the physical world, working to obtain data from one's surroundings. The eye, and all of the parts that make it up, function as a tool for the brain. This tool has the ability to shape the ways in which we think and interact with our environment. Alongside the eye and other biological tools, humans develop methodologies for thinking that are used to process raw data and turn it into something we can understand. I have shown that Fourier Analysis can be thought of as one of these methods. It can be used not only as an illuminating mathematical model for vision, but also as a gateway into how the mind deciphers complicated information. In this project, the Fourier methodology and the results it can yield became the most prevalent link between my work in the realm of physics and in the realm of art.

Now, with a better handle on the vocabulary pertaining to this subject, I can shed more light on the details of my art installation. In the beginning, my interest in perception stemmed from the moments where human experiences do not perfectly align with what "actually" exists in the physical environment. This initial interest led me to the part of perception where humans are just starting to construct an image of the outside world. This section of human perception is believed to be where edges become distinct and go on to define distinct objects. In this moment, the link between Fourier Analysis and the artwork is rather utilitarian. As I have previously addressed, Fourier Transforms are required to drastically decrease the computing power necessary to implement the Gabor filter. My installation utilizes the Gabor function to do exactly what it was designed to do, that is, highlight the edges found in an image.

Out of this utilitarian integration of the Gabor filter, the more conceptual aspects of my project began to fall into place. But, before I go on to explain the conceptual side of my installation I will describe, in more detail, the technical aspects of the piece. I chose to use the Gabor filter on a set of forty photographs taken of me during my early childhood, which are projected on one wall in the space. The Gabor filter and the images are controlled through computer code which was written in a java based language called Processing. A more in depth conversation on how this code was written can be found after the conclusion of this project. The strength of the filter, that is the amount of distortion on the images, is dictated by the amount of sound in the space. When the room is silent almost all of the details are indiscernible and the images become clearer with more sound. Through two pairs of headphones the audience is given a set of prompts, all of which are different in an attempt to create a somewhat unique experience for each pair of listeners. The prompts are delivered through a recording of my voice and include questions for the listener to answer out loud or directions to make other types of sounds or movements in the space.

Within this installation there are, of course, many layers of personal meaning. The most conscious concept or topic I am trying to address is the ephemerality of my own past and how the memories I have gathered up to this point in my life directly inform who I am. The sounds made by my audience function as brief moments of clarity, moments where I become aware of the most defining parts of my life and their direct influence on my current state. To bring Fourier back into the picture, I have been thinking of my own past as one of those complicated waveforms. As you would probably agree, one's entire life is messy and hard to take in all at once. Those defining moments, whether they are captured in a photograph or put away somewhere in my mind, are the sine or cosine functions that are buried in the complexity of the whole. By taking the time to sift through these moments and analyzing them as their own separate entities, one can more fully understand the result of their combination.

Throughout the process of completing this project, I found that the Fourier methodology is able to exist across fields within both Art and Physics. Like any good process of thinking, this method proved to be extremely adaptable to the problems I was dealing with or the questions I asked myself. On the more macroscopic level of my project, the methods used within Fourier Analysis allowed me to build a conceptual bridge between art and physics. Then, within the written part of my project, Fourier transforms became a necessary component in the development of a model, or conceptual analogy for understanding one part of human vision. From there I was able to solidify this bridge between art and physics by using Fourier Transforms as a technical tool within the Gabor filter to explore image processing in theory and then put that theory into action.

In the beginning of this paper I mentioned the common disbelief people have when I explain my major in both Physics and Art. Throughout my college career I continually assured myself that a joint project could be done, even though I did not exactly know how most of the time. Even my advisors showed (understandable) doubt that these two fields could really come together. But, for me, the turning point came when I had the realization that I do not have to make an object that employs principles of Physics, nor do I have to make really artistic graphs for data I found in a lab. Instead, I found the most important thing for me to do is change my frame of mind from applying one field to the other to instead finding a true synthesis.

**Help from Friends and the Internet**

When I decided exactly what I wanted to do for my installation I knew I would need to use a computer. I'm relatively well versed in the digital image creation side of computers, but for this project I needed actual coding skills. Upon learning this I approached my long-time friend, Diana Ruggiero, who happens to be a computer science major. She knew almost immediately I would need to use Processing, which was actually created to help artists ease into the world of coding. Diana was gracious enough to show me all of the cool things Processing had to offer. Being fluent in this language, Diana was able to help me implement my artistic ideas on a level I would have never been able to achieve with the amount of time I had this year. Over the course of about five meetings together, Diana and I were able to complete the code to where it was exactly what I imagined it to be in my head.

Alongside Diana's help, the internet became a source for key parts of the code we wrote. The Gabor Filter itself was published by Patrick Fuller on his blog where he guided his reader through his process and encouraged experimentation. This code was taken and adapted by Diana and myself to fit the needs of this specific project. Other incorporations from the internet include bits and pieces from Processing's very own forum where users supply advice to each other to help complete their individual projects. Both Diana and I feel that this way of code directly aligns with the open source ethos of the Processing community. That being said, this section is included to make it clear that the coding aspect of my installation involved more than myself and give others the recognition they deserve.

**Artist Statement**

Understanding is a hard thing to do, but we try to do it anyway.

How is the *now* connected to the *then*?

How is the *then* different than the *was*?

How is the *was* showing up in the *now*?

Understanding is a hard thing to do, but we try to do it anyway.

I've always tried to live in the present, but it's hard to separate the present from where I've been and where I'm going. I still don't fully understand my art or art in general, but I have come to understand that my work, when it is at its best, is personal. I can marvel at the work of artists that access the most abstract, formal ideas in art, but I have yet to produce that kind of work myself. To make something worthwhile I have decided I need to look into rather than around myself. The idea that the things I am trying to convey are recognizable and accessible is what is important to me. I am a person. You are a person. The feelings, thoughts, and ideas I have are probably close to your own, but if they're not then that's even more exciting.

These ideas, like the ones I'm trying to write right now, are huge and complicated and hard to understand. Even our daily lives prove to be more than enough food for our minds to devour year after year. If I stick with the food analogy, some of the food we consume doesn't digest very well and it remains stuck deep within the gut of our mind. These thoughts and feelings, the ones that have come and passed, can be stirred up again at any time. You hug a friend, but it feels like you're hugging your grandma because "wow, they smell exactly the same." Or maybe you walk outside and the sky looks just like it did when you had nothing to do but lay in the grass and watch the clouds. Memories like these are just the remnants of things once touched, tasted, smelled, seen, or heard. These are the moments that make us who we are, but the details of our lives can get lost in heads too busy to slow down. Sometimes we need to go back and rummage through the memories that got left behind, and with this piece I invite you to do just that.

**Invitation**

ANALOGY: A DECOMPOSITION OF SPACE AND TIME
A SENIOR PROJECT EXHIBITION
BY DAVID SHOEMAKER

OPENING RECEPTION: 5/7/2016 2:00 - 5:00PM
UBS EXHIBITION CENTER
RED HOOK, NY 12571
ON VIEW 5/7 - 5/21/2016

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