

Spring 2016

Persistent Asset Mispricing: Variations on a Simultaneous Move Game

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Recommended Citation

Brara, Arjun Kavi, "Persistent Asset Mispricing: Variations on a Simultaneous Move Game" (2016). *Senior Projects Spring 2016*. 214.

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Persistent Asset Mispricing: Variations on a Simultaneous Move Game

A Senior Project to be submitted to
The Division of Social Sciences
Of Bard College

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Annandale-On-Hudson, New York

May 2016

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Abstract

The purpose of this paper will be to demonstrate instances of persistent asset mispricing. By persistent we mean the existence and achievement of a stable equilibrium where market prices deviate from an underlying fundamental value. These equilibria will be demonstrated through three variations on a general model in which agents make investment decisions on an initial public offering (IPO), given imperfect information on a stock's true, fundamental value. The first variation holds agents' beliefs fixed, and trades are conducted "blindly" through a market maker who clears the market by adjusting the stock price. The model establishes a case for initial mispricing in the market, resulting from a sub-optimal quantity of outstanding shares. The second variation allows for agents to update beliefs on price movements, and ultimately identifies an unstable equilibrium where the asset is correctly priced. The third and final variation removes the "blind" game conditions, and allows for agents to update beliefs on public signals generated by other agents' investment decisions. This latter most model successfully demonstrates persistent asset mispricing, given an initial mispricing in the market.

[JEL A10, C62, C68, C73, G12]

Chapter I: Introduction

Introduction To The Model

This paper aims to demonstrate the existence, and achievement of, stable equilibria wherein a financial asset is mispriced¹ in the market. A focus will be placed on demonstrating instances of overvaluations. Specifically, we model an initial public offering (IPO) of stock, and introduce a non-profit driven market maker through which a finite set of agents make trades. Three variations on this basic model are presented:

¹ The market price does not accurately reflect some true value determined by the underlying fundamentals.

1. Agents' have heterogeneous beliefs on the fundamental value of the stock, which are assumed fixed for the duration of the game. Agents make trades through the market maker in a random, sequential order to buy or sell a single unit of stock. Agents are allowed to short sell. These trades are not public information, but private, and known only to the agent in question and the market maker. Thus, the game is essentially simultaneous in nature. The market maker adjusts the price after all agents have made a single trade, given the level of excess demand or supply. These price revisions are imperfect, and the quantity of stock is fixed. This is the Base-Case Model (BCM).
2. The assumptions of the BCM hold with the exception that agents' beliefs on the fundamental value are now updated in instances where price movements contradict initial, or prior, beliefs. The updating of beliefs occurs *post* price movements after a single round² is completed. This is the Price-Case Model (PCM).
3. The assumptions of the BCM hold with the exception that beliefs on the fundamental value are now updated on public signals³; trades are no longer private, but public, information. As in the PCM, beliefs are updated in instances where public signals contradict a private prior belief. This updating takes place within the round *pre* price adjustments. This game loses the simultaneous nature of the BCM and PCM. This is the Action-Case Model (ACM)

The BCM model serves to develop a theoretical foundation for an initial mispricing of a stock unit, given fixed beliefs. The PCM and ACM focus on demonstrating stable equilibria where the stock is mispriced, given an initial mispricing in the market. The assumptions behind each model will be rigorously developed, and the equilibrium outcomes mathematically modeled and, for the ACM, simulated. We now progress to a brief

² A round is completed once every agent has made a single trade through the market maker.

³ Public signals are those informations on the private signals of other agents conveyed through a history of actions.

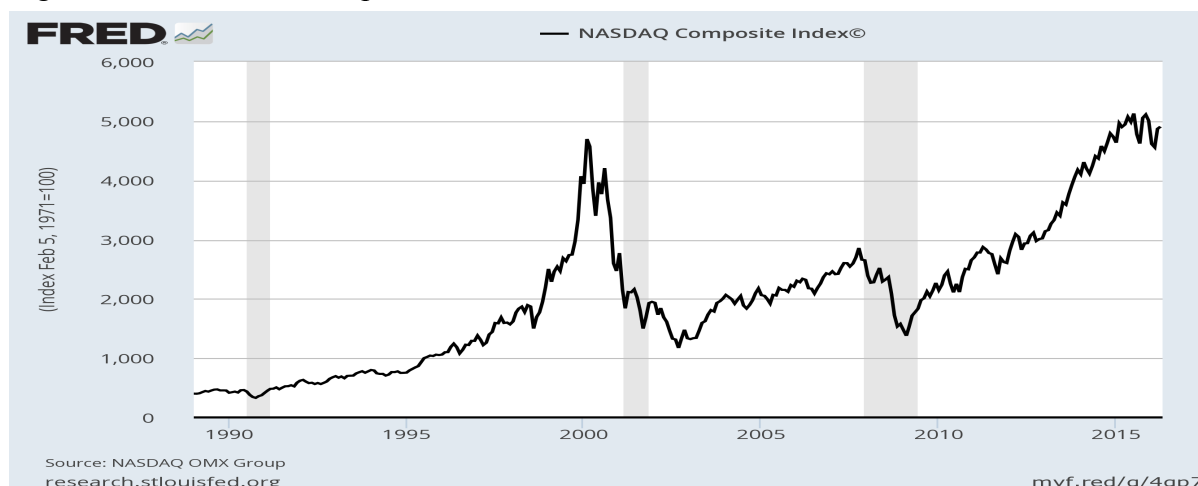
introduction on the background of asset mispricing. Note that the majority of literatures address a specific type of mispricing: asset bubbles. Thus, while the background and following literature reviews narrow in on explaining asset “booms and crashes”, the reader should maintain the broader perspective of considering mispricing generally. With this in mind, we continue onwards.

A Background On Asset Mispricing

Chamley (2004) nicely summarizes asset bubbles as a state where “an asset that yields no real dividend has a positive price”. The definition is specific to single asset with a zero fundamental value. The definition for mispricing can be generalized however to a state where the market price of an asset falls out of line with its true, or underlying, value as determined by the fundamentals. The classical argument against the existence of bubbles with rational agents, Chamley also describes well from a purely game theoretic perspective: that is that any price unsustainable at some time t^* is, by backward induction, unsustainable at any and all times $t < t^*$. The basis of neoclassical economics similarly rejects market mispricing; the efficient market hypothesis assumes prices to reflect all relevant information and accordingly all assets to be priced fairly – at their fundamental value in the long run. Yet the existence of extended mispricing, and market bubbles particularly, is unquestionable. The recent housing bubble of the 2000s is a prime example. Skidelsky (2009) cites U.S. housing prices having risen 124% between 1997 and 2006, later plummeting by 33% in 2007. The Dot-com bubble of the late 1990’s is another famous example, and is shown below by the NASDAQ Composite Index charted from 1989 to 2016.

(See Figure 1.1 on the following page).

Figure 1.1: NASDAQ Composite Index, Dot-com Bubble



The majority of models explaining mispricing resort to psychological explanations for irrational behavior, or alternatively to bounded rational agents⁴. We begin the literature review by briefing a number of texts that, while offering distinct interpretations on the causal mechanisms behind mispricing, do so largely without mathematical models. These will serve as a general introduction to the core literature behind this paper. To reiterate, these texts focus largely on asset bubbles rather than asset mispricing generally. However, given the game structures developed in this paper and the emphasis placed on demonstrating over valuations, the chosen literature is acceptable.

The General Literature

Papadimitriou and Wray (2010) explore a multitude of circumstances capable of generating market bubbles with rational agents. An initial model explains market bubbles as a result of limited efficiency of market arbitrage. An initial short-run undervaluation of an asset is assumed, leading agents to rush into purchase orders and consequently appreciating the market price above some fundamental value. Liquidity constraints on short selling however, result in medium-term continued asset overvaluations. In the long

⁴ Rational agents bounded by imperfect information.

run, these constraints are overcome, agents sell, and the market price falls to realign with the fundamental value. Papadimitriou and Wray term this a *rational bubble*.

The text also covers the Minskyian explanation of market bubbles that, while similarly stressing the importance of structural inefficiencies, also emphasizes the importance of the bounded rational characteristics of economic agents. Minsky's model begins with agents systematically underestimating risk⁵ and expecting an appreciation of market prices. As in the model of Papadimitriou and Wray (2010), agents consequently rush into assets, the market prices of which accordingly appreciate above associated production-cost prices – a proxy for fundamental values. These optimistic market conditions generate increased competition between firms, and corresponding capital expansion results in greater credit exposure. Minsky suggests that once industrial expansion exhausts, declining output induces illiquidity, which in turn creates insolvency. Assets are sold off and pessimistic markets ensue, characterized by agents exiting their market positions and falling market prices. While Minsky's perspective is larger in scale than a simple stock bubble analysis, the structural layout is still applicable to the coevolution of a single company's capital structure and stock price history during a market boom.

Papadimitriou and Wray also consider how an open economy model might explain market bubbles. The chief example presented focuses on international money flows; a scenario is presented where a weakening in the economy of one Country A prompts capital flight to another Country B, wherein asset prices appreciate without any justifiable change in asset fundamentals. The case of the U.S. dollar during the onset of the 2007-8 financial crisis is a prime example. Note that the scenario presented here can also be

⁵ The underestimation of risk can be thought to result from either imperfect information or some combination of rational calculation and sentiment.

reduced to understanding a single stock price bubble, if we liken the two countries described by Papadimitriou and Wray to two competing companies within an industry⁶.

Earl et al. (2007) propose an alternative explanation for market bubbles. Contrary to Minsky's emphasis on structural inefficiencies with particular regard to imperfect information, Earl et al. argue that the availability of information today allows for markets to be approximated as perfectly efficient. Instead, market bubbles are accounted for by loosely categorizing agents into a spectrum with well informed and poorly informed investor extremes, and the introduction of *decision-rule cascades*. As described by Earl et al., decision-rule cascades occur when complex investment strategies created by well informed investors experience an *entropic degradation* as they are traded socially, largely due to opportunism tacit knowledge. These reductions in complexity correspondingly result in lower investor uncertainty. Earl et al. stress that the time lags present in decision-rule cascades mean rules-of-thumb strategies adopted by investors are "*less fit to the changing environment than when they were first employed*" (Earl et al. 2007). When a market inevitably runs out of steam, they propose that experienced professionals can no longer beat the market, and that these rules of thumb are abandoned ensuing a market crash. Similarly to the Minskyian model then, a decision-rule cascade theory is reliant on a leveling out of production capacity.

Earl et al. (2007) briefly propose another explanation for market bubbles towards the end of their paper. Taking a more behavioral approach, they describe an instance where some psychological threshold is met, as nervous investors recognize that in order to fully realize capital gains, they must be among the first to exit the market. However, we should question where this threshold comes from. When and why is it met? How is it overcome?

⁶ A critique to this parallel is that a fall in one of two competing company's performance may lead to an expected improvement in the fundamentals of the other company as a result of lower competition.

Tuckett and Taffler help resolve these lingering questions, lending further insight into a purely psychological explanation for bubbles.

Tuckett and Taffler (2008) begin by making the argument that all expectations are inherently uncertain and that there is much room for sentiment in the market. In this regard, their perspective is closely aligned with the Keynesian conception of animal spirits. The paper likens innovations, specifically financial assets, to *phantastic objects* generative of excitement and a feeling of omnipotence among investors. The innovation makes earlier rules, whether theoretical or regulatory in nature, seem breakable. The hype, or *irrational exuberance* as coined by Schiller (2000), boosts demand for the new asset class whose expected appreciation is overestimated due to both the adoption of vague or new age valuation techniques and wishful thinking. Increases in market prices only serve to justify earlier expectations, which are self-fulfilling, and according to Schiller, a “*naturally occurring Ponzi process*” (Schiller 2000). As market prices deviate further from their “true value”, Tuckett and Taffler suggest that an anxiety builds within the investor. They further propose that the unconscious rids of this anxiety by “splitting off from awareness”, allowing for the agent to continue living in the phantasy. Ultimately, the paper acknowledges that “*bubbles form when reality-oriented thought, including the capacity to be anxious about potential losses in risky situations, is overridden*” (Tuckett 2008).

Their paper also addresses why professional investors, thought to act rationally, behave similarly with respect to their investment decisions. Tuckett and Taffler suggest that institutional pressures such as reputation and compensation-based herding play a part. So do widespread notions that “markets do not lie” and that positions in the market can be ideally timed to maximize capital gains. Their explanations for professional investor

behavior fits neatly with Shleifer and Vishny (1997) who propose that betting against sentimental investors can be both risky and costly.

Baker and Wurgler (2007) lay out a similar reasoning for asset mispricing. Namely, they point to changes in sentiment among irrational investors and limitations to arbitrage among rational investors, the latter of which had been seen in the models of Papadimitriou and Wray (2010). Their contributions worth noting are two-fold. Firstly, Baker and Wurgler suggest that harder to value stocks will be more subject to shifts in sentiment – consistent with the works of Tuckett and Taffler (2008). Specifically, they point to small-cap stocks, lower in trading volume and with less publically available information, as prime candidates for price bubbles. They back this hypothesis with empirical data. Secondly, the paper develops a *sentiment seesaw* model illustrating the point that in times of high sentiment assets that are more speculative in nature generally see valuations that exceed some fundamental value, while less speculative assets see valuations below said fundamental value. The reverse is true during times of low sentiment. Baker and Wurgler similarly support this model with empirical evidence. Importantly, these findings generally support the rationale behind the supposed correlation between high sentiment and high demand.

This concludes our introduction to the general literature on mispricing. With this background, we progress into a more rigorous discussion of the core contributing texts to this paper and the general model it develops.

The Core Literature

We begin with Chamley (2004), who in his last chapter on financial frenzies develops a rather simple model explaining booms and crashes. The model introduces an asset on which some initial good news is received. The asset's market price and

fundamental value grow at the same rate until some time θ , after which the market price growth rate exceeds that of the fundamental value. Time θ is treated as a random, Gaussian (normally distributed) variable. For all times $t > \theta$ agents receive private signals⁷ on the value of θ . The variance of prior beliefs on θ allow for the market price to increase above the fundamental value. At some time $\theta + \sigma$ all agents become aware of θ , however a lack of common knowledge sets the stage for a continued disparity between the market price and fundamental value, as agents believe they can ride the bubble a little longer. The model offers two explanations for a crash. The first is an exogenously set multiple of the fundamental value which the market price can not exceed; if the market price rises above the multiple there is a crash and the price falls to the fundamental value with a probability one. The second is endogenous, where some critical number of sales is reached as agents' expected capital gains fall below expected losses. Again, the result is a collapse in market prices to the fundamental value.

Chamley introduces the model of Caplin and Leahy (1993). As described by Chamley, the model introduces a finite (or countably infinite) set of agents, each of whom receive a Gaussian private signal on the state of nature that is a binary good or bad, with regards to some investment. Agents convey their private signals through their actions that take on a similarly binary one-zero value, depending on whether the agent does or does not invest, respectively. The history of actions is known to all agents but imperfectly represents the aggregate belief with some degree of noise⁸, also represented by a random Gaussian variable. Agents base investment decisions on the relative likelihoods of each state of nature given private signals and public signals. Caplin and Leahy estimate the aggregate investment level with a manipulation of the cumulative distribution of private

⁷ These are identical signals that are not public information.

⁸ The noise results from the introduction of noise traders, necessary to prevent a no-trade result, briefed in greater detail shortly (see CARA-Gauss Model, pg.7).

beliefs on the state of nature. We borrow this method with some alteration to derive the demand and supply curves in our model.

Caplin and Leahy describe a scenario where the true state of nature is bad, but optimism allows for agents to perceive the likelihood of a good state as higher than that of a bad state. Gradual social learning allows for agents' beliefs to converge on the true state of nature. A crash occurs when the aggregate of public signals overcome the noise, and the shift in public beliefs evolves quickly due to the “*nonlinearity information content of individual actions*” (Chamley 2004). The model nicely captures the gradual boom and sudden crash typical of market bubbles.

Chamley also introduces the CARA-Gauss setting, which as in the model of Caplin and Leahy (1993) has random Gaussian variables, but further specifies agents to have a constant absolute risk aversion (CARA). Chamley builds several variations on a standard CARA-Gauss model that features a large number of agents each with a net demand and an auctioneer clearing the market with an equilibrium price⁹. Agents are categorized into three groups, similar to what we have seen in other models: informed agents with private signals, uninformed agents relying solely on public signals, and noise traders that buy or sell randomly for a private (non-profit) motives. Note that the introduction of noise traders in the CARA-Gauss model serves a similar function as the introduction a random Gaussian noise variable in model of Caplin and Leahy; noise traders serve to prevent a no-trade result, a characteristic outcome of zero-sum games with perfect information, assuming all agents completely rational. Chamley focuses largely on evaluating rates of social learning under variations on this basic structure, specifically for varying elasticities

⁹ “In a CARA-Gauss setting, the demand for the asset is similar to the demand for investment in a quadratic model, with the important difference that the cost is endogenous to information revealed by others” (Chamley, 330).

of demand¹⁰ and market vs. limit orders. His detailed discussions on rates of social learning have been sacrificed here for a fuller discussion on the structures of the models presented here, key to the development of our the model presented in this paper.

The model of Genotte and Leland (1990) is then presented. Genotte and Leland build from the CARA-Gauss model, giving all informed agents identical private signals on the fundamental value of the asset but incorporating agents that rely on alternative price-based strategies¹¹ lending a net upwards-sloping demand curve. The model demonstrates that a net demand curve with a positive slope is conducive to generating multiple market equilibria with a similarly positively sloped supply curve; demand and supply curves are non-linear. Genotte and Leland argue that a shift in the fundamentals of the asset, and therefore a shift in the net demand curve, can eliminate an existing equilibrium and thereby cause large, sudden price fluctuations: crashes and booms.

Chamley (2004) develops another model that – similarly to the model of Genotte and Leland (1990) – explains asset bubbles through multiple equilibria of the demand and supply curves. Chamley establishes two sets of agents, informed and noise agents. An asset pricing equation is presented that is a linear function of the mass of agents, a random Gaussian variable whose variance is known. Agents estimate the mass of agents with the known equilibrium price. Estimates are distorted however by the relative number of noise to informed agents. The smaller the share of noise agents, the lower the variance on the mass of agents, which if significant, results in a higher demand and market price. A higher share of noise agents increase the variance on the mass of agents and an uncertainty of the demand keeps prices low. These are the two equilibria established by the model.

¹⁰ Chamley demonstrates that, *ceteris paribus*, the net demand and supply curves for some asset are downwards and upwards sloping respectively (Chamley (2004), pp. 342, 352).

¹¹ Incorporating the complexity of portfolio insurance strategies that recommend selling or purchasing assets when prices fall or rise, respectively (Chamley (2004), pp. 350).

We now brief the models of Glosten and Milgrom (1985) and Avery and Zemsky (1998), as told by Chamley, the latter of which is variation on the former incorporating herding effects and generative of an asset “boom and crash”. The model of Glosten and Milgrom begins with a financial asset whose value is to be realized in a later period after all agents have made their respective investment decisions. A risk-neutral broker is introduced in a competitive setting so that expected profit from the sale and purchase of the financial assets from agents is zero. Similar to other models discussed thus far, agents are categorized as informed or noise agents. Informed agents base their decisions on distinct private signals and noise agents purchase, hold or sell the asset with equal probabilities. In each period a single agent meets with the broker and buys or sells a single unit of the asset. The model demonstrates that the bid and ask prices set by the market maker under a zero-profit condition leads to the convergence of the public beliefs on the true value of the state of nature in the limit. It is also shown that noise agents do not significantly slow the rate of social learning. The structure behind our Base-Case Model is largely taken from that of Glosten and Milgrom.

Avery and Zemsky adapt the model of Glosten and Milgrom with a specific structure of private signals, which they denote *nonmonotone*. Briefly, the nonmonotone structure assumes there to be a normal state of nature $\theta = \frac{1}{2}$ and two possible alternative states $\theta = \{0,1\}$ that have experienced a negative or positive shock, respectively. Agents are categorized as highly informed, poorly informed, or noise traders. All informed agents know exactly when a shock has occurred, receiving some private signal. The precision of the signal with respect to the *type* of shock is imperfect however, and lower for poorly informed agents. The model further specifies a low proportion of highly informed agents and a market maker who not only assumes a high proportion of highly informed investors, but whose expectations of a shock are incredibly low. An exogenous sequence of agents is

set with several buy orders taking place within the first few periods by informed agents whose private signals suggest a positive shock, or by noise agents or some combination. These initial purchases cause poorly informed agents to raise their expectations of a positive shock, and consequently make purchase orders.

Given the market maker's low likelihood on a shock, (s)he does not update beliefs on the state of nature. After some large number of periods however, the beliefs of the market maker quickly adjust to account for number of purchases that contradict a prior belief on the normal state. Because the true proportion of informed agents is small, the increased price significantly reduces the number of trades made. This low trade volume again contradicts the market maker's assumption of a positive shock that would otherwise have justified the increased price. Having realized the error, the market maker drops the price significantly. Chamley credits this model for its ability to aptly mimic the sudden price movements in the market. However, he points to the unrealistic assumptions about the sequence of agents and the probabilities on different states of nature as a significant limitation to the models applicability and practicality.

Bikhchandani and Sharma (2000) similarly focus on a behavioral understanding of market bubbles, emphasizing the role of herd behavior amongst bounded-rational agents. As in many of the models presented by Chamley (2004), agents face an investment decision whose payoff is defined by the state of nature that can take on a binary one-zero value. Agents receive distinct private signals on the state of nature. Bikhchandani and Sharma present several reasons an agent may to abandon their own private signal, following public signals instead: the agent is aware that other players might know information that (s)he is not aware of¹²; the agent may have an "intrinsic preference for conformity" (*Bikhchandani 2000*); should the agent be an asset manager, there may be

¹² If we consider Keynes's notion that all information is held with uncertainty, this explanation is all the more influential.

compensational benefits for following the herd. This latter most argument aligns with the works of Tuckett and Taffler (2008), and is further supported by Maug and Naik's discussions, focusing on compensational structures where an asset manager's performance is measured relative to a benchmark (*Maug and Naik, 1996*). Such structures are often put in place when moral hazards and adverse selection are risks. Scharfstein and Stein's theories on reputation-based herding further bolster the point (*Scharfstein and Stein, 1990*).

Bikhchandani and Sharma model herding using symmetric public signals: all public signals are treated with equal weight *regardless of when they occur*. The model also specifies a binary action set $\{Buy, Not Buy\}$. The game begins with Player 1 acting on his or her private signal that may or may not reflect the true state of nature. Following the Player 1 is Player 2, making an investment decision based on a private signal and the public signal generated by Player 1. Should the public signal support Player 2's private information, Player 2 will follow the private signal, and so will a following Player 3 *regardless* of his or her private information. Should the public signal contradict Player 2's private information, Player 2 is rendered indifferent. Depending on whether Player 2 opts to buy or not buy, Player 3 will herd or follow a private signal, respectively. Note that if all agents abandon private for public signals, such that public signals no longer generate new information, an information cascade is said to have occurred.

The model comes to several conclusions about a bubble's nature. Chiefly, that bubbles are path dependent and idiosyncratic; the initial few players set the majority trend. According to Bikhchandani and Sharma, even if the public signal is "noisy"¹³ there remains a high probability of a cascade developing amongst the first few players. Accordingly, Bikhchandani and Sharma note that information cascades may lead to an

¹³ The history of actions is not clearly dominated by one signal over another.

incorrect majority investment decision. They also remind the reader that because public signals need only be slightly more informative in order for agents to abandon private signals, cascades are sensitive to shocks. Intuitively then, bubbles are fragile, and can collapse quickly. Importantly, the authors suggest that the conclusions of the model are in fact robust to relaxing the assumptions of symmetric signals. Our Action-Case Model will borrow the use of symmetric signals for its simple structure and intuitive explanations for herding.

Lastly, we take a brief moment to touch upon another model introduced by Bikhchandani and Sharma (2000). Their model just described assumed the costs of investment constant. However, this does not fit the cost structure associated with stock investments, where the cost of investment (the price of the asset itself) fluctuates depending on net market orders. The authors make the argument that if markets are perfectly efficient, public signals will lead agents to be indifferent about an investment. Instead, players will rely to private signals, preventing the development of herds and consequentially stock bubbles.

Here we conclude our discussion on the core contributing literature. The following section will serve to a.) restate specific contents borrowed from the core literature, and b.) highlight the distinctive features of our model.

A Comparison of Models

Similar to other models, this paper will deal with bounded rational agents. Agents will be divided into informed and noise agents, the former receiving private signals and the latter making investment decisions randomly. Rather than private signals being informative on a binary state of nature, the signals in this model are informative on the fundamental value of an IPO stock unit, which is exogenously determined. These signals are distinct random

Gaussian variables as in the majority of models, and their joint distributed is assumed to be normally distributed around the fundamental value, should the mass of agents be sufficiently large.

The game structure closely mimics that of Glosten and Milgrom (1985), with agents meeting a risk-neutral market maker and limited to trading a single unit of a stock per round. A key distinction however is that our market maker is concerned only with clearing the market and not with any expected profit function. This will allow for a better focus on the behavior of agents and the firm within the game; an essential condition in the Price and Action-Case Models, where the updating of beliefs on price movements and public signals are explored in depth.

The Gaussian distribution of beliefs around the fundamental value allow for the paper to borrow also from Caplin and Leahy (1994). Specifically, we take a similar approach to deriving aggregate demand functions from the cumulative distribution of beliefs, as did Caplin and Leahy in deriving an aggregate measure of investment level. The exact methodology differs however, and our use of an approximation function for the cumulative distribution not only allows for a simplification of our equilibria expressions, but also for a better representation of private beliefs that are unlikely to extend to negative infinity. The results are accordingly more intuitive.

The Action-Case Model, the second and final variation on the Base-Case Model, takes from Bikhchandani and Sharma (2000). Their use of symmetric signals in nicely demonstrating herd behavior will be used, almost without revision, to explain the updating of beliefs on public signals. The primary revision is the specification that only agents whose initial beliefs contradict public signals engage in the updating process. Within any round then, while information cascades are ruled out, herd behavior is prevalent.

The chief contributions of the model presented here are four-fold: the Base-Case Model's integration of investor and firm behavior in the determination of initial market mispricing; the unique *sequence* of game structures from the Base-Case through Action-Case Models; the endogenously determined sequencing of agents and the robustness of results to changes in these sequences; the simplicity and intuitive nature behind the assumptions and results of each model.

We now advance to Chapter 2, which introduces the model.

Chapter 2: The Model

This chapter will begin by addressing general model. By general model, we mean the assumptions that apply across the Base-Case, Price-Case and Action-Case Models.

Assumptions specific to each model will then be addressed separately in respective sections.

These sections will be brief. Following Chapter 2, the results of the BCM, PCM and ACM will be discussed in the order listed here.

The General Model

1. The Firm and Asset

- i. Firm j launches an IPO. All stock units are homogenous and have the same market price P_t , determined endogenously by a market maker, with the exception of P_0 denoting the starting IPO price that is set by Firm j , but similarly endogenous.
- ii. The fundamental value of each stock unit is denoted F . Let F be the expected value of $\theta_0(F)$, a exogenously set Gaussian probability distribution, representing the total population of information on the fundamental value of a stock unit at time $t = 0$.

Thus the fundamental value F is fixed for all times $t \geq 0$.

- iii. Let $\theta_t = X_t$ for all $t > 0$, where X_t is the joint distribution of agents' prior beliefs¹⁴ in the game.
- iv. Firm j samples θ_t 's' times to estimate F and σ_t^2 , the latter of which denotes the variance of θ_t . Any updated, or posterior, beliefs held by Firm j are the result of changes in θ_t .
- v. The number of outstanding shares, Q_0 , is set by Firm j and fixed for the duration of the game. Specifically, let $Q_0 \propto \frac{1}{p_0}$, given an expected downwards sloping demand function.
- vi. Firm j incurs both fixed costs and variable costs in setting Q_0 ; the total cost is a function of Q_0 .
- vii. The objective of Firm j is to maximize equity gain from the IPO¹⁵.

2. The Agents

- i. There exists some finite set of agents $A = \{1, \dots, N\}$, where N is fixed.
- ii. Let there be two subsets of A , A_I and A_N , that represent informed and noise agents, respectively. Noise agents are assumed a small share of the total mass of agents.
- iii. The fundamental value F is unknown to all agents. Informed agents receive a distinct and imperfect private signal¹⁶ on the value of F at time $t = 0$, call it x_0 . Noise agents receive no such signal.
- iv. Let x_0 be the mean of a distinct sample drawn from θ_0 by the i^{th} agent. Specifically, $x_0 = F + \epsilon_i$, where $\epsilon \sim N(0, \sigma^2)$ denotes an unknown error.

¹⁴ See assumption 'iii' through 'vi' in the following subsection on agent assumptions.

¹⁵ The end discussion of Chapter 3 briefly considers a case where, alternatively, Firm j attempts to price fairly.

¹⁶ This signal is synonymous with a prior belief.

- v. By the central limit theorem, the distribution of sample means – and therefore the joint distribution of prior beliefs denoted X_0 – is approximately normal if the number of informed agents is sufficiently large, and roughly symmetrical around F . Thus, we can say $X_0 \sim N(F, \sigma^2)$, where σ^2 is unknown to all agents.
- vi. Agents may, or may not, update their initial beliefs (x_0, X_0) depending on the specific model. For all models where agents do, posterior beliefs and their joint distribution are denoted x_t and X_t , respectively, where $t > 0$. The distribution of X_t is deterministic of the distribution of θ_t , such that $\theta_t = X_t$.
- vii. Informed agents are homogenous in all respects other than in their prior beliefs x_0 .
- viii. Agents face a single decision in all rounds, which are assumed to continue infinitely: to purchase, hold, sell or short-sell firm i 's stock. Noise agents choose amongst investment alternatives randomly, with equal probabilities.
- ix. All agents are restricted to holding or short-selling a single unit of the stock over the entirety of the game. This is the unit restriction condition (URC), tantamount to imposing homogenous wealth restrictions on all agents, so long as $\frac{P_0}{2} < P_t < 2P_t$ holds and stock units are indivisible.
- x. The URC synonymizes purchasing and holding decisions for all agents holding a stock unit, and the same for selling and short selling decisions for all agents with a short position.
- xi. Informed agents attempt to maximize a common profit function $\pi_i = E_i(F) - P_t$ for the i^{th} agent, where $E_i(F) = x_t$.
- xii. Let π_i be the sole basis for all informed agents' decision-making that is determined by

$$\text{a common decision rule } R_i = \begin{cases} 1, & \pi_i > 0 \\ 0, & \pi_i = 0 \\ -1, & \pi_i < 0 \end{cases}, \text{ where } 1 \text{ and } -1 \text{ reflect the decision to}$$

purchase (hold) or sell (short) respectively, and 0 an indifference between investment

alternatives. This decision rule is not assumed to be common knowledge in the strictest sense¹⁷.

3. The Story and Structure

- i. At time $t = 0$ Firm j launches an IPO at a price P_0 and quantity Q_0 , attempting to maximize equity given the expected demand. The number of agents N is assumed to be known by Firm j .
- ii. Agents make trade sequentially and in a random order through a market maker. Depending on the specific model, these trades may or may not be public knowledge.
- iii. The spread charged by the market maker is allowed to fluctuate, but assumed to have an insignificant effect on the investment decision of any agent as determined by the common decision rule. The practicality of this assumption is questionable, but accepted here for simplifying the model.
- iv. The market maker's sole responsibility is to clear the market by adjusting P_t to P_{t+1} . Price adjustments are made at the end of a round, after all agents have made a trade, given the level of excess demand or supply. To avoid ambiguity, by clearing the market we mean only that the number of stock for sale is equal to the number of stock demanded.
- v. The market maker is not aware of F , σ^2 or any agent's x_t . However, (s)he understands that generally $X_0 \sim N(F, \sigma^2)$. Because σ^2 is unknown to the market maker, price adjustments are imperfect.
- vi. The manner in which agents update beliefs is specific to the model in question.

¹⁷ The assumption that the common utility function is not common knowledge is validated by the existence of noise traders that invest for private motives.

The BCM

1. Agents' trades are private information to the agent and the market maker; investment decisions are not public information. Thus, despite the sequential order of trades, the game is essentially simultaneous.
2. Agents do not update beliefs. Because $X_0 = X_t$ for all $t \geq 0$, θ_t is similarly fixed at θ_0 .
3. Firm j does not update beliefs as θ_t is fixed. Accordingly, $Q_0 = Q_t$ for all $t \geq 0$.

The PCM

1. Agents' trades are similarly private information to the agent and the market maker, as in the BCM.
2. Agents update initial beliefs on price movements. Agents revising beliefs do so simultaneously and at the end of each round, after the market maker adjusts the price. Thus $X_0 \neq X_t$ and $\theta_0 \neq \theta_t$ for all $t > 0$; X_t and θ_t adjust simultaneously at the end of each round.
3. Firm j may or may not update beliefs, as θ_t is variable. However, for modeling purposes and practical considerations, we assume $Q_0 = Q_t$ for all $t \geq 0$, as in the BCM¹⁸.

The ACM

1. Agents' trades are public information in a limited fashion. Specifically, agents are aware of trades made by those agents that precede them in a random sequencing of agents' interactions with the market maker.

¹⁸ Our reasoning for holding shares outstanding fixed can be found in the opening of Chapter 4, which begins the analysis and construction of the PCM.

2. Agents update initial beliefs on public signals generated by other (earlier) agents' investment decisions. Agents revising beliefs do so within the round, before the market maker adjusts the price. Again, $X_0 \neq X_t$ and $\theta_0 \neq \theta_t$ for all $t > 0$. X_t and θ_t are perfectly flexible.
3. As in the PCM, Firm j may or may not update beliefs, as θ_t is variable. For modeling purposes and practical considerations however, we assume $Q_0 = Q_t$ for all $t \geq 0$.

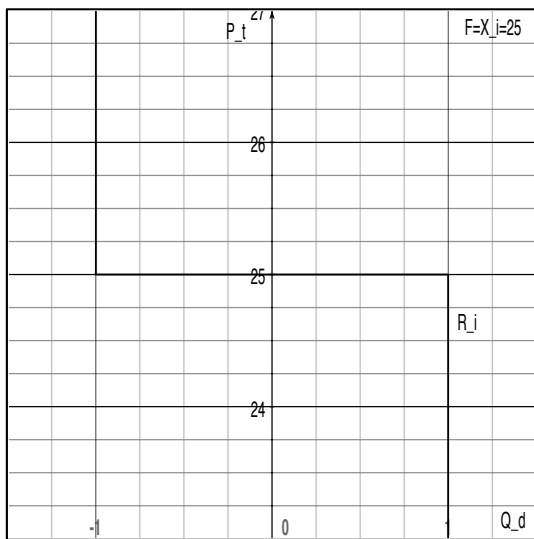
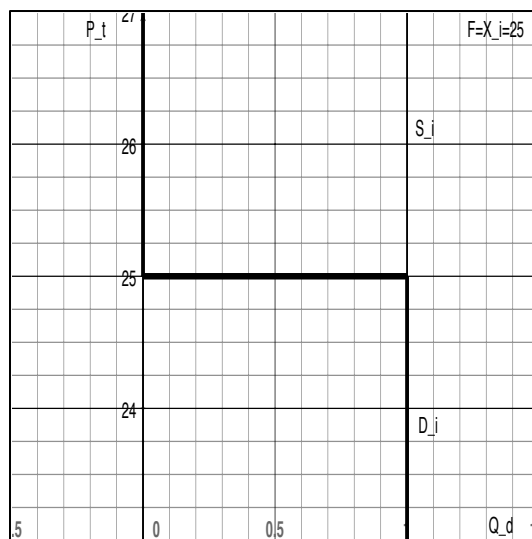
Chapter 3: BCM Results

Construction and Analysis

Note: As the distributions of X_t and θ_t are fixed at X_0 and θ_0 under the BCM, all time subscripts are neglected.

Let us begin the construction of the BCM with a simple analysis of the decisions rule for any one informed agent i with a private signal x_i . We have shown under the assumptions of the BCM that the joint distribution $X \sim N(F, \sigma^2)$. For purposes of modeling, let us take the “average” or expected agent i^* whose $x_{i^*} = F$. It follows that if $P_t < x_{i^*}$ the agent will place a purchase order or hold, and analogously that if $P_t > x_{i^*}$, the agent will place a sell or short-sell order. The decision rule for agent i^* , call it R_{i^*} , is illustrated in Figure 3.1. Note that R_{i^*} can be transformed into the demand and supply functions of agent i^* , D_{i^*} and S_{i^*} , shown in Figure 3.2.

(See Figures 3.1 and 3.2 on the following page).

Figure 3.1: $R_i (X_{i^*} = F = 25)$ Figure 3.2: D_i (Bolded) and $S_i (X_{i^*} = F = 25)$ 

The aggregate demand and supply functions of all agents, D and S , can be easily derived. By the assumptions of the model, informed agents with a $x_i > P_t$ will each demand a single unit of Firm j 's stock. Informed agents with an $x_i < P_t$ will each *be willing*¹⁹ to supply a single stock unit. Because $X \sim N(F, \sigma^2)$, the probabilities of a randomly selected informed agent demanding and or supplying a stock unit can be written as:

$$P(x_i > P_t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_i=P_t}^{x_i=\infty} e^{-\left(\frac{P_t-F}{\sigma}\right)^2} \quad (3.1)$$

$$P(x_i < P_t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_i=-\infty}^{x_i=P_t} e^{-\left(\frac{P_t-F}{\sigma}\right)^2} \quad (3.2)$$

Assuming a small share of noise agents that choose across investment alternatives with equal probability, the equations (3.1) and (3.2) roughly hold for all agents. The aggregate demand and supply functions are then derived by multiplying respective probabilities by the number of agents, N , which is assumed sufficiently large to justify $X \sim N(F, \sigma^2)$.

$$D = \frac{N}{\sigma\sqrt{2\pi}} \int_{x_i=P_t}^{x_i=\infty} e^{-\left(\frac{P_t-F}{\sigma}\right)^2} \quad (3.3)$$

$$S = \frac{N}{\sigma\sqrt{2\pi}} \int_{x_i=-\infty}^{x_i=P_t} e^{-\left(\frac{P_t-F}{\sigma}\right)^2} \quad (3.4)$$

¹⁹ Recall that the aggregate supply is fixed at Q_0 ; for some P_t , $D = S > Q_0$, the market would not clear.

Despite a sufficiently large N , it is unlikely in practice and distortive in modeling for $X = [-\infty, \infty]$. We restate equations (3.3) and (3.4) using the Bowling et al. (2009) best-fit logistic approximation for cumulative normal distributions.

$$D = N \left(1 - \frac{1}{1 + e^{-1.702 * Z}} \right), \quad (3.5)$$

$$S = \frac{N}{1 + e^{-1.702 * Z}} \quad (3.6)$$

Where $Z = \frac{P_t - F}{\sigma}$. The equilibrium market price P^* can be determined by equating $D = Q_0$ and solving for P_t .

$$N \left(1 - \frac{1}{1 + e^{-1.702 * Z}} \right) = Q_0$$

$$\ln \left(\frac{1}{1 - \frac{Q_0}{N}} - 1 \right) = -1.702 * Z \ln(e)$$

$$\ln(1) - \ln \left(\frac{N}{Q_0} - 1 \right) = -1.702 * \frac{P_t - F}{\sigma}$$

$$P^* = \frac{\ln \left(\frac{N}{Q_0} - 1 \right) \sigma}{1.702} + F \quad (3.7)$$

Note that P^* need not equal F , and that for $P^* = F$, $\ln \left(\frac{N}{Q_0} - 1 \right) = 0$ assuming $\sigma > 0$. Thus, we can identify some Q^* number of shares that results in the stock being correctly priced at its fundamental value F .

$$\ln \left(\frac{N}{Q_0} - 1 \right) = 0$$

$$\frac{N}{Q_0} - 1 = e^0 = 1$$

$$Q^* = \frac{N}{2} \quad (3.8)$$

The result is intuitive. Given $X \sim N(F, \sigma^2)$ an equal number of buyers and potential sellers are expected if $P^* = F$. By the URC, this is synonymous with an equal number of shares demanded as are supplied. It follows that if $Q_0 = \frac{N}{2}$, exactly as many are being

demanded as are being sold. Equations (3.5) through (3.8) are illustrated in Figures 3.3 and 3.4 below.

Figure 3.3: D, S
 $N = 50, F = 25, \sigma = \{1, 3, 5\}$

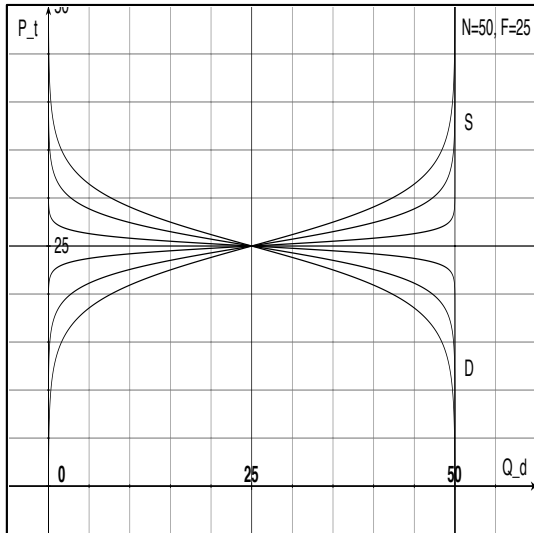
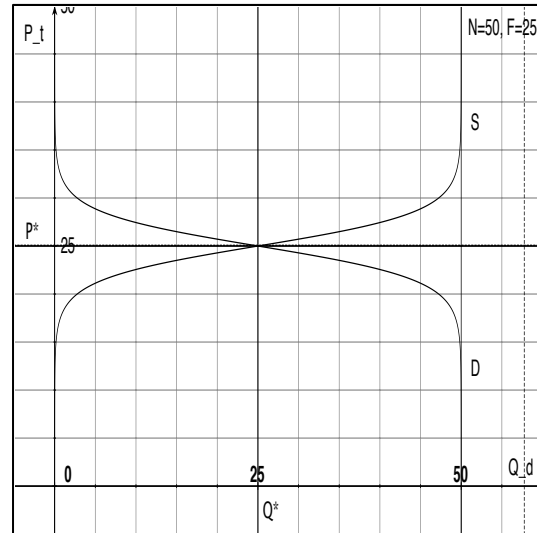


Figure 3.4: D, S, P^*, Q^*
 $N = 50, F = 25, \sigma = 3$



As Q_0 is considered fixed, the equilibrium price P^* follows the path of the demand function D for varying *possible* levels of Q_0 . When $Q_0 < Q^*$ or $Q_0 > Q^*$ it will be the case that $P^* > F$ or $P^* < F$, respectively. Importantly, note that if $Q_0 = Q^*$, $P^* = F$ for varying σ . Again, σ is fixed in the BCM by the assumption that θ is fixed.

Recall that Firm j sets Q_0 and P_0 to maximize equity gain, and suppose that while F and σ are unknown, Firm j is aware that the distribution of X is normal. Firm j estimates F and σ from its sampling of θ , $E_j(F) = x_j$ and $E_j(\sigma) = \sigma_j$ respectively. The inverse expected demand curve could be written as:

$$P^* = \frac{\ln\left(\frac{N}{Q_0} - 1\right)\sigma_j}{1.702} + x_j \quad (3.9)$$

The equity gain π_j from the IPO, assuming a fixed cost C and variable cost c^{20} , can be

²⁰ The dominant variable cost in an IPO is the underwriting discount (\$/Share) that is often set to roughly 7% of the IPO price, or P_0 .

derived. The derivative, marginal equity gain π_j' , is used to identify the equity maximizing quantity where $\pi_j' = 0$.

$$\pi_j = \left(\frac{\ln\left(\frac{N}{Q_0} - 1\right)\sigma_j}{1.702} + x_j \right) Q_0 - C - cQ_0 \tag{3.10}$$

$$\pi_j' = \frac{d\pi_j}{dQ_0} = \left(\left(\frac{\ln\left(\frac{N}{Q_0} - 1\right)\sigma_j}{1.702} + X_j \right) Q_0 - C - cQ_0 \right) dQ_0 \tag{3.11}$$

We graph the expected demand curve (3.9) and marginal equity gain (3.11) against the actual market demand curve (3.5) in Figures 3.5 and 3.6 below. The actual market supply curve has been removed as the equilibrium price is ultimately set where $D = Q_0$ rather than $D = S$.

Figure 3.5: D, D_j, π_j'
 $N = 50, X_j = F = 25$
 $\sigma_j = \sigma = 3, c = \{0, x, 2Q_0\}$

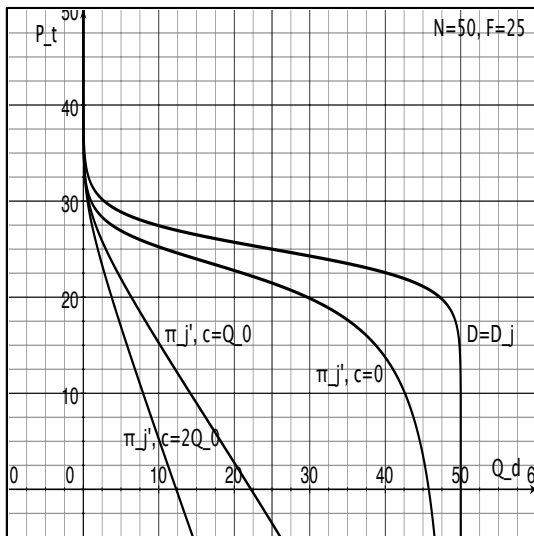
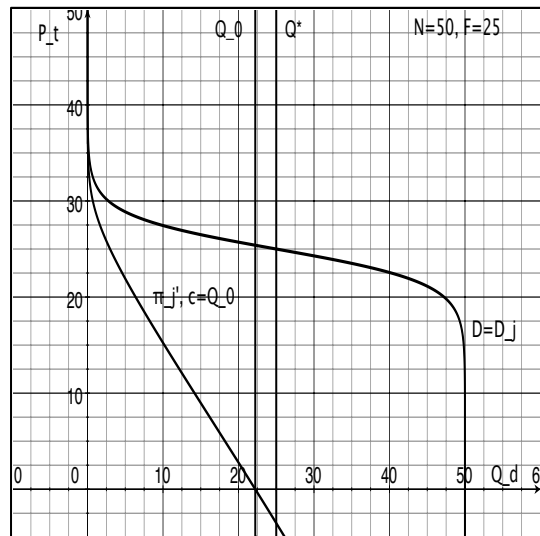


Figure 3.6: $D, D_j, \pi_j', P^*, Q^*, Q_0$
 $N = 50, X_j = F = 25$
 $\sigma_j = \sigma = 3, c = Q_0$



In Figures 3.5 and 3.6 we have assumed that Firm j correctly identifies the actual market demand curve, $D_j = D$. Given a marginal cost $c = Q_0$, we find that $Q_0 < Q^*$, and accordingly $P_0 > F$. Because $D_j = D$ and Q_0 is fixed, this is a stable equilibrium where $P^* = P_0 > F$. We now examine a case where Firm j incorrectly assesses the market demand curve D ; any

instance where $x_j \neq F$ or $\sigma_j \neq \sigma$. Figures 2.7 and 2.8 illustrate both possible error types, respectively for a marginal cost $c = Q_0$.

Figure 3.7: $D, D_j, \pi_j', P^*, Q^*, Q_0$

$N = 50, F = 25, X_j = 20$

$\sigma_j = \sigma = 3, c = Q_0$

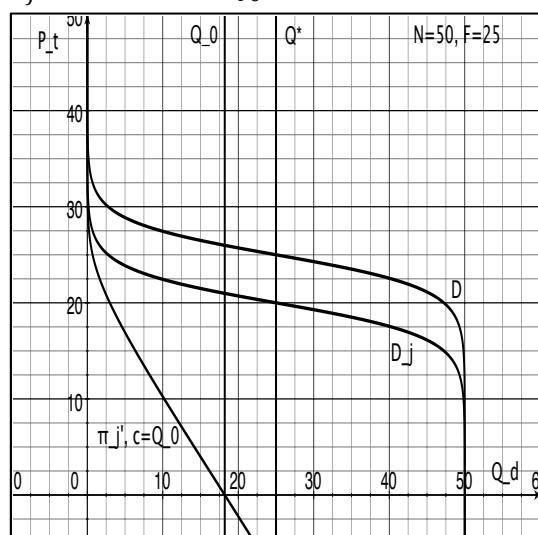


Figure 3.8: $D, D_j, \pi_j', P^*, Q^*, Q_0$

$N = 50, X_j = F = 25$

$\sigma = 3, \sigma_j = 7, c = Q_0$

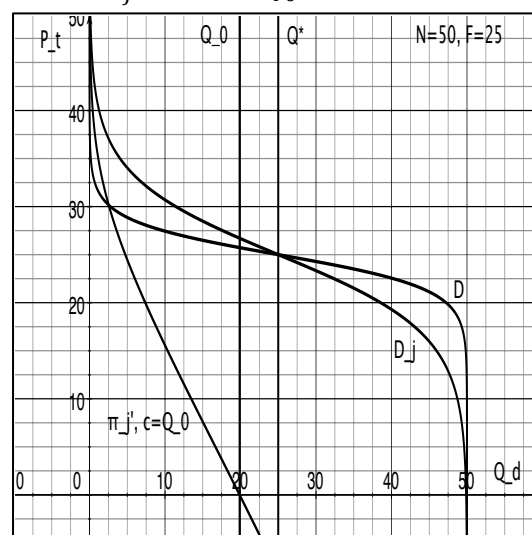


Figure 2.7 holds $\sigma_j = \sigma$ and deals with an instance where Firm j underestimates F , $x_j < F$. It produces at Q_0 and prices at P_0 where $Q_0 = D_j$. This is not a market equilibrium however, since at P_0 , $D > D_j$; there is excess demand. Because Q_0 is fixed, the market maker will raise the price over successive rounds until the market clears at a price P^* where $Q_0 = D$. A stable equilibrium is reached where the stock is overvalued. Note however that if the marginal cost were such that $Q_0 = Q^*$, the equilibrium price $P^* = F$ and the stock would be fairly priced.

Figure 2.8 is similarly intuitive. Firm j overestimates σ , and as a result overprices at $Q_0 = D_j$. This can be paralleled to simple neoclassical models where an increasingly inelastic demand function lends a firm higher pricing power. Continuing with the analysis, the market maker will lower prices until some P^* is reached where $Q_0 = D$. Importantly, understand that

unlike in Figure 2.7, an incorrect assessment of σ would allow for $P_0 = P^*$ assuming marginal costs are such that $Q_0 = Q^*$. The implications of this conclusion are intriguing: if Firm j is reasonably certain that $X_j \approx F$, it would benefit the firm to underestimate σ , assuming some marginal cost $c > 0$.

BCM End Discussion

The examples shown thus far have illustrated the existence and achievement of stable equilibria where an asset is mispriced relative to its fundamental value. Specifically, this has been done for a fixed θ and X . Note also that by assuming Q_0 – set by Firm j – fixed, we have made any changes in the beliefs of the firm irrelevant as the market maker determines the price for all $t > 0$. Before ending our discussion of the BCM, we pause briefly to address several points worth mentioning.

Recall that the market maker, like Firm j , is unaware of F and σ . Thus, price adjustments are imperfect and are thought to be some function of the excess demand (supply) in the market. In our discussion, we arrived at our equilibrium P^* smoothly, making two assumptions:

1. The magnitude of price adjustments (say α) is smaller or equal to the precision of the price P_t (say β). If we allow for $\alpha > \beta$, then $\lim_{t \rightarrow \infty} P_t = P^* \pm (\alpha - \beta)$. Generally the assumption that $\alpha = \beta$ is valid. However, we should carefully consider how the simultaneous nature of the game influences α . Might α be smaller if prices were to adjust after *each* agent conducts a trade, rather than after *all* agents have done so?
2. The demand and supply curves are continuous functions, allowing for excess demand to be a sufficiently fine signal for price adjustments, so that $\lim_{t \rightarrow \infty} P_t = P^*$ if $Q_0 = Q^*$. The validity of assuming continuous rather than step functions depends on the number of agents, N , within the game. As $N \rightarrow \infty$, the assumption of continuous

functions increasingly represents the market demand and supply curves *ceteris paribus*.

Thus far, there is little to suggest that $\lim_{t \rightarrow \infty} P^* = F$ unless $Q_0 = Q^*$. We have also found that it is unlikely for Firm j to set $Q_0 = Q^*$; doing so could largely be attributed to chance under the assumptions of the BCM. Namely, the following three assumptions are responsible for these conclusions:

1. Firm j sets Q_0 and P_0 attempting to maximize equity gain rather than to price fairly, $P_0 = X_j$. Assume instead that Firm j attempts to price fairly and sets the quantity Q_0 accordingly, with respect to its expected demand function D_j . We consider four possible outcomes:
 - a. The market demand function D is correctly identified (see Figure 2.6), $D_j = D$: Firm j will set $P_0 = P^*$ and accordingly set $Q_0 = Q^*$ so that $P^* = F$. Our assumption that $D_j = D$ is rather unrealistic however. We relax this assumption in outcomes 'b', 'c' and 'd' that follow.
 - b. The market demand function D is incorrectly identified as a result of underestimating the fundamental value (see Figure 2.7), $x_j < F$: Firm j will price $P_0 < P^*$, but sets $Q_0 = Q^*$. Thus the market maker will act to eliminate excess demand by raising prices to $P^* = F$.
 - c. The market demand function D is incorrectly identified as a result of overestimating the variance of beliefs around F (see Figure 2.8), $\sigma_j > \sigma$: Firm j will set $P_0 = P^*$ and $Q_0 = Q^*$ so that $P^* = F$.
 - d. The market demand function D is incorrectly identified as a result of underestimating the fundamental value and overestimating the variance of beliefs around F , $x_j < F$ and $\sigma_j > \sigma$; the outcome is identical to that of outcome 'b'.

Interestingly, we find that regardless of any error in Firm j 's ability to assess the market demand, so long as the firm attempts to price fairly, $Q_0 = Q^*$ and $\lim_{t \rightarrow \infty} P^* = F$.

2. The quantity of outstanding shares Q_0 is fixed. This is not an unreasonable assumption in the short run, given the costs involved with issuing new shares or pursuing share buybacks. In the BCM, this assumption is supported by the duality of θ being fixed, and Firm j restricted from adjusting beliefs on price movements. The persistence of this assumption in the PCM and ACM is discussed in greater detail in the opening of the following chapter.
3. The size of the market is correctly identified by Firm j . While there are a number of practicable methods by which firms can arrive at an estimate of the market size, non are guaranteed. Thus a critique on this particular assumption warranted, but limited by the accuracy of current market size approximation methods.

Having briefed our results and questioned our assumptions, we here conclude our discussion on the BCM. The tendency towards an initial market mispricing, given an equity-maximizing firm, will take on an integral role in the development of the following two models. We now progress to Chapter 4 on the Price-Case Model (PCM).

Chapter 4: PCM Results

Construction and Analysis

Note: As the distributions of X and θ are variable under the PCM, all time subscripts are reintroduced.

In the BCM we assumed all agents' beliefs to be fixed. Accordingly, the distributions of X_0 and θ_0 applied for all $t \geq 0$, and Firm j had no incentive to adjust Q_0 . In the PCM, we allow for agents to update beliefs on price movements in the market. Agents that revise

beliefs do so simultaneously and, by assumption, after the market maker adjusts prices. Recall that for all times $t > 0$, θ_t is determined by and equal to the joint distribution X_t . Thus, θ_t is variable in the PCM. In an ideal scenario, Firm j responds by re-sampling θ_t , updating its beliefs on market demand and adjusting Q_0 to some $Q_t \neq Q_0$ where equity gain is maximized given $X_t \neq X_0$. There are, however, a number of both practical (real-world) limitations and theoretical (modeling) dilemmas that arise when assuming perfect flexibility of outstanding shares:

1. Shares outstanding have upper limits specific to each company, regulated by the SEC. These are referred to as authorized shares. Liquidity constraints may also impose lower limits on outstanding shares by restricting share buybacks.
2. How outstanding shares are adjusted depends on a.) Firm j 's expected market demand D_j , which may differ from the true market demand, $x_j \neq F$ and $\sigma_j \neq \sigma$, b.) Firm j 's *confidence* in that accuracy of D_j , and c.) Firm j 's perception of its own ability to affect market prices, partially determined by 'b'.
3. The process by which θ_t is updated on X_t for all $t > 0$ is likely subject to time lags; the total population of information of the fundamental value may not instantaneously reflect shifts in the joint distribution of beliefs on said fundamental value.
4. Stable equilibria in the model that might otherwise exist were Q_t fixed at Q_0 could be lost, as the distribution of X_t in the PCM is similarly variable.
5. We risk shifting the focus towards the behavior of Firm j , rather than focusing on the influence of agents' behavior over price movements.

For these reasons, while the assumptions of the model assume θ_t variable, we will consider Q_t fixed at Q_0 for all $t \geq 0$.

Before proceeding with the mathematics however, rules must be developed that are deterministic of how agents revise expectations. In the PCM, these rules can be summarized as follows:

1. Agents update beliefs simultaneously at the end of a round, after prices adjustments are made by the market maker.
2. Agents are concerned strictly with the direction of price movements. Accordingly, expectations are not affected by P_0 , e.g. Agents with an $x_0 < P_0$ will not revise expectations at time $t = 0$ as they expect $P_1 < P_0$ and $P^* = x_0$.
3. Agents update beliefs strictly when the direction of price movements contradicts their private x_t for all $t \geq 0$, e.g. if $P_{t+1} > P_t \geq x_t$, agents will revise expectations. Price movements are used as an imperfect signal on the investment behavior of other agents.
4. Updated, or posterior beliefs, are assumed equal to the market price that triggered the revision, e.g. if $x_t \neq x_{t+1}$ then $x_{t+1} = P_{t+1}$. This is justified by the following logic: no agent revising expectations should adjust their beliefs any more that what is suggested by the market price. Note that the PCM is robust to relaxing this assumption, such that all prior beliefs are updated by a common magnitude m in the direction of P_{t+1} .

Rules 1-3 outlined above make explicit that the distribution of X_t cannot be assumed normal throughout the game. We begin however by presenting a specific case in which X_t retains its normal form for all $t \geq 0$.

As seen in Chapter 3, there are a number of possible combinations of P_0 and Q_0 set by Firm j , assuming its goal to maximize equity gain given an expected demand function. Here, we consider a case where Firm j correctly identifies the market demand curve, $D_j = D$, and marginal costs are such that $Q_0 = Q^* = \frac{N}{2}$. The result is that $P_0 = F = P^*$. We know this

outcome to hold at time $t = 0$, as X_0 is normally distributed. Because the IPO price and outstanding shares are such that the market is in equilibrium, the market price $P_0 = P_t$ holds for all $t \geq 0$. Furthermore, as $P_0 = F$, there will be an equal number of buyers and sellers for any time $t \geq 0$. The result is that all agents identified by $x_0 < P_0$ and by $x_0 > P_0$ update beliefs upwards and downwards, respectively. As agents update beliefs at the same rate, under Rules 1-3, the normality of X_t is retained. The variance of X_t , σ_t , decreases as the beliefs of agents converge upon the fundamental value F . We can express this process formally with several variations on the equilibrium price condition, equation (3.7), rewritten below.

$$P^* = \frac{\ln\left(\frac{N}{Q_0}-1\right)\sigma}{1.702} + F \quad (3.7)$$

Recall that because Q_0 had been fixed in our original analysis, the equilibrium price condition is no different than an inverse demand function. Equation (3.7) can be transformed for any i^{th} agent as follows.

$$x_t = \frac{\ln\left(\frac{N}{Q_i}-1\right)\sigma_i}{1.702} + F \quad (4.1)$$

Where Q_i and σ_i are the quantity demanded and the standard deviation of x_t at any time t , particular to the i^{th} agent. The case discussed above, can then be expressed as equation (4.2).

$$\lim_{\sigma_i \rightarrow 0} x_t = F \quad (4.2)$$

We specify in the limit, rather than at x_1 , allowing for agents to adjust beliefs at rate m per round. For the aggregate of all informed agents we can state:

$$\lim_{\sigma_t \rightarrow 0} X_t = F \quad (4.3)$$

Figures 4.1 and 4.2 below illustrate equations 4.1 through 4.3, with the limit result bolded in Figure 4.2. Note that the σ range used is different between graphs for better visual interpretation.

Figure 4.1: $\lim_{\sigma_t \rightarrow 0} X_t = F$
 $F = 25, \sigma = \{.25, .5, 1\}$

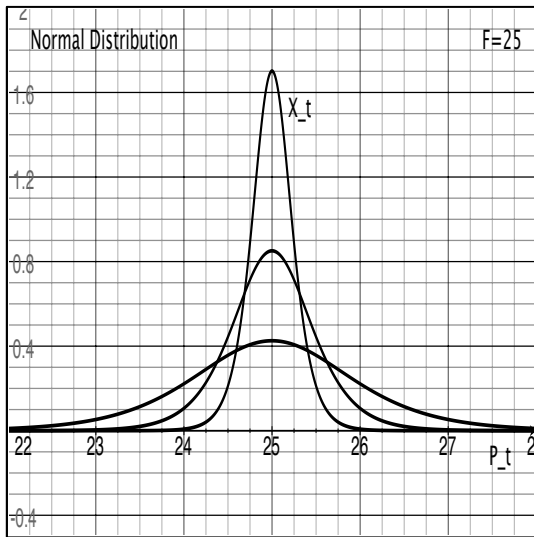
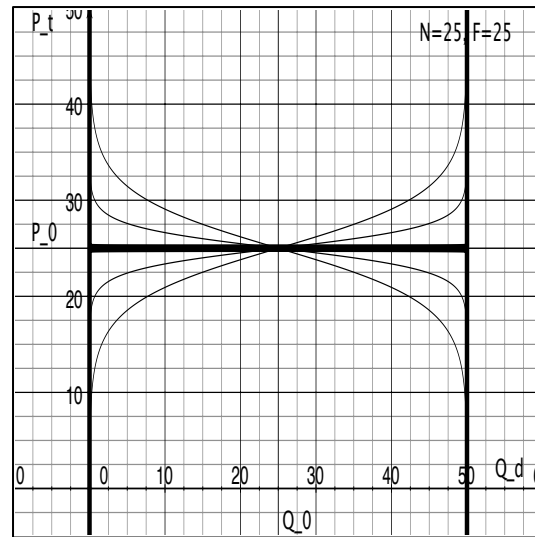


Figure 4.2: D, S, P_0, Q_0
 $N = 50, F = 25, \sigma = \{.025, 2, 5\}$



Interestingly, we can note that if beliefs converge to F within a single period, rather than over successive periods at a rate of m , allowing for a perfectly flexible Q_0 has no effect on equilibrium price P^* . If $X_1 = F$, all agents are indifferent between investment options. Thus, the market maker can clear the market for any quantity of stock $Q_0 < N$. Accordingly we expect Firm j to increase $Q_0 = \frac{N}{2}$ to $Q_t = N$. In cases where agents do not revise beliefs as depicted here, allowing for a perfectly flexible Q_t adds considerably greater complexity and is beyond the scope of this paper for reasons discussed in the opening of this chapter.

We now consider a case where X_t loses its normal distribution for all $t > 0$. Specifically, we begin with a scenario where $Q_0 = Q^* = \frac{N}{2}$, but $P_0 < F$; the stock is undervalued relative to its fundamental value. The market maker responds to excess demand in the market by raising P_0 to some $P_1 > P_0$. Recall that P_1 does not necessarily clear the market, as the market maker has imperfect information on both F and σ_t . Rather than all agents updating beliefs, only those agents characterized by $x_0 < P_1$ will revise their expected values of F . For simplicity, assume that all revised expectations equal P_1 , rather increasing at some common rate m . While X_t may no longer be normal, Rules 1-3 allow for partial use of

our market demand and supply functions determined in Chapter 3; these equations hold for all agents characterized by $x_0 \geq P_1$. Thus, we can construct our market demand curve and supply curves as piece wise functions.

Begin by considering the demand functions for agents $x_0 > P_1$ and $x_0 < P_1$, represented by equations (4.4) and (4.5) respectively. Note that because all agents characterized by $x_0 < P_1$ share a common $x_1 = P_1$, the demand function is similar to that of a single agent.

$$D_{>P_1} = N \left(1 - \frac{1}{1+e^{-1.702*Z}} \right), \quad P_t > P_1, \quad \text{Where } Z = \frac{P_t - F}{\sigma} \quad (4.4)$$

$$D_{<P_1} = \begin{cases} \left(\frac{N}{1+e^{-1.702*Z}} \right), & P_t < P_1 \\ 0, & P_t > P_1 \end{cases}, \quad \text{Where } Z = \frac{P_1 - F}{\sigma} \quad (4.5)$$

Equations (4.6) and (4.7) below are the analogous supply functions. While these supply curves will not ultimately affect the equilibrium price – determined by $D = Q_0$, they are shown here to better understand the behavior of agents.

$$S_{>P_1} = \frac{N}{1+e^{-1.702*Z}}, \quad P_t > P_1, \quad \text{Where } Z = \frac{P_t - F}{\sigma} \quad (4.6)$$

$$S_{<P_1} = \begin{cases} \frac{N}{1+e^{-1.702*Z}}, & P_t > P_1 \\ 0, & P_t < P_1 \end{cases}, \quad \text{Where } Z = \frac{P_1 - F}{\sigma} \quad (4.7)$$

We have yet to develop the aggregate demand and supply functions, however. Before aggregating, we consider Figure 4.3 below, illustrating equations (4.4) through (4.7).

Equations (4.4) and (4.6) have been shown for all P_t and (4.5) shifted to the right by

$N \left(1 - \frac{1}{1+e^{-1.702*Z}} \right)$ where $Z = \frac{P_1 - F}{\sigma}$, for interpretive ease.

(See Figures 4.3 and 4.4 on the following page).

Figure 4.3: $D_{>P_1}, D_{<P_1}, S_{>P_1}, S_{<P_1}$
 $N = 50, F = 25$
 $\sigma = 3, P_0 = 18, P_1 = 22$

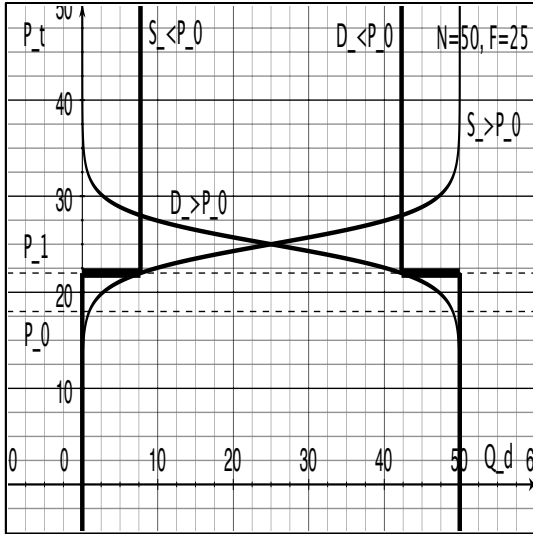
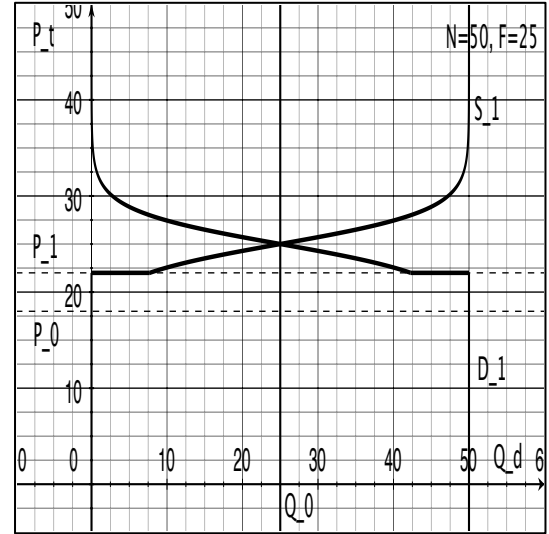


Figure 4.4: D_1, S_1
 $N = 50, F = 25$
 $\sigma = 3, P_0 = 18, P_1 = 22$



From Figure 4.3, developing the aggregate demand and supply functions becomes a more intuitive process. Agent's characterized by $x_0 < P_0$ that would not have previously demanded the asset for *all* $P_t < P_1$, now do. The same set of agents that would have sold for P_1 will now not. The aggregate demand and supply functions, D_1 and S_1 , are then written as equations (4.8) and (4.9) below. Figure 4.4 above illustrates (4.8) and (4.9), and introduces $Q_0 = \frac{N}{2}$.

$$D_1 = \begin{cases} N \left(1 - \frac{1}{1 + e^{-1.702 * Z_1}} \right), & P_t > P_1 \\ N \left(1 - \frac{1}{1 + e^{-1.702 * Z_2}} \right) + \frac{N}{1 + e^{-1.702 * Z_2}}, & P_t < P_1 \end{cases}, \text{ Where } Z_1 = \frac{P_t - F}{\sigma} \text{ and } Z_2 = \frac{P_1 - F}{\sigma} \quad (4.8)$$

$$S_1 = \begin{cases} 0, & P_t > P_1 \\ \frac{N}{1 + e^{-1.702 * Z_1}}, & P_t < P_1 \end{cases}, \text{ Where } Z_1 = \frac{P_t - F}{\sigma} \quad (4.9)$$

As seen from Figure 4.4, excess demand persists in the market for P_1 , given Q_0 . Accordingly, the market maker raises the price P_1 to some $P_2 > P_1$. Figure 4.5 below illustrates a scenario where P_2 is set equal to F .

Figure 4.5: D_2, S_2
 $N = 50, F = 25$
 $\sigma = 3, P_0 = 18, P_2 = 25$

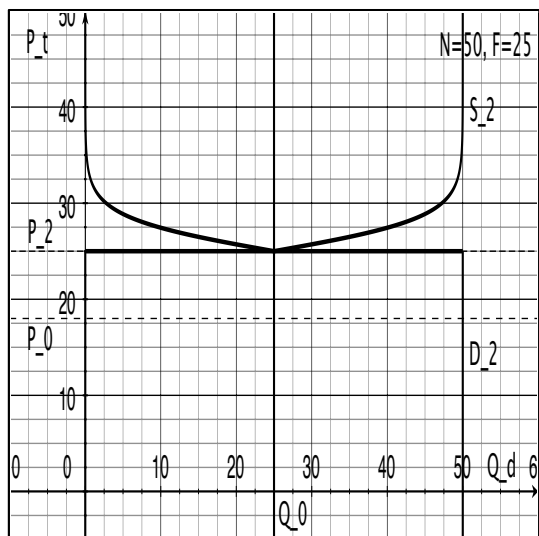
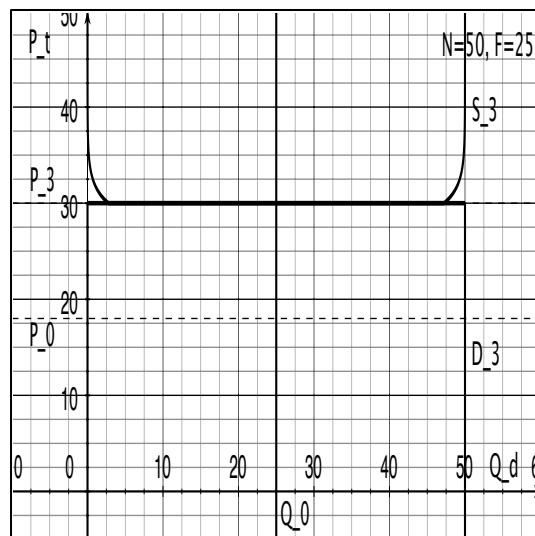


Figure 4.6: D_3, S_3
 $N = 50, F = 25$
 $\sigma = 3, P_0 = 18, P_3 = 30$



Where $P_2 = F$ we find the market at equilibrium; excess demand is zero and the market maker holds the price constant. Because the price is stable, all agents characterized by $x_0 > P_2$ will update beliefs downwards in following rounds so that $\lim_{t \rightarrow \infty} X_t = F$.

Seemingly, a similar conclusion has been reached as in the first PCM scenario, where $P_0 = F$ and the normality of X_t was retained. Note however, a distinct feature of this latter case: there is a *growing* indifference between investment alternatives for all agents characterized by $x_0 < P_t$ for any $t > 0$.

Recall the end discussion of Chapter 3, which briefed our assumption that the magnitude of price adjustments is sufficiently fine, such that – in this particular case – P_t never exceeds F for any $t \geq 0$. Consider an instance in which the market maker raises the price from P_1 to some $P_3 > P_2 = F$. The majority share of agents is now indifferent, and the market clears for P_3 at Q_0 , as illustrated by Figure 4.6. Thus, our market equilibrium where $P_2 = F$ is only stable if assume sufficiently fine price adjustments. While these results can be mathematically demonstrated, a graphical depiction is sufficient here.

PCM End Discussion

Our brief end discussion will focus on Rule 3, which assumed all posterior beliefs equal to the going market price; the assumption is largely responsible for the results in this chapter. In practice however, Rule 3 is rather unrealistic with respect to how real-world agents may adjust their expectations. Two obvious disparities stand out:

1. Agents likely update beliefs in the direction of the new market price at some rate m , which is not identical across investors. This weakens the applicability of the first case and second case results, the latter more so than the first where results proved robust against *identical* revisions towards the market price at a rate m .
2. Agent's beliefs are not perfectly flexible. There may be some price range within which agents are willing to adjust their expectations. If these assumptions had been introduced to the models of this chapter, we may find some upper limit of price equilibria above which agents would sell off Firm j 's stock.

It should also be stated once more that the results found in this chapter rely not only on a fixed quantity of shares outstanding, but specifically a $Q_0 = Q^* = \frac{N}{2}$. Looking at Figure 4.5, it is relatively straightforward that if $Q_0 < Q^*$, the equilibrium price $P^* > F$, and visa versa. With these points in mind, we progress to Chapter 5, which introduces the Action-Case Model (ACM).

Chapter 5: ACM Results

Construction and Analysis

In the PCM, we had agents update beliefs on price movements in the market and therefore *post* price adjustments. As in the BCM, agents' investment decisions were private, rendering the sequential move game essentially simultaneous. In the ACM, agents no longer update beliefs on price movements. Rather, we allow for investment decisions to be public

knowledge; each trade conducted with the market maker acts as a public signal to all *following* investors. Agents' use these signals to update their beliefs on the fundamental value of the asset. Note that while the game has lost its simultaneous nature, the market maker continues to adjust prices after all agents have made a single trade.

Let us begin with a detailed discussion on the public signals generated in the ACM.

Recall that under the URC, investment decisions for holding and buying, or selling and shorting are synonymized. Thus, the public signals generated from investment decisions are reduced to a binary $\{0, 1\}$ signal. The limited vocabulary of social communications is prime for generating herd behavior, which will naturally be a feature of social learning in a behavioral model where asset mispricing is demonstrated. The exact learning rule used here to justify herding amongst agents will be Bikchandani and Sharma (2000)'s symmetric signals. The symmetry of signals implies that agents place equal weight on each public signal, as they do on their own private information²¹.

Symmetric signals have an interesting implication: if in some sequential ordering of agents, the signals generated by the first two agents are identical, the third agent will herd *regardless* of his or her private information. More generally, we can say if by some time t the number of '1's in the history of actions exceeds the number '0's by two, a buy herd will occur by $t + 1$.

Understanding the joint distribution of prior beliefs X_0 to be normal, we can determine the probability with which a buy or sell herd forms by the third agent, at time $t = 0$, represented by equations (5.1) and (5.2) respectively.

$$p_{Buy} = \left(1 - \frac{1}{1 + e^{-1.702 * Z}}\right)^2, \text{ where } Z = \frac{P_0 - F}{\sigma} \quad (5.1)$$

²¹ A valid criticism is that in games where the existence of noise agents is common knowledge, symmetric signals are unlikely. However, we assume symmetric signals for their simplicity and interpretive ease.

$$p_{Sell} = \left(\frac{1}{1+e^{-1.702*Z}} \right)^2, \text{ where } Z = \frac{P_0-F}{\sigma} \tag{5.2}$$

Equations (5.1) and (5.2) are restrictive however, in that they rule out instances where herding may occur by some fourth, or fifth (and so on) agent in the game. Equation (5.3), accounts for such instances. The equation for a sell herd by the n^{th} agent has an analogous form.

$$p'_{Buy} = \sum_k^n \binom{n}{k} \left(1 - \frac{1}{1+e^{-1.702*Z}} \right)^k \left(\frac{1}{1+e^{-1.702*Z}} \right)^{n-k}, \text{ Where } k = \frac{N+2}{2} \text{ and } Z = \frac{P_0-F}{\sigma} \tag{5.3}$$

Note that n represents the n^{th} agent by which a buy herd occurs. It is easily seen that the probability of some herd occurring depends on the difference between the fundamental value and the IPO price, given a normal X_t .

Figures 5.1 below, illustrates D_0 and S_0 with P_t 's that equate equations (5.1) and (5.2) to .5. Also below is Figure 5.2, illustrating the derivative of equation (5.3) for varying P_t .

Figure 5.1: D_0, S_0
 $N = 50, F = 25$
 $\sigma = 3, P_0 = 18, P_2 = 25$

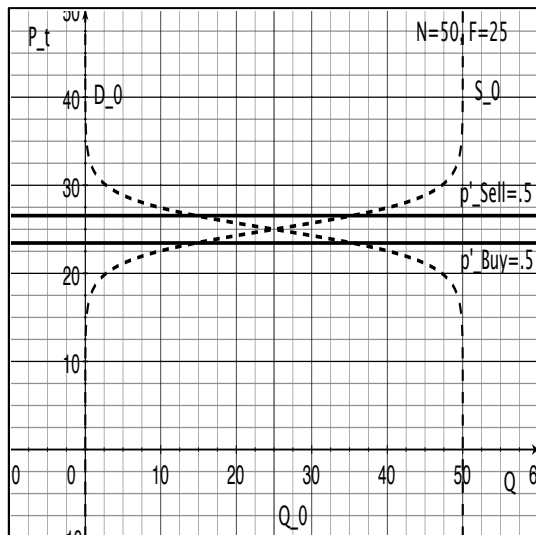
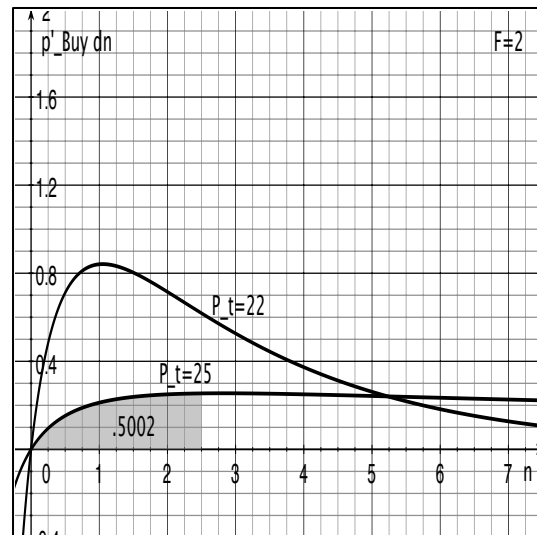


Figure 5.2: $p'_{Buy}, n = 5$
 $F = 25, \sigma = 3, P_t = 25, P_t = 22$



From Figure 5.2, we can see more clearly that the probability with which a herd occurs by any n^{th} agent diminishes as $P_t \rightarrow F$. Specifically, the likelihood of a herd taking place

increases at a decreasing rate as $n \rightarrow \infty$. For $P_t = F$, we can see that the probability of herd occurring by the third agent, at 2.5, is roughly .5 which is to be expected. To reiterate, the exact equations (5.1) through (5.3) and their results depicted in Figures 5.1 and 5.2, hold only if X_t is normally distributed for all $t > 0$. The general characteristics however, hold for any distribution of X_t where the probabilities of a randomly selected agent purchasing or selling the asset can vary for a given P_t . The ACM will exploit probability to determine when herds occur. If for a given n , $P'_{Buy} > .5$, we assume a buy herd to occur, and the same for sell herds. For instances where P'_{Buy} or P'_{Sell} equal .5, we assume no herd to form. Relying on likelihood allows for our sequencing of agent to become irrelevant, and thus the results of the model are robust to any endogenous ordering of agents.

Having better understood public signals and their capacity to generate herd behavior in the ACM, we now examine more closely the process by which agents update beliefs. The updating rules in the ACM are similar to those laid out in the PCM, only more intuitive in a herding scenario. The rules are summarized as follows:

1. Agents update beliefs sequentially within the round, before prices are adjusted by the market maker.
2. Beliefs are updated when, and if, a buy or sell herd takes place that contradicts some prior belief x_t for all $t > 0$, e.g. if a buy herd occurs at time t , all agents characterized by $x_t \leq P_t$, will revise expectations upwards.
3. Agents' posterior beliefs x_t' follow the form of equation (5.4),

$$x_t' = P_t \pm \varepsilon, \text{ where } \varepsilon \text{ is any small positive real number.} \quad (5.4)$$

The value ε , is added or subtracted respective to whether a buy or sell herd triggers the updating. The rule is justified by the following logic: if some agent characterized by $x_t \leq P_t$ observes a large majority of preceding agents placing purchase orders,

said agent must infer that the true fundamental value $F > P_t$, by the principle of symmetric signals.

4. The value ε is minimized by all agents given some confidence interval each agent has on their respective x_t . For purposes of modeling, we assume an identical ε across all posterior beliefs.

As herding, in whichever direction, triggers select agents to update beliefs the distribution of X_t loses its normality for all $t > 0$. Interestingly, note that because ε is minimized the distribution of X_t is similar to that in the PCM. Thus for any $t > 0$, the demand and supply functions of the PCM can be considered an approximation for those in the ACM. We emphasize once more that unlike the PCM, the demand and supply schedules in the ACM shift before price adjustments in the market. Regardless, three points can be inferred at this stage:

1. As in the PCM, posterior beliefs x_t' likely converge to $P_t \pm \varepsilon$ for all $t > 0$.
2. Considering ε as the *degree* to which agents update beliefs, the equilibrium price is sensitive to relative magnitudes of ε and α , the magnitude of price adjustments made by the market maker. We assume α to be similarly constant across all adjustments. Alternatively, a model might assume α to be correlated with the absolute value of either excess demand or supply.
3. If we allow for ε to be sufficiently greater than α so that herding is present by $P_t = F$, an equilibrium price $P^* > F$ is *expected*.

With these points in mind, we progress onwards to the chief example of asset mispricing demonstrated by the ACM. Unlike in previous models, these results will be simulated rather than mathematically derived. This is necessitated by the complexity introduced by herding; to determine the net effect of ε , α , P_t , and X_t on X_{t+1} , p_{buy}' given some n and accordingly P_{t+1}

mathematically, would be unnecessarily tedious. Before a discussion on the simulation results however, we specify the example laid out in this chapter.

The game will begin almost identically to the PCM, where the quantity of outstanding shares $Q_0 = Q^* = \frac{N}{2}$, and $P_0 < F$. Additionally, we will assume that $F - P_0$ is such that $p'_{Buy} > .5$, given $n = 5$; we deem limiting the possibility of herding to the first two agents overly restrictive. The contextual updating rules for all agents characterized by $x_t \leq P_t$ is $x'_t = P_t + \varepsilon$, if $p'_{Buy} > .5$. For all agents identified by $x_t \geq P_t$, the specific updating rule is $x'_t = P_t - \varepsilon$, if $p'_{Sell} > .5$. The value of ε is assumed constant across agents and the magnitude α of the marker maker's price adjustments is similarly constant.

The scenario described above has been simulated, and the results found in Table 5.1 and Figure 5.5. Importantly, note that the simulations were conducted in Excel with an initial distribution of $X_0 \sim N(F, \sigma)$, where $F = 25$ and $\sigma = 3$, for 50 agents. The distribution was created using StatPlus, which allows for the generation of discrete random values following a specified distribution, mean and standard deviation. The exact distribution of X_0 and corresponding demand and supply curves can be seen below in Figures 5.3 and 5.4, respectively.

Figure 5.3: X_0
 $N = 50, F = 25, \sigma = 3$

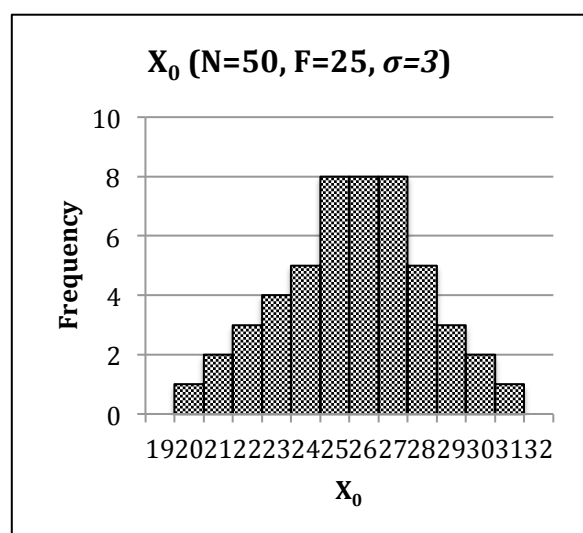
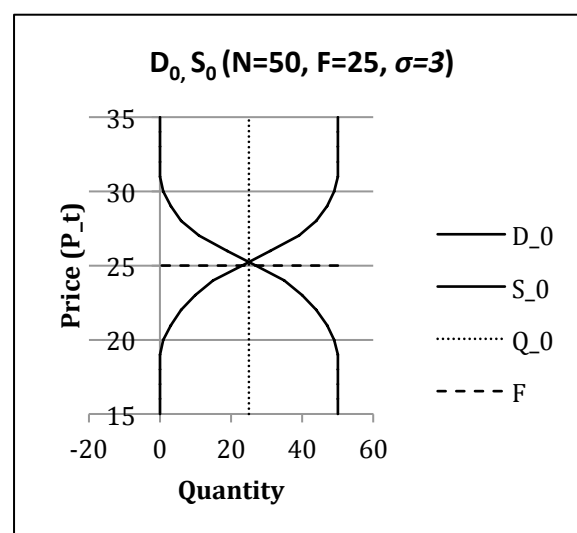


Figure 5.4: D_0, S_0
 $N = 50, F = 25, \sigma = 3$



The distribution of X_0 is evidently an approximation of a normal distribution, and the demand and supply curves generated follow almost identically the figures presented earlier in the Chapters 3 and 4. Figure 5.5 shown later will demonstrate the convergence of all x_t' to $P_t + \varepsilon$ at some $t > 0$; a flattening out of the lower half of the demand and supply curves that closely resembles Figures 4.4 and 4.5 from Chapter 4. The results from the simulation are now shown below. Specifically the relevant time (t) data and a summary chart in Table 5.1 and Figure 5.6, respectively.

Table 5.1: ACM Simulation Data
($N = 50, F = 25, \sigma = 3, P_0 = 18, n = 5$)

| N | 50 | Time | D_t | S_t | Q_0 | P(Buy) | P(Sell) | P_t |
|-------|----|------|-----|-----|-----|------------|------------|-----|
| F | 25 | 0 | 50 | 0 | 25 | 1 | 0 | 18 |
| E | 2 | 1 | 50 | 0 | 25 | 1 | 0 | 19 |
| Alpha | 1 | 2 | 50 | 0 | 25 | 0.99804768 | 0.00041568 | 20 |
| | | 3 | 50 | 0 | 25 | 0.99237888 | 0.00172288 | 21 |
| | | 4 | 50 | 0 | 25 | 0.97099008 | 0.00734208 | 22 |
| | | 5 | 50 | 0 | 25 | 0.93795328 | 0.01744128 | 23 |
| | | 6 | 50 | 0 | 25 | 0.87089248 | 0.04196448 | 24 |
| | | 7 | 50 | 0 | 25 | 0.72741888 | 0.11001088 | 25 |
| | | 8 | 50 | 0 | 25 | 0.63109728 | 0.16748128 | 26 |
| | | 9 | 50 | 0 | 25 | 0.46814208 | 0.28900608 | 27 |
| | | 10 | 28 | 22 | 25 | 0.01744128 | 0.93795328 | 28 |
| | | 11 | 1 | 49 | 25 | 0.375 | 0.375 | 27 |
| | | 12 | 25 | 25 | 25 | 0.375 | 0.375 | 27 |
| | | 13 | 25 | 25 | 25 | 0.375 | 0.375 | 27 |

The simulation parameters are specified in the upper left-hand corner of Table 5.1.

Note that “E” and “Alpha” correspond with ε and α , respectively. Furthermore, we state here that the probabilities for a randomly selected agent demanding or supply the asset were calculated by setting the number of agents whose $x_t > P_t$ or $x_t < P_t$ over the total number of agents, N . Thus, the accuracy of our probabilities is maintained despite the changing distribution of X_t . The Excel computation codes can be found in the Appendix.

The data fits neatly with our expected results, outlined briefly above. At time $t = 0$, $P_0 = 18$ such that the probability of a buy herd “P(Buy)” – *by the fifth agent* – is greater than

.5. In this particular case the probability is 1, as the minimum x_0 is roughly 20 (see Figure 5.3). This is also evidenced by $D_0 = N$. Thus while a herd may have occurred, no agents update beliefs. The excess demand relative to Q_0 calls for the market maker to raise prices by α , or 1, at $t = 1$. By time $t = 2$, an updating of beliefs begins. Despite the updating process, we see the probability of a buy herd diminish. This is specific to instances where ε does not exceed α significantly. In a trial case where $\varepsilon = 3$, holding α constant, the probability of a buy herd remained stable at 1 for all $t > 0$ and the price P_t rose linearly without restraint. A high ε however is unlikely, as agents would generally allow only for some small positive ε . It is less clear whether $\varepsilon > \alpha$; if we assume price revisions per round to be a fine signal, the value of α must be sufficiently small. In a trial case where $\varepsilon = \alpha = 1$, the probability of herding diminished to less than .5 by $P_t = F = 25$, and according $P^* = 25$.

The sustained demand seen in Table 5.1 for all $t < 10$ is a product of the updating of beliefs due to herding. We find that for time $t = 9$, the probability with which agents herd falls below .5, where the price $P_t = 27$. As the market demand continues to exceed the outstanding share quantity at time $t = 9$, the market maker raises prices. Note that at $t = 10$, the market demand falls drastically as agents no longer update their beliefs. Specifically, we can see that demand fell by roughly half, representative of agents whose $x_0 < F$. These agents all dispose of the stock simultaneously, as their updated beliefs converged to $P_t + \varepsilon$. Figure 5.5 (below) illustrates the corresponding “flattening out” of the lower half of the demand and supply schedules at time $t = 9$. Interestingly, as the posterior beliefs of all agents characterized by $x_t < F$ converged, prices do not see a similar precipitous fall; by lowering the price only slightly the market maker raises demand significantly enough to bring the market to equilibrium. Note that the equilibrium price $P^* = 27 > F = 25$. The model successfully demonstrates the existence and achievement of a stable equilibrium where the market price exceeds the fundamental value.

Along with Figure 5.5 below, we have Figure 5.6, a summary chart of Table 5.1. As with Table 5.1, the simulation parameters have been listed. The time period over which Figure 5.6 extends is greater than that in Table 5.1 to clearly illustrate the stability of the equilibrium market price achieved. Additionally, when looking at Figure 5.6, it is important to recall that the market is brought to equilibrium where $D_t = Q_0 = 25$. A line for Q_0 , and the probabilities of buy and sell herds have been excluded from the summary chart so that the demand, supply and price movements can be clearly seen.

Figure 5.5: D_9, S_9 ($N = 50, F = 25, \sigma = 3, t = 9$)

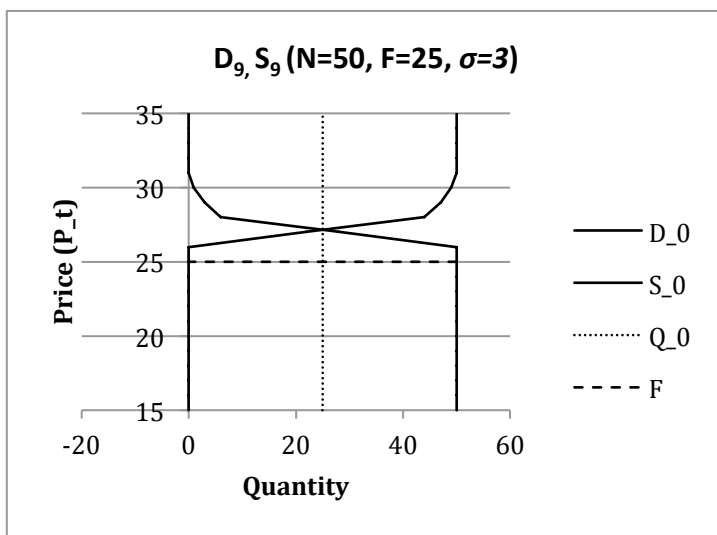
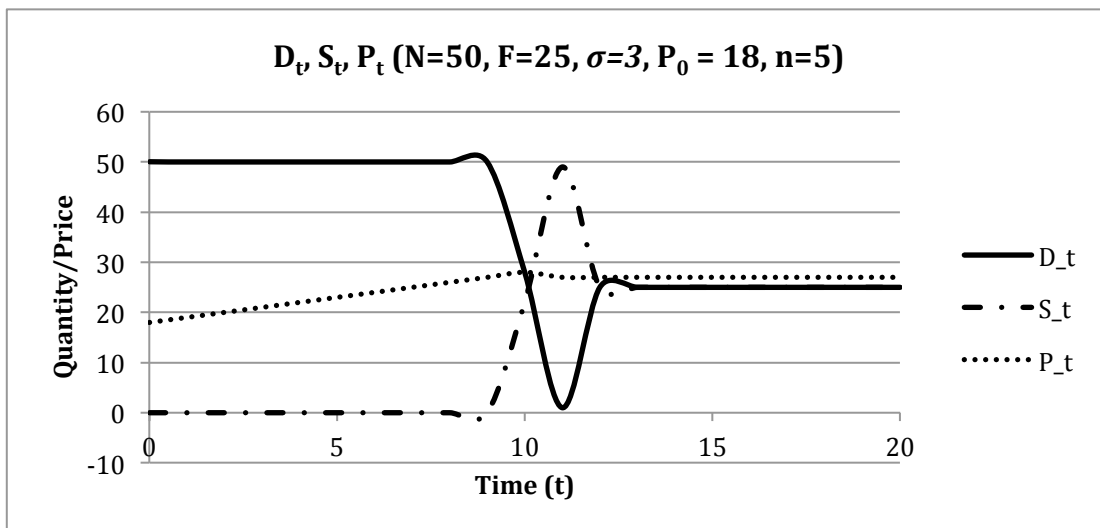


Figure 5.6: ACM Simulation Summary ($N = 50, F = 25, \sigma = 3, P_0 = 18$)



Here we conclude our construction and analysis of the ACM. We now progress the end discussion of the chapter.

ACM End Discussion

As in the BCM and PCM, the ACM successfully demonstrates the existence and achievement of a stable equilibrium where an asset is mispriced relative to its fundamental value. However, as with our earlier models, several points worth mentioning remain. In particular, two assumptions must be discussed:

1. Limiting the probability of herding to the fifth agent. To be sure, allowing only for herding by the third agent is restrictive. Because herding probabilities increase at a decreasing rate with n , for a given P_t (see Figure 5.2), there is no clear reasoning for what exactly the upper limit on n should be. In this model we set $n = 5$, as it represents a relatively small share of the total number of agents. This is important, because we use the probability of herding to determine whether or not all [updating] agents should update – thus, we escape relying on any endogenous sequence of agents. Regardless, a critique of the assumption itself is warranted.
2. Allowing for a constant α across price adjustments by the market maker. As mentioned earlier, prices may alternatively adjust as function of the amount of excess demand or supply in the market. Seemingly, the chief issue is that in a model where the market maker adjusts prices at the end of the round, there is a lacking distinction between the magnitude and rate of price changes. This distinction is perhaps clearer in the practice.

Lastly, we brief what may be a limitation or advantage to this chapter: simulating rather than mathematically deriving the existence of equilibria. The inherent risk is that some equilibrium, which may exist, is not identified. Furthermore, we lack the generality of results

seen in previous chapters, which may prove more useful in practical application. That said, the demonstration allowed the chapter to neatly handle a quantitatively tedious scenario and in doing so, support the models illustrated in previous figures. Here, we end our discussion, closing Chapter 5, and proceed to the conclusion.

Chapter 6: Conclusion

Summary Of Results

The intended purpose of this paper has been to demonstrate the existence and achievement of stable equilibria where an asset is mispriced relative to its fundamental value. A general model was developed, within which these equilibria were sought out. The model – or game – consists of a finite number of agents making investment decisions on an IPO, where the quantity of stock is assumed fixed. A market maker attempts to match demand with the number of outstanding shares (clearing the market) by adjusting the market price. Agents make trades through the market maker in a random, sequential order and price adjustments occur after all agents have made a single trade. Several variations on this general model were developed.

The Base-Case Model (BCM) assumes the beliefs of all agents on the fundamental value of the stock to be fixed, and further that all trades are private information. Accordingly, the game is essentially a simultaneous move one. The model demonstrates that a correct pricing of the asset depends, *ceteris paribus*, on an optimal quantity of stock in the market. Under assumptions of an equity-maximizing firm, the BCM model nicely demonstrates that the optimal quantity of stock is offered only by chance. The model established a base for initial mispricing of an asset in the market.

The Price-Case Model (PCM) begins assuming the optimal quantity of stock to prevail in the market place, and holds the assumptions of the BCM constant with the

exception of allowing for agents to update beliefs. Specifically, agents update beliefs simultaneously at the end of a round, based on price movements in the market should they contradict prior beliefs about the fundamental value. The PCM demonstrates that in instances where an asset is initially correctly priced, the equilibrium price in the limit is the fundamental value itself. Allowing for an initial mispricing, as supported by the BCM, the PCM reveals the equilibrium price to be inline with the fundamental value only if price revisions are sufficiently fine. The convergence of beliefs in the model generally allows for any number of price levels to clear the market.

The Action-Case Model (ACM) holds the assumptions of the PCM, with the exception that agents do not update beliefs on price movements, but rather the actions of other agents; the assumption that trades are private information is abandoned. Specifically, agents update beliefs sequentially and within the round, before price movements. Investment decisions act as public signals to all following agents and the market maker continues to adjust the price at the end of the round. The model successfully demonstrates the achievement of an equilibrium price above the fundamental value, given a number of assumptions over the probabilities of herding and the relative magnitudes by which the prices and beliefs are updated.

All models seem effective in demonstrating potential persistent asset mispricing, the PCM perhaps less so than the BCM and ACM. The general game structure and underlying mathematics holds well across models and are all the more supported by the simulations conducted for the ACM. Accordingly, the efforts of the paper have been met with success. That said, and as briefed in the end discussions of each model, there are without doubt limitations to each model and valid critiques on the assumptions made throughout. The following section discusses areas for further work, beginning with a discussion on the practical application of the models generated in this paper.

Areas For Further Work

An analysis of the practical application of this paper lends itself well to generating a discussion on areas for further work. In considering the calculability of the general model, we find the underlying mathematics to be rooted in the classical assumptions of econometrics. Almost all measurements used throughout the model can be at the very least estimated, given a sufficiently large sample of estimates on the fundamental value of a stock. Two potential exceptions may be ε and α , the degree of updating and the magnitude of price adjustments respectively. While these values may not be grounded in econometrics per say, they may be estimated by other means. In this respect, we find room for further work: identifying real world estimators for the variables used throughout the models.

Theoretically, whether these models have any practical application is less certain. The general assumption of a fixed quantity of shares may or may not hold in the short run. Considerations must accommodate for instances where firms intentionally buybacks shares to create shareholder value. The assumption that firms attempt to maximize equity gain rather than price fairly is similarly questionable. A further exploration into IPO strategy could lend itself either to support or weaken this paper. Regardless, the area presents itself as a natural extension of this paper.

The game structures themselves for the PCM and ACM are likely weak with respect to practical applications. In real-world scenarios agents do not update beliefs perfectly with prices as in the PCM, nor do they have perfect information on the investment decisions of all agents. A related critique is that symmetric signals, while useful in modeling, are perhaps an overly simplified expression of reality. Two areas for further work are then identified. Firstly, the development of a game that draws both from the PCM and ACM, with agents' information and beliefs bounded more so than in the models presented here. Secondly, the introduction of a more complex herding mechanism to the game just proposed.

As mentioned briefly in the results summary, a weakness of the ACM is its lack of mathematically derived equilibrium conditions due to quantitative complexities. Work in establishing these equilibria, and consequentially adding to the generality of the model, is another significant area for further works. That said, the importance of this extension is perhaps second to that of integrating the PCM and ACM.

Lastly, any exploration into the theoretical application of these models to explaining asset bubbles may prove to be a fascinating extension of this work.

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Appendix: Excel Code

Raw Data Range: P2:BM50 (Only P2:BM22 was used in Table 5.1 and Figure 5.6 in Chapter 5).

1. Demand
=COUNTIF(P2:BM2,">"&I2)
2. Supply
=COUNTIF(P2:BM2,"<"&I2)
3. Probability For Buy Herd
=((COUNTIF(P2:BM2,">"&I2)/B\$1))^2+2*((COUNTIF(P2:BM2,">"&I2)/B\$1))^3*
((COUNTIF(P2:BM2,"<"&I2)/B\$1))
4. Probability For Sell Herd
=((COUNTIF(P2:BM2,"<"&I2)/B\$1))^2+2*((COUNTIF(P2:BM2,"<"&I2)/B\$1))^3*
((COUNTIF(P2:BM2,">"&I2)/B\$1))
5. Updating Beliefs (Herding)
=IF(\$G2>0.5,IF(P2>\$I2,P2,P2+\$B\$3),IF(\$H2>0.5,IF(P2<\$I2,P2,P2-\$B\$3),P2))
6. Price Adjustments
=IF(D3>F3,I2+\$B\$4,IF(D3=F3,I2,I2-\$B\$4))