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Sample Study of Economic Ordering Quantities

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by

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I. Introduction

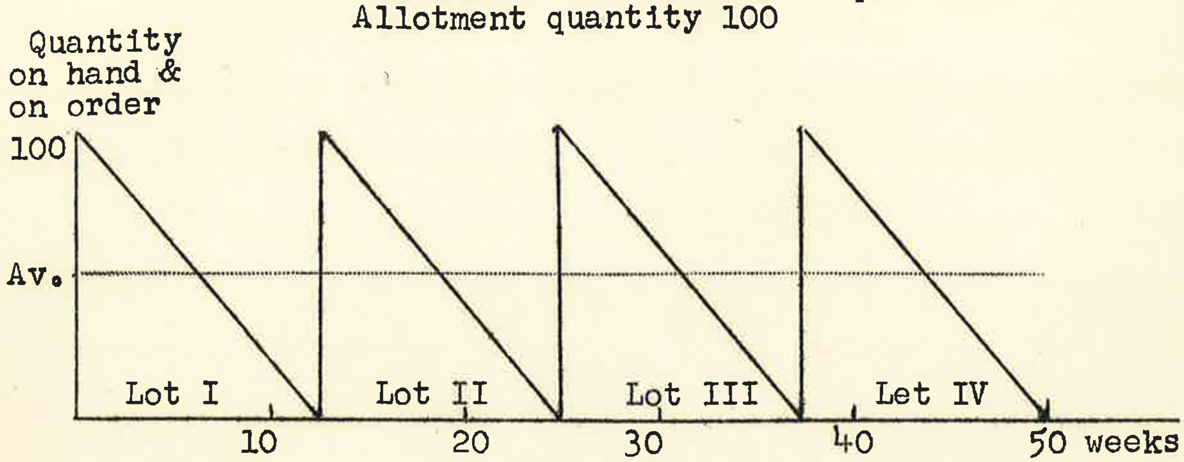
This report investigates the savings available to Brown and Sharpe by adopting an Economic Ordering Quantity System rather than the present Ordering System for three lines of parts used in the 00 Screw Machine. This report therefore deals with but a small part of the total ordering and scheduling problem. The result of this investigation indicates that substantial savings are available to Brown and Sharpe by changing its ordering and production technique.

II. The Alternative Ordering Systems

The Allotment Ordering System does not separate parts ordering and the ordering of the assembly of machines. Under the Allotment Ordering System a management committee decides that a specific number of machines are to be assembled beginning at a date determined by the lead time. During this lead time, the required number of each part for the specific number of machines to be assembled are produced. Hence if things work smoothly, an inventory of parts of a size determined by the allotment quantity is on hand at the date of the assembly of the first machine. Assuming an even rate of assembly of machines, this inventory of parts is drawn down at a constant rate during the assembly period. The present calculations are based upon an annual production of 400 machines, and an allotment ordering quantity of 100 machines. Hence over the year, on the average, the inventory of parts is sufficient for the assembly of 50 machines, assuming a smooth assembly of the 400 machines. This is illustrated in diagram I.

Diagram I

Allotment Ordering System Inventory
400 machines assembled at a rate of 8 per week
Allotment quantity 100

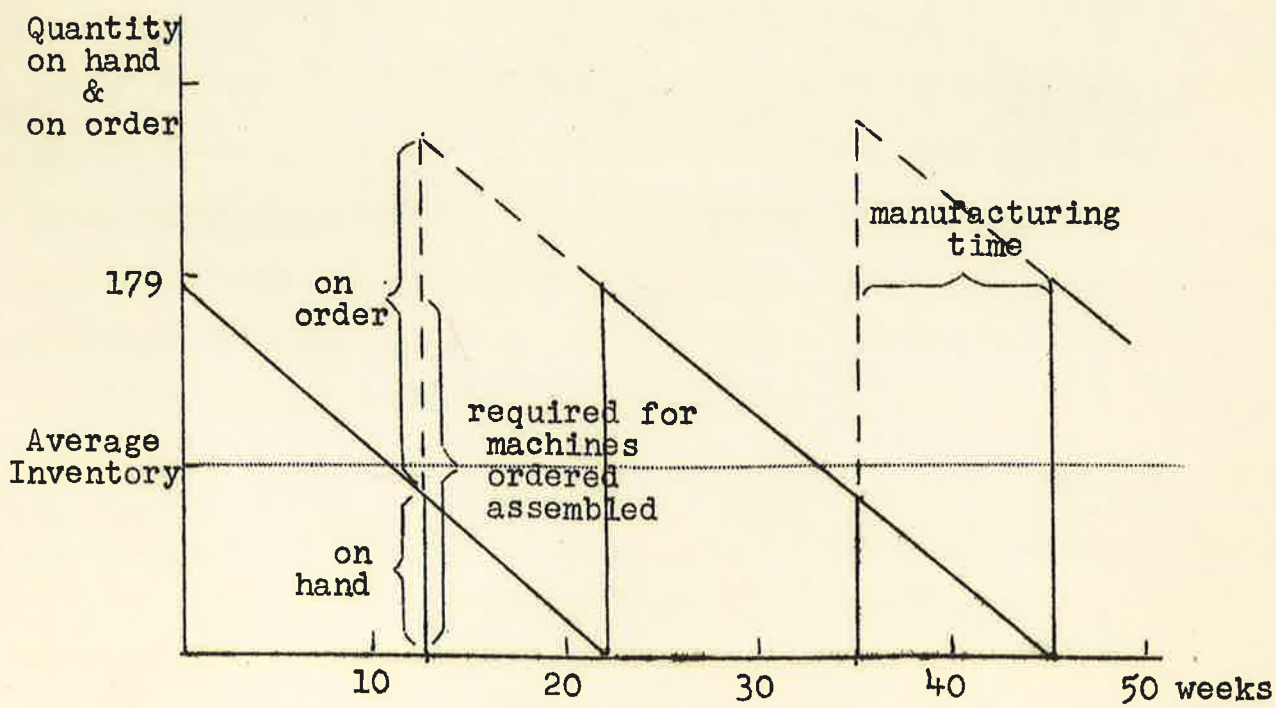


The alternative ordering system investigated, the Economic Ordering Quantity System, separates parts ordering and the ordering of the assembly of machines. The rate of assembly of machines is determined, as under the allotment system, by a management committee. The machines are assembled by drawing the required number of each part out of an inventory. For each part, an Economic Ordering Quantity, which depends upon a set of factors to be specified later, is determined. At the date upon which the existing inventory is just sufficient (allowance for contingencies is made by a safety factor) to supply the number of parts required for the assembly of machines during the manufacturing time of the part, the part is ordered in the Economic Ordering Quantity.

As the members of the Brown and Sharpe management committee we conferred with stated that the present allotment system will be maintained for major parts, the lead time between the decision to assemble and the beginning of assembly will be the major parts fabricating time. It is expected that for the 00 Screw Machine this will be four months. If we assume a one month assembly period, the quantity of each part that will be required over a five month period will be known. Hence except for parts with a production period in excess of twenty weeks a part will never have to be ordered solely on the basis of an extrapolated rate of assembly. This ordering system is illustrated in Diagram II using data for part number 42-16239. The Economic Ordering Quantity is 179 units and the manufacturing time is 9-1/5 weeks.

Diagram II

Economic Ordering System Inventory for Part #42-16239
400 machines assembled at the rate of 8 per week



Economic Ordering Quantity 179
Manufacturing Time 9-1/5

Assuming that the machine is being assembled at the rate of eight per week, and there is a twenty week lead time between ordering and assembling, only 19 of the pieces on hand at the 0 week are not required for machines already ordered. In the 13th week, when 75 pieces are on hand, 179 additional pieces are ordered. Pieces on hand and on order will total 254 of which 160 will be required for machines whose assembly has already been ordered. This indicates that even for a piece with a rather long manufacturing time, the risk of obsolescence applies to only a portion of the pieces of any part on hand or on order.

III. Nature of the Study.

This study is a sample study of costs under alternative ordering systems. The sample used is a random sample of 10 parts from each of three production lines. The elements of the sample were selected by numbering each of the parts and then selecting the sample by use of a table of random numbers. The average savings for each production line is computed, and an estimate of the total savings from each line is made. In addtion two estimates of the overall savings available are made: one weighting the line averages by the number of pieces, the other weighting the line average by the relative cost of a run of 210 pieces for the 00 Screw Machine through the line. Confidence limits are established for each estimate made.

IV. The Relevant Formulas:

The Economic Ordering Quantity for each part is that ordering quantity for which the total cost of a given annual requirement is minimum. Total cost is the sum of the following:

1. ordering costs
2. set up costs
3. material costs
4. manufacturing costs
5. inventory carrying costs

Two things are to be determined: the economic ordering quantity and the difference in total costs between using the economic ordering quantity system and the present allotment system.

The symbols used:

A = annual requirement of the part

Q_E = economic ordering quantity

Q_A = allotment ordering quantity

O = costs per order

s = costs per set up

m = material plus manufacturing costs per unit of output

I = inventory carrying costs (as a % of the value of inventory)

p = safety stock

(1) Economic Ordering System Total Costs =

$$TC = \frac{A}{Q_E}(O + s) + Am + \left(\frac{Q_E m + O + s}{2}\right) I$$

$$\frac{dTC}{dQ_E} = -\frac{A}{Q_E^2}(O+s) + \frac{mI}{2} = 0$$

(2)
$$Q_E = \sqrt{\frac{2A(O + s)}{mI}}$$

$$Q_E = \sqrt{\frac{2A(O+s)}{mI}}$$

(3) Allotment Ordering System Total Costs =

$$\frac{A}{Q_A}(O + s) + Am + \left(\frac{Q_A m + O + s}{2}\right) I$$

(4) Savings = Allotment System Total Costs - Economic Ordering System Total Costs =

$$(Q_E - Q_A)\left[\frac{A(O + s)}{Q_E Q_A} - \frac{mI}{2}\right]$$

The above estimate of the savings is on the basis of no precautionary stock of parts on hand. It is customary in an economic ordering quantity ordering system to carry some safety stock on hand. The estimate of the safety stock used was the quantity that would be consumed in one quarter of the manufacturing time given in weeks. Using a 50 week work year for each part the

(5) p = Safety Stock = 1/4 A/50 x Manufacturing time in weeks.

This is a very large safety margin and in a smoothly running production process it can be reduced significantly. This safety stock is valued at the manufacturing costs, and multiplying by inventory carrying costs yields the costs of carrying a Safety Stock. If a safety stock is carried the

(6) Savings = Total Cost Allotment System - Total Cost Economic Ordering System - Safety Stock Carrying Costs =

$$(Q_E - Q_A)\left[\frac{A(O + s)}{Q_E Q_A} - \frac{mI}{2}\right] - p\ m\ I$$

V. Nature of the Data and the Limiting Assumptions.

Little of the information needed to determine the Economic Ordering Quantity and the savings available to Brown and Sharpe from changing its ordering technique is available with such precision that we do not have to inquire as how variations in their value would affect the results obtained. Hence the specific assumptions made for each item, and the source of the data will be discussed.

Annual Requirement: It was assumed that 400 "00 Screw Machines" would be assembled each year. This assumption was in part based upon sales in recent years; in part it was adopted as a convenient number to use. The Annual Requirement for each part is the number of times the part is used in the "00" times 400. No allowance is made for the "sales" demand for a part. In actual application of the Economic Ordering Quantity system allowance would have to be made for the sales demand. An increase in the annual requirement increases the Economic Ordering Quantity and would tend to increase the available savings more than proportionately to the increase in the annual requirement.

The Allotment Ordering Quantity is 100 times the number of times each part is used in the machine except for part number 42-16362. For this part current practices would split the lot of 100 pieces into two lots of 50 pieces each; hence an allotment quantity of 50 was used for this part.

An increase in the Allotment Ordering Quantity, for a given annual requirement of the part, would decrease the savings resulting from adoption of an Economic Ordering Quantity system, up to the point where the allotment quantity equals the Economic Ordering Quantity. Further increase in Allotment Quantity would result in higher annual costs than with the Economic Ordering Quantity system. In the sample study made the Economic Ordering Quantity was smaller than the Allotment Quantity for two parts.

Ordering Costs: the cost of processing an order for a part was not available. Part ordering is so intertwined with the ordering of the assembly of machines in the present organization of paper work that their separation was deemed to be too difficult for the present experimental study.

The inclusion of ordering costs would increase the size of the Economic Ordering Quantity and would increase the estimate of the savings that could be realized by shifting to an economic ordering quantity system. In the actual application of the Economic Ordering Quantity technique to Brown and Sharpe, these costs would have to be included.

Set-up costs, material costs and manufacturing costs: the number of hours of set-up costs, the unit manufacturing time in hours and the material costs for each part in the sample was furnished by Mr. Newton.

Both Set-Up Costs and Unit Manufacturing Costs were obtained by multiplying the number of hours required by $6.00 per hour. The $6.00 per hour is the standard costing price for man and machine combinations used by Brown and Sharpe. It was not clear whether or not the $6.00 per hour costing figure allows for the job times being standard times fixed under the incentive system so that a worker can achieve 120% to 125% of the standard work. It is obvious that if the actual price of man and machine hours were different the savings available would change significantly, although the size of the ecomomic ordering quantity would not be significantly changed in our analysis.

Every man and machine combination is priced at the same price, $6.00 an hour. This is obviously not true. The production of some parts require more expensive machines and labor than the production of others. Ideally each machine used in the production process should have its own price per hour, and the set up and manufacturing time of each machine used in the production of a part should be multiplied by this price to determine costs. The refinement of treating set-up and manufacturing time for each part at each machine separately will be necessary in solving the scheduling problem. At such time the pricing of each machine separately may be possible. This greater refinement will tend to increase the Economic Ordering Quantity of those parts whose production requires a long set-up time at expensive machines. It will tend to decrease the Economic Ordering Quantity of those parts which use a large amount of expensive manufacturing time.

Another abstraction made is to consider the manufacturing costs per piece of a part as being independent of the quantity produced in a run. An important element in cost is spoilage. Apart from the question of whether manufacturing time per piece varies with the size of the run, if the spoilage rate for parts in manufacturing depends upon the length of the manufacturing run, then costs per piece would vary with the length of the run. If manufacturing costs per piece decrease with the length of the manufacturing run, then the economic ordering quantity increases and the savings available from adopting an economic ordering quantity system would increase.

Inventory Carrying costs consist of the cost of financing the inventory, the cost of physically storing and handling the inventory and the costs of the risk of obsolescence and spoilage while the part is held in inventory. The cost of financing the inventory was arbitrarily put at 10% by Mr. Gordon. This is high for a firm with the financial position of Brown and Sharpe: the costs of financing the inventory should be some small premium over the rate at which Brown and Sharpe could borrow for inventory holding purposes from banks. As the Economic Ordering Quantity varies inversely with the rate used in determining inventory carrying costs, changes in the borrowing rate would affect the optimum size of an order and the size of the inventory on hand. The use of a fixed 10% figure does not result in such adjustment of the inventory to financial market developments. However, in order to avoid an investigation at this stage into what funds cost Brown and Sharpe, a 10% figure was used for financing inventory.

The parts under consideration are small in size. They are not expensive to handle, they will not normally deteriorate while being stored for the time periods under consideration. Under these assumptions, it was estimated that 5% of the average value of the inventory would cover the storage and handling costs.

As mentioned earlier, it is contemplated that the major parts will still be ordered on the Allotment System. This will result in a long lead period between an order to assemble machines and the actual assembly of the machines. Hence for all except parts with a very long manufacturing time, a significant portion of an order of parts will be used in machines whose assembly had already been decided upon when the part is ordered. There is no risk of obsolescence for this portion of the parts ordered. A 5% obsolescence and spoilage allowance is equivalent to saying that the value of parts made obsolete or spoiled while in inventory during each year will average out to be worth one-twentieth of the average value of the inventory. This seems to be a liberal estimate of the obsolescence risks run by the company. However, 5% was assigned to this element of inventory carrying costs.

The 20% which is used as the inventory carrying costs is therefore a very liberal estimate of these costs for Brown and

Sharpe. A reduction in the inventory carrying costs would increase the Economic Ordering Quantity and would further increase the savings by lowering the cost of carrying the inventory.

In what follows the estimates of the savings available to Brown and Sharpe from use of an economic ordering quantity system are made upon two bases: in one no safety stock is carried in the other a safety stock equal to the estimated demand for the part in one-fourth of the manufacturing time is carried. This is a large safety factor and as no equivalent cost is assigned to the present system, this tends to lower the estimate of savings available through adopting an Economic Ordering Quantity System. The discussions we had with members of the management of Brown and Sharpe indicate that "Crises" occur regularly in both parts manufacturing and the availability of parts for assembly under the present system. No attempt was made to include the costs of such regular events under the present system. In addition, the lead time between the ordering of the machines and the beginning of assembly included under the present system a long tolerance period.

The longer the lead time between machine ordering and machine assembly the greater the inflexibility of the organization. The safety stock is a factor making for flexibility which is not available under the present system and as such is worth something to the organization. As a result of the above factors it is suggested that the true savings available to Brown and Sharpe may even be greater than the amount estimated by ignoring the cost of carrying a safety stock.

In what follows estimates are made of the Economic Ordering Quantity and the Savings available for each of three lines of part for the "00 Screw Machine". To estimate total savings available, two different weighting schemes are used. In one, the average saving per part for each line is weighted by the number of parts in the line. In the second the average saving per part is weighted by the relative cost of the parts that are made on that line, when every part is being made in quantities of 210. The two weighting systems result in considerably different estimates of savings. The "value" weighting schemes would in general be more accurate, but the estimate of total savings available weighted by the number of pieces in each line can be taken as a very conservative estimate of the savings available.

VI. The Findings:

Samples of 10 parts were selected at random from each of Line I, Line III and Line IV and for each part the Economic Ordering Quantity was determined. Table I gives the Average Economic Ordering Quantity for each line and the average quantity ordered under the present allotment system for each line. For lines I and IV the Average Economic Ordering Quantity is 1.6 the Average Allotment Quantity, for line III the Average Economic Ordering Quantity is 2.3 the Average Allotment Quantity. Because the average set-up costs are higher for line III, and the high ratio between economic ordering quantity and the allotment quantity, the savings on set-up costs are much higher for this line. The charges against the savings in set-up costs are given in the average increase in inventory carrying costs and in the average

Table I

Average Economic Ordering Quantity and Average Cost Items
Lines I, III and IV

"00" Screw Machine

	I	III	IV
Average Economic Ordering Quantity	191.0	217.0	189.0
Average Order: Allotment System	120.0	95.0	120.0
Average Set-Up Costs $	27.8	80.9	42.2
Average Savings: Set-Up Costs per year $	44.1	208.1	61.8
Average Increase in Inventory Carrying Costs per year $	25.2	85.8	34.7
Average Safety Stock Carrying Costs per year $	10.7	28.5	29.7

safety stock carrying costs rows in Table I. Note that including the costs of carrying the safety stock results in the Allotment System being cheaper for Line IV. However, a slight reduction in the average safety stock, or a slight reduction in the inventory carrying charges would result in substantial savings being available from this line.

As Table II shows, the largest potential savings is in Line III, the Gear Line. If no safety stock carrying costs are assessed, the potential savings is $11,494.32 ± 2,307.70*.

* The meaning of the confidence limits is that if we assume normality in the distribution of the savings, we can assert with a confidence of 2/3 that allowing only for sampling errors the true savings will lie within the range $11,494.32 ± 2,307.70 for Line III.

Table II

Savings Available - Lines I, III and IV
Weighting by Number of Pieces in the Line

Line	I	III	IV
No. of Pieces in Line	94.0	94.0	112.0
Savings/pieces: no safety stock $	18.87	122.28	27.15
Savings/piece: safety stock $	8.18	93.77	-3.77
Estimated Savings: no safety stock $	1773.78	11494.32	3040.80
± Standard Error of Estimate $	±392.45	±2307.70	±723.60
Estimated Savings: Safety stock	768.92	8813.06	-422.02
± Standard Error of Estimate $	+523.40	±1868.7	±959.84

If estimated costs of carrying the safety stocks are assessed the predicted savings from changing the ordering system for Line III will be $8,813.06 ± 1,868.7, with the same confidence as stated previously.

In addition to an estimate of total available savings arrived at by weighting each line by the number of parts in the line, an estimate of the total available savings was made based upon the total cost (in hours of making a run of 210 pieces of each part in each line). The cost in hours of Line I was 14,442, of Line III 24,544 and of Line IV, 17,544, or in percentage terms the weight of Line I was 25.55%, Line III 43.42% and Line IV 31.03%. Table III presents the estimated total savings available from changing the ordering systems for Lines I, III and IV using both set of weights and with and without provision for a safety stock. The most favorable estimate of the savings available is approximately $20,000 per year. On a run of 400

Total Estimated Savings
Lines I, II, III "00" Screw Machine

	Weighting by No. of Pieces	Weighting by relative costs of producing 210 pieces
No Safety Stock	16308.90	19902.00
± Standard error of Estimate $	±2384.30	±2748.60
Safety Stock	9159.96	12488.90
± Standard error of Estimate $	±2085.30	±2167.00

"00 Screw Machines" per year this is equal to a savings of almost $50.00 per machine. Under the set of assumptions least favorable to the Economic Ordering Quantity System, it is estimated that somewhat more than $9,000 will be saved. On a run of 400 "00 Screw Machines" per year, this is equal to a savings of more than $22.50 per machine.

VII. Recommendations:

1. Line III, the Gear Line, should be placed on an Economic Ordering Quantity system as soon as possible. The sample study indicated that very large savings could be made on this line, so large that the firm would save significant sums even if a large safety stock is carried. The scheduling of the transition to the Economic Ordering Quantity System should not

be difficult for this line.

2. The savings shown to be available on Line III are so large, that a systematic investigation of the savings available by adopting an Economic Ordering Quantity system should be undertaken for every part ordered in quantity during the year.

3. The savings available from changing the ordering system for Line III were so large that it is suggested that the present plans to continue ordering the major parts by the present allotment system be reviewed. A shift to an inventory of major parts would increase the flexibility of operations but would also increase the risk factor due to obsolescence.

4. The problems involved in the scheduling of production under an Economic Ordering Quantity system should be studied. The immediate adoption of the Economic Ordering Quantity system for Line III will provide the company with a "pilot plant" which would aid in the study.

5. A study of the costs of ordering should be undertaken so as to permit greater precision in the determination of the Economic Ordering Quantity.

6. A study of the costs of financing and handling inventory is needed, again in order to give greater precision to the determination of the Economic Ordering Quantity.