Gaussian Cosmology: A New Model for the Accelerated Expansion of the Universe

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by

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Abstract

In this paper I lay the groundwork for an alternative model for the contemporary expansion of the universe. The current model states that the universe is growing exponentially due to the vacuum of space pulling on it. This model states that the growth rate of the universe is linear in time. However in 1998, researchers suggested that the expansion rate of the universe is accelerating. This means the universe is not expanding logarithmically, not linearly. In my model, I lay the groundwork for future research and suggest that the universe develops in time in accordance to a Gaussian scale factor and gives a contemporary inflation. This theoretical change has many consequences. It may seem that the universe should continue to expand at an accelerating rate forever; however, my paper suggests that after expanding the universe will eventually slow, stop and reverse, and compress back into a single point after expanding. Second, my paper claims that a scalar field driven by dark energy is the causal mechanism for the universe's inflation. This scalar field is homogeneous and grows linearly in time. Finally, my paper claims that primordial inflation fits within my model, as it can predict CMB temperature fluctuations. My model challenges the theory that vacuum energy drives the universe's current expansion and that quantum perturbations drive primordial inflation.

Acknowledgements

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Contents

1 Introduction and Brief History of Cosmology

1.1 Einstein's Equations

In this section, I will introduce general relativity and cosmology and I will also provide a short history of the development of cosmological ideas pertaining to this paper. I will begin with a discussion of general relativity. The very nature of spacetime has been questioned since the publication of Albert Einstein's breakthrough theory of general relativity. The principal contribution of the theory of General Relativity was the idea that objects bends the space and time surrounding them and energy and mass were equivalent. The theory has allowed us to make leaps in our understanding of the natural universe beyond our planet. It has allowed us to make theoretical predictions of exotic objects such as black holes, and it has challenged the Newtonian absolutist paradigm of space, time and gravity. As powerful as this theory is, researchers who work on it know that there are several unsolved problems in the field; such as, what is driving the accelerated expansion of the universe and what theoretical model could best predict this expansion? This is the problem my project addresses. I will propose a solution and see how it fits into the existing literature. I aim to build on the existing literature in the subject, as this is only the beginning of a new way of doing cosmology.

To begin, I will start with the famous Einstein equation, relating the curvature of spacetime $G_{\mu\nu}$ with the stress-energy tensor $T_{\mu\nu}$. The equation states that

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1}$$

where G is Newton's Universal Gravitational constant and c is the speed of light in vacuum. This equation uses the stress energy tensor to show that spatial curvature, $G_{\mu\nu}$ is a direct result of matter, represented by $T_{\mu\nu}$. Matter bends spacetime such that the shortest path taken between two moving objects changes from a straight line to a curved path. This effect can be observed in the bending of light when the path that light takes as it travels around an object, or when a planet orbits its host star.

The stress energy tensor is a mathematical encapsulation of energy density and pressure. My paper describe the universe as a perfect fluid,[1] that was the and first term of the stress energy tensor, T_{11}, represents the universe's energy density, and the remaining elements, T_{22}, T_{33}, and T_{44} describe its pressure.

Depending on which model of spacetime one uses, and based upon certain conditions, one can receive accurate predictions for observed phenomena like black holes, cosmological inflation, the bending of light due to gravity, the precession of Mercury, etc. In the following subsections, I will describe in detail what has been said on the subject and the existing models for the expansion of the universe, its history, and where it is now. I will also introduce the theory of primordial inflation because, even though treated as a separate problem, has relevance to the development of cosmology and provides the context for the second half of my argument.

1.2 Robertson-Walker Cosmology

After Albert Einstein published his theory, as early as the 1920s, the standard cosmological framework of the universe developed. The standard cosmological tool developed at this time was the Robertson-Walker(RW) metric, the current, most-

[1] This is a system that assumes that the constituent particles interact.

commonly used model to describe a changing universe in time. Various solutions have been proposed that can fit the Robertson-Walker model to describe various phenomena. This paper also will assume my own variant to the Robertson-Walker metric. The proposed solution that does work on the system is added in to the formulation after $G_{\mu\nu}$ has been worked out and the Einstein equation solved, and both occupy a pseudo-Riemannian manifold.[2]

Because equation (1) is so general, cosmologists in the 1920s developed an exact solution for a metrical framework of the universe. That solution has become the standard cosmological model that is still used today. A generalized FRW metric is [18], [20], [25], [6], and [9],

$$ds^2 = -a^2(t)(dx^i dx^i) + dt^2, \tag{2}$$

where $a(t)$ refers to any function of time. Now in equation (1) I did not specify either the Einstein Tensor's nor the Stress-Energy Tensor's exact form. To do this, I would make certain assumptions about the universe beforehand. I will define them now to set up the equations of the motion of spacetime which are called the Friedmann equations. Both the RW metric and Friedmann Equations assume that the universe acts like an isotropic[3], homogeneous[4] perfect fluid[5], that exist in a vacuum, Λ, pulling them apart. The form of equation (1) becomes

[2]This leaves the possibility open to have their geodesics solve timelike, null, and spacelike curves which is instrumentally helpful in solving various astronomical problems such as gravitational lensing.

[3]density of particles are the same in all directions

[4]particles are indistinguishable

[5]particles are interacting

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{3}$$

I have introduced two new objects $R_{\mu\nu}$, the Ricci tensor and R which is the Ricci curvature scalar. The Ricci Tensor is a contracted rank two tensor of the Riemannian Curvature tensor, derived from the Christoffel symbols, symbols which are in turn derived from the metric and its various partial derivatives of changing variables. The scalar is simply $R = R_{\mu\nu}g^{\mu\nu}$ which is a scalar quantity. These definitions will be revisited later when I compute them for my new metric. In both the Friedman equations and in the de Sitter model, Λ is used to represent the vacuum of space pulling on the universe making it expand. It was the de Sitter solution that explained the expansion of the universe to a convincing degree. Before I go introduce the de Sitter Model, however, I would like to present the following Friedmann equations, after computing the necessary details from the Einstein equations:

$$\begin{gathered} \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho + \Lambda c^2}{3}; {}^6; \\ \left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}. \end{gathered} \tag{4}$$

Unfortunately this form is not invariant. I have make a new definition for the Hubble constant[7] as $H = \frac{\dot{a}}{a}$ and replacing

$$\begin{gathered} \rho \to \rho - \frac{\Lambda c^2}{8\pi G}; \\ P \to p + \frac{\Lambda c^4}{8\pi G}. \end{gathered} \tag{5}$$

Then we can simply the other equation and receive the following invariant set of Friedmann Equations

[7]Or parameter if you want it to depend on time, which is allowed in this sort of framework.

$$\begin{aligned} H^2 &= \tfrac{8\pi G}{3}\rho \\ \dot{H} + H^2 &= -\tfrac{4\pi G}{3}\left(\rho + \tfrac{3p}{c^2})\right) \end{aligned} \tag{6}$$

If treated as a system of differential equations with an unknown a, then

$$a(t) = a_0 t^{(\frac{2}{3})}. \tag{7}$$

The scale factor has an explicit beginning at $t = 0$. This suggests that the universe had a definite origin, a Big Bang, and that the universe has always had decelerated growth because the scale factor's second derivative is always negative. The metric would later prove insufficient to describe Hubble's universe where his data suggested that the universe was expanding, detailed in the following subsection.

1.3 The Expanding Universe and the de Sitter Model

Now that I have introduced General Relativity: I can go on to cosmology via inflation, and in the subsection following talking about the standard cosmology developed in this era. In 1929 Edwin Hubble observed that the distances from galaxies were directly proportional to their redshift. In his paper, Edwin Hubble, when measuring the luminosity of various extra stellar objects determined that data he collected indicated "a linear correlation between distances and velocities, whether the latter are used directly or corrected for solar motion, according to the older solutions."[13] When measuring the spectra of light emitted from the stars and nebulae, Hubble discovered that it had a slightly lower frequency[8] than expected. To explain this, he had to make corrections given to him by Einstein's Special Theory of Relativity. In

[8] redshifted

accordance to the Special theory of Relativity, the frequency of light emitted from a source moving away appears to decrease, or redshift, to an observer measuring it. This phenomena is known as the relativistic Doppler effect. This same principle can be applied to moving extrastellar objects moving away from us. This is not cosmology; Hubble's discovery was based in astrophysics. Hubble asked for a cosmological solution.

Since "distances of extra-galactic nebulae depend ultimately upon the application of absolute-luminosity criteria to involved stars whose types can be recognized," Hubble added these relativistic corrections, and plotted his data and found a linear relationship in luminosity measured and the distance of the object away from us. After plotting the measured distances and velocities, below on figure 1, the results of such analysis confirm "a roughly linear relation between velocities and distances among nebulae for which velocities have been previously published, and the relation appears to dominate the distribution of velocities." [13]

Hubble called for geometrical solutions, stating that "the outstanding feature, however, is the possibility that the velocity-distance relation may represent the de Sitter effect, and hence that numerical data may be introduced into discussions of the general curvature of space." [13] The geometrical model that was proposed to solve it was the de Sitter metric, where a simple exponential drives the linear expansion of the universe.

At the time of the metric's development, there were two competing ideas in cosmology to the question, can the universe evolve of time? Did it have a definite beginning? With the Robertson Walker metric, anything was possible since $a(t)$ could theoretically equal a constant, but due to Hubble's discovery, it shook the cos-

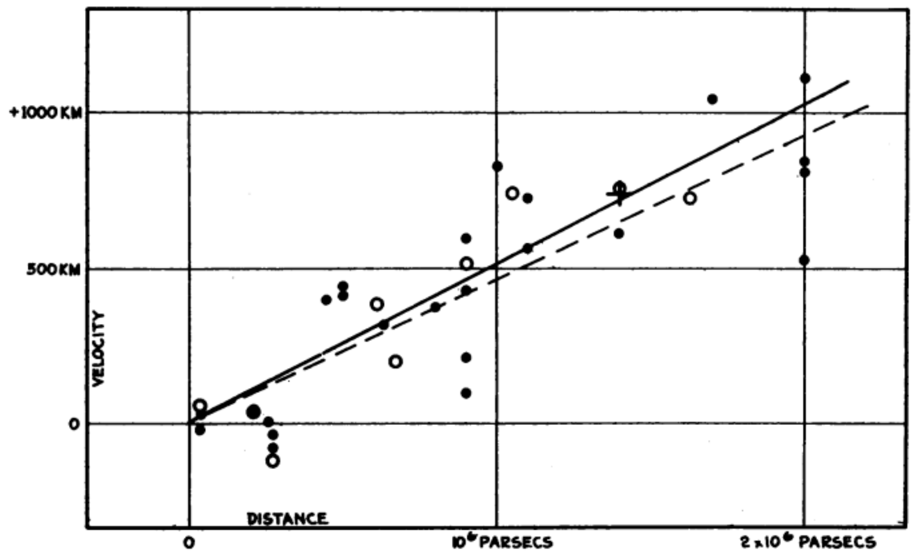

Figure 1: This graph illustrates the “Velocity-Distance Relation among Extra-Galactic Nebulae.” In it, “Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.” [13]

mological discussions even further. The first idea was a Steady State Universe[9] and an expanding universe which had a definite origin, namely the Big Bang. Hubble's data seemed to confirm the latter model of the universe. However, at the time, it also seemed possible that the expansion of the universe was due to quantum mechanical effects yet to be fully understood and did not necessarily *prove* that the expansion began immediately following the Big Bang. However, a successful model that described the inflation was the de Sitter metric where [8], [9]

$$a(t) = a_s e^{H(t-t_s)}. \tag{8}$$

This is a simple exponential in time and H, the Hubble constant for redshift calculations, is related to the cosmological constant, Λ, where[9]

$$\Lambda = 3H^2. \tag{9}$$

This is sufficient to describe the linear inflation Hubble observed, and had been considered as early as 1930 by Tolman [26] and Robertson. [25] However, it is imperfect and over-approximates the redshifts as time continues. Following the addition of the cosmological constant, cosmologists placed densities of its constituent elements of the universe, including the vacuum energy density, into the Friedmann equations. Cosmologists defined and additional density called the critical density, ρ_c as the density in which if observed, would mean that it would be so low that the vacuum energy would become so dominant, that there is no hope for the universe to ever stop expanding. With the current FRW the Hubble parameter is a constant, so if the total

[9]This model is an unchanging model in which even Einstein flirted with the possibility of the universe existing as one; however, in his model, it failed with $\rho = 0$ and $P = 0$.[21]

densities from matter radiation, and from the vacuum to be

$$\begin{aligned} \Omega_m &= \tfrac{\rho_{m0}}{\rho_c}, \\ \Omega_r &= \tfrac{\rho_{r0}}{\rho_c}, \\ \Omega_v &= \tfrac{\rho_v}{\rho_c}, \\ \Omega_k = 1 - (\Omega_m + \Omega_r + \Omega_v) &= \tfrac{K}{H_0^2}. \end{aligned} \tag{10}$$

Ω_m refers to the baryonic matter density, Ω_r refers to the radiation density, and finally Ω_v refers to the vacuum or dark energy density, and Ω_k is the sum of the three densities subtracted from 1 to maintain the fatness of the universe. When the universe is flat, K is equal to zero and the sum of the ratios cancel. The vacuum density takes on the value of the cosmological constant. Then the equations of motion can be derived based on this metric via the Friedman equation[18]:

$$\dot{a}^2 - \frac{8\pi G}{3}(\frac{\rho_{m0}}{a^3} + \rho_{r0} + \rho_v)a^2 = 0. \tag{11}$$

In terms of our Hubble parameter [18],

$$\left(\frac{1}{H_0}\frac{da}{dt}\right)^2 = \frac{\Omega_m}{a} + \frac{\Omega_r}{a^2} + \Omega_v a^2. \tag{12}$$

With this I can insert the current values for each ratio. This is typically the standard equation of motion in de Sitter cosmology. The Ω_v accounts for the vacuum energy, or what is believed to be the vacuum energy, giving the universe the needed push for accelerated inflation. This theory was developed in the 1930s and was challenged later in the 1980s via the1940s possibility of scalar fields, rather than the ratio of matter and vacuum energy driving the expansion. However, before we get there the in 1940s and 60's experienced several experimental developments completely unanticipated at this time that must be considered.

1.4 Cosmology from 1948 to 1981

By 1948 George Gamow had the idea that the static universe model should be thrown out and replaced in favor of a new model. When Ralph Alpher and George Gamow published their paper in 1948, they suggested that with a Big Bang origin, the early universe would have the appropriate conditions to create the correct ratio of hydrogen and helium that is currently in the universe. [11]. This model considered, according to Alan Guth in 1981 in his paper, the *The Inflationary Universe*, an "adiabatically expanding radiation-dominated universe described by the Robertson-Walker metric," [12], and its structure fit with the de Sitter model.

The next major step towards the adoption of the Big Bang Model was in 1964 when two astronomers, Penzias, A. A., and Wilson, R. W, discovered that the extra stellar static picked up by their satellite antennae had "yielded a value of 3.5 K higher than expected." [22] In fact this static was "isotropic, unpolarized, and free from seasonal variations." [22] When mapped out, to an advanced modern analysis, the Cosmic Microwave Background (CMB) has discernible patterns of temperature variations on the millikelvin scale.

With the discovery of the CMB and Hubble's discovery of the expanding universe, the Steady State Model was struck and gave more evidence in favor of a Big Bang. However, Gamow's solution for an adiabatically expanding universe did not solve two key issues that only became highlighted with the discovery of the CMB. This is where Alan Guth proposed his solution to both problems via primordial inflation. The first problem is the horizon problem. Guth explains,

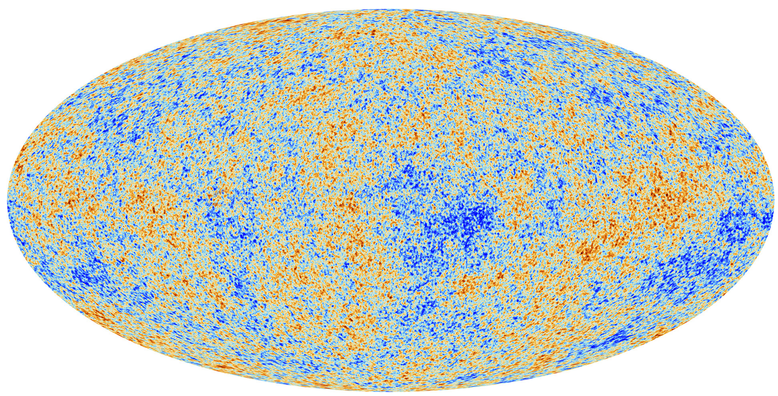

Figure 2: The diagram is the currently the most detailed map yet of "the anisotropies of the Cosmic microwave background (CMB) as observed by Planck." The CMB is an exciting "snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380,000 years old. It shows tiny temperature fluctuations that correspond to regions of slightly different densities, representing the seeds of all future structure: the stars and galaxies of today." [1]

> The initial universe is assumed to be homogeneous, yet it consists of about 10^{83} separate regions which are causally disconnected i.e. these regions have not had time to communicate with each other via light signals. ... Thus, one must assume that the forces which created these initial conditions were capable of violating causality. [12]

The second problem is the flatness problem. This problem relates to the geometrical structure of the universe and its lasting effect on the current energy density of the universe. The spatial curvature term included in the metric assumes the universe is flat. With the idea of the Big Bang now taking hold of the cosmological community, then, how does the Big Bang theory account for the universe's flatness since it is expanding? Why is the current energy density close to the universe's critical density? While related, neither the Big Bang model nor the Steady State model could explain it. The Robertson Walker metric allows for the possibility for the value of k, the curvature parameter of the universe, to be anywhere from -1 to 1[12], so why is it specifically zero? The current energy density of the universe is

$$0.01 < \Omega_0 < 10, \tag{13}$$

where the subscript 0 denotes the present time, having it ≈ 1 "renders the condition unstable". [12] Guth continued,

> Furthermore, the only time scale which appears in the equations for a radiation-dominated universe is the Planck time, $\frac{1}{M_p} = 5.4 \times 10^{-44}$ sec. A typical closed universe[10] will reach its maximum size on the order of

[10] when the geometrical constant, K encoded within the Ricci tensor = 1

> the time scale, while a typical open universe[11] will dwindle to a value of ρ much less than ρ_{cr}. A universe which can only survive 10^{10} only by extreme fine tuning of the initial values of ρ and H, so that ρ is very near ρ_{cr}. For the initial conditions taken at $T_0 = 10^{17}$ GeV, the value of H_0 must be fine tuned to an accuracy of 10^{-55}.[12]

This incredible level of precision required to determine the relationship between ρ, H_0 and T_0, he explains, "must be assumed without explanation." [12]

In addition to the primordial inflation model, in 1973, Steven Hawking proposed the anthropic principle to solve both the flatness and the horizon problem. In in Hawking explained in the beginning of time there were infinite universes created before the Big Bang; but, because each individual universe had different initial conditions, all of them except ours fell apart by creating the incorrect ratios of matter and radiation. By assuming our universe has correct density, $\Omega \approx 1$, our universe has been allowed to exist. [7] As fascinating as this idea is, the anthropic principle is not easily testable. In 1981 Alan Guth proposed a more experimentally verifiable solution to the two problems. This was his theory of primordial inflation. Aland Guth suggested that before the universe came to be, when all of the universe's energy was compressed into a tiny point in space, the soup of particles were constantly changing. Quantum mechanical subtleties dominated, and by its random nature particles were constantly being created and destroyed, and eventually at one point, there was a antisymmetric amount of matter and radiation. Due to the symmetry breaking, the equilibrium was broken and thus caused the energy to dissipate from the singularity. Since matter and spacetime are related, the universe was born with the expanding

[11] when the geometrical constant, K encoded within the Ricci tensor $= -1$

mass and energy. Since the total amount of mass and energy in this universe are finite, the energy density (and average temperature[12] is as a result) lowered and slowed down to values measured for the current temperature today. That way the expansion of the early universe would cool off and over time settle down in one that, even though is expanding, is not accelerating. Unfortunately he could not find "a smooth ending to the period of exponential expansion." [12]

Even though he could not receive a perfect solution, this opened up an unseen investigation of the CMB.

1.5 Quintessence Since the 1980s

After Guth's initial challenge to the de Sitter and Gamow Model, other cosmologists in the 1980s had alternative theories that the universe's expansion due strictly to the ratio of matter, and radiation, and now dark energy in lieu of just the vacuum energy. Cosmologists were considering other possibilities such as scalar fields; since scalar fields have the ability to do work on systems, like in electromagnetism[13], could

[12] From thermal physics, the temperature is the change in entropy with respect to the internal energy of the system, or average kinetic energy of a gas. So if the particles expand, the energy density decreases and so does the temperature.

[13] A scalar field is a mathematical object that is defined at each point in space. Physical quantities can be associated with certain fields, e.g., a magnetic field or a temperature map. The development of quantum field theory sparked the idea that a scalar field drives primordial inflation. It can be shown that a gravitational field acts like an electric field, obeying a sort of "Gravitational Gauss's Law" and Poisson's Field Equations. However, there is no known "Gavitomagnetic" field known. It could be possible I could use a vector potential $\mathbf{A}$ to make a "Gavitomagnetic" four vector $\tilde{\mathbf{A}} = (\phi, A_x, A_y, A_z)$, but that is still a hot topic of research unrelated to this paper.

there be some sort of inflationary scalar field driving the universe's contemporary expansion? This would replace the cosmological constant with something that operates just like it, namely dark energy.

In 1988, Bharat Ratra and P. J. E. Peebles first investigated quintessence in cosmology. They investigated the "the cosmological consequences of a pervasive, rolling, self-interacting, homogeneous scalar field." [23] With the scalar field they were looking at both contemporary and primordial inflation to have their estimates "agree with dynamical estimates of the density parameter, $\Omega_{\mathrm{dym}\approx 0.2}$." [23] In fact, determined that

> The energy density in the scalar field is not appreciably perturbed by nonrelativistic gravitational fields, either in the radiation-dominated, matter-dominated, or scalar-field-dominated epochs. On the basis of this effect, we argue that these models could reconcile the low dynamical estimates of the mean mass density with the negligibly small spatial curvature preferred by inflation. [23]

With this, their work provided evidence that a quintessence model of the universe, that is a model that makes use of the scalar field and dark energy, drives its evolution.

The next two major developments in quintessence happened after the contemporary inflation was discovered in this century, but take the most importance becuase these two models both consider quadratic exponentials. Even though these models consider quadratic, exponentials, they are not Gaussian exponentials. This means their quadratic terms are *positive* to model the extremely fast primordial inflation the universe experienced. Linde's model today is pretty much the reigning model of primordial inflation.

Andrei Linde, in his 2007 paper *Inflationary Cosmology* [14], built off of Guth's and Peebles' model to set out to provide a quantum-mechanical evolution of the early model under the influence of a scalar field. This is important because Linde also gives a brief history of the theoretical developments in the 1980s where cosmologists believed that there was a scalar field that was caused the universe's primordial inflation. This field was created from the various quantum mechanical perturbations and uncertainty that was happening at the time. This inflationary field ϕ would give the universe a push, would give the universe a potential energy, and would give us a causal mechanism for the changes in the density of matter and energy over time. Linde's model incorporated the vacuum energy into his field.

By first assuming a quadratic potential, $V(\phi)$, it had allowed for the possibility of a linear, time-dependent scalar field ϕ whose origins were a result of quantum perturbations. He considered a polynomial potential of every order, but reduced the model to a simple quadratic potential. He even developed a quadratic exponential for the early inflation of the universe, however the scale factor he drove was a *positive* quadratic exponential, not a Gaussian exponential. According to Andrei Linde, "inflation is an exponential expansion of the universe in a supercooled false vacuum state. False vacuum is a metastable state without any fields or particles but with large energy density. Imagine a universe filled with such 'heavy nothing.' When the universe expands, empty space remains empty, so its energy density does not change," [14] and with a constant energy density we have an exponential expansion.

As interesting as this model is, the simple exponential-growth model of the universe has its qualifications, explaining that the random bubbles of particles that are spontaneously created or annihilated during the early universe could be the objects that determined the initial conditions for Linde's inflation. However, it comes with qualifications that during the early universe that "if the probability of the bubble formation is large, bubbles of the new phase are formed near each other, inflation is too short to solve any problems, and the bubble wall collisions make the universe extremely inhomogeneous." If formed too far away, then "each of these bubbles represents a separate open universe with a vanishingly small Ω." [14] Linde would have to precisely determine to a high degree of accuracy the appropriate ratio of quantum bubbles.

To compensate for these glitches in the popular exponential growth model for primordial inflation, in 1981-2, the slow-roll potential was developed. These slow roll parameters, from the slow-roll potential, are small terms that would vanish around our time, but play a significant role during the Big Bang. These would do increasingly less work on the universe. This potential began "either in the false vacuum, or in an unstable state at the top of the effective potential. Then the inflation field ϕ slowly rolls down to the minimum of its effective potential." With this, the energy density of the universe can accurately describe inflation, but the problem is that near $\phi = 0$ the solution would imply inhomogeneity in the model, in which the model assumed that the early universe was homogeneous. With an accelerating universe, there is a possibility that the potential of the universe could evolve in time.

Linde proposed that previous models assumed that the universe was in a state of constant thermal equilibrium and which requires the particles of the universe to interact, and suggests that the field governing the expansion plateau around $\phi = 0$. [14] This is where the modern theory of inflation left off. Linde continues to suggest a chaotic inflation model with the potential of the inflation field being proportional to

ϕ^2 and having that develop into a more accurate quadratic exponential development of the universe, complete with a linear scalar field. However, this is for the primordial inflationary model where it is assumed that there is a quantum mechanical origin of the universe. The random perturbations, giving off random bits of radiation, randomly decays, particles creating, all within the singularity (or near singularity) of space would cause certain initial condition to be met and thus spark an inflation. Apparently as a result of these quantum fluctuations, the inflationary field is created. Unfortunately, as solid as this model is at predicting the CMB, Linde could also be erring in the causality of the quantum fluctuations. It is possible that due to the compressed conditions of spacetime the quantum fluctuations were a result rather than the other way around in which the quantum fluctuations cause the compressed spacetime. This is the current reigning explanation for the early primordial inflation.

In 2000, Valerio Faraoni proposed an alternative, albeit weaker to Linde's, explanation for primordial inflation proportional to $e^{(H_0 t + H_1 t^2)}$ with a roll over and constant potential governing the universe's expansions, figuring out what sort of field driving it [14]. Unfortunately Faraoni's model does not describe a *Gaussian* evolution and only considers a constant $V(\phi)$, that which in an accelerated inflation, the energy density will change over time, and thus the vacuum energy will subsequently change as a result.

[14]While he did consider that a(t) took the form $e^{(H_0 t + H_1 t^2)}$ which would imply that there is a runaway universe. He threw out the idea that the universe was governed by a constant scalar field, set up the Klein Gordon equation, and solved for $\dot{\phi}$, stating that $\dot{\phi} = \pm \frac{C}{a^3}$.[9] Setting $\Lambda = 0$ in an intermediate step gave him the final resultant scalar field of $\phi = \phi_0 \ln[\tanh(\sqrt{\frac{3\Lambda}{2}} t)]$. He then found that $H^{-1} = \sqrt{\frac{3}{\Lambda}} \tanh(\sqrt{3\Lambda} t)$. His solution is interesting, but it is not Gaussian. [9]

1.6 Discovery of Contemporary Inflation

To add to the experimental contributions to Hubble's 1929 discovery and the discovery of the CMB, Adam Reiss et. al published a paper in 1998, *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant.* This paper went on to receive the Nobel Prize in 2011. It experimentally revised the idea that the expansion Hubble observed in 1929 was accelerating. In fact, the first sentence of the paper states, "This paper reports observations of 10 new high-redshift Type Ia supernovae (SNe Ia) and the values of the cosmological parameters derived from them." [24] Reiss et. al.'s universe model looked at the mass-energy balance solution where the vacuum energy density pushes the universe out. They used the matter-determination model that looked for a measured value of Ω_v.

The group looked at the redshifted light observed from telescopes at supernovae because they were showing patterns that seemed to defy Hubble's law that the universe's redshift for faraway objects was linear. Instead, they found a logarithmic relationship in their measured values. to figure out the densities, they began by studying the intrinsic luminosity of the extra stellar object,

$$D_L = (\frac{\mathcal{L}}{4\pi\mathcal{F}})^{\frac{1}{4}}, \tag{14}$$

and related it to how far away it was based on the signal that they were measuring.[24] And in the FRW metric, substituting $a(t) = (1 + z)$, the redshift, z, with the appropriate densities are determined that [24]

$$D_L = cH_0^{-1}(1+z)|\Omega_k|^{-1/2}\sin^{-1}[|\Omega|^{\frac{-1}{2}}]\times\int_0^z dz[(1+z)^2(1+z\Omega_m)-z(z+2)\Omega_v]^{\frac{-1}{2}}. \tag{15}$$

Following the established equation, and then after using the data collected then they developed a model that would best predict then the ratios of the matter densities on an axis which then suggested all possibilities for the universe's expansion. Their findings suggested that Ω_v was around 0.7 which, when placed within their models with a 0.4 for Ω_m, being lower than expected, assuming a constant inflation. Therefore, the data suggested that the universe's expansion was accelerating.

Today, we are left with two very pressing issues: what precisely is causing the accelerated inflation and what, precisely, is going on during the primordial inflation that caused the universe's initial expansion and the inflation of today. Though seemingly unrelated, we still are in the same universe; and whatever accounted for universe's expansion then should influence what inflation is happening now. The cosmological constant and quantum fluctuations cannot explain both, but a scalar field *that exists for all time* can.

2 Deriving The Scalar Field of Gaussian Inflation

2.1 Tolman's Rejected Model for Gaussian Expansion

Apart from Faraoni's contribution, all the way back in 1930 before cosmologists knew of inflation, and when the current exponential growth of the universe was still being theoretically considered, a quadratic exponential was considered by Robert Tolman. My solution is directly tied to a solution proposed by R. C. Tolman in his paper *The Effect of the Annihilation of Matter on the Wave-Length of Light from the Nebulae* (1929). [26] In this ambitious paper, Tolman proposed that the redshifted light emanating from the annihilation of matter in distant nebulae was due to the expanding universe. He claimed that the previous solutions that were locally bound to any given body were insufficient to model the universe's expansion. He proposed a new metric, figured out from the metric a way to model the nebulae's rate of annihilation, and applied it to measured redshift data at the time. His results were only successful for a short term before quickly diverging.

Tolman proposed that his new geometrical framework of the universe was to be modeled to redefine the "large-scale metrical properties of the universe as a whole, neglecting local deviations due to the neighborhood of stars or stellar systems," because Einstein's and de Sitter's "proved entirely unsatisfactory." [26] Therefore, Tolman deduced that "the form of line element which would correspond to the transformation of matter into radiation...and by introducing reasonable simplifications and assumptions, shall actually be able to obtain as a first approximation for the form of the line element the expression" [26]

$$ds^2 = \frac{-e^{2kt}}{1+4r^2}(dx^2 + dy^2 + dz^2) + dt^2, \tag{16}$$

where k is the rate at which matter transfers into radiation and $r = \sqrt{x^2 + y^2 + z^2}$ is the distance from the origin. Tolman began with a general form of the expression that [26]

$$ds^2 = e^{\mu}(dx^2 + dy^2 + dz^2) + e^{\nu} dt^2, \tag{17}$$

where μ and ν are functions of space and time. He looked at his geodesic equations and determined that e^{ν} had to be 1, that is $\nu = 0$, starting with the geodesic equation:

$$\frac{d^2 x_a}{ds^2} + \Gamma^a_{bc}\frac{dx_b}{ds}\frac{dx_c}{ds} = 0, \tag{18}$$

where a, b, and c are indices corresponding to the coordinates. “And considering for example accelerations in the x-direction, for particles having zero spatial velocities,

$$\frac{dx}{ds} = \frac{dy}{ds} = \frac{dz}{ds} = 0, \text{”} [26] \tag{19}$$

then equation (18) becomes

$$\frac{d^2x}{ds^2} + \Gamma^1_{44}\left(\frac{dt}{ds}\right)^2 = 0 \tag{20}$$

“Thus, if the acceleration $\frac{d^2x}{ds^2}$ is to be equal to zero, we are led to the conclusion that the line element must be such as to make the Christoffel 3-index symbol Γ^1_{44} itself equal to zero.” [26] To remove the dependence on e^ν on dt^2 in the metric, we need to look at the symbol

$$\Gamma^1_{44} = \frac{1}{2}e^{\mu+\nu}\frac{\partial \nu}{\partial x} = 0. \tag{21}$$

By setting Γ^1_{44} equal to zero, we remove the dependence of ν and x “and by symmetry, y and z as well.” [26] We have freedom to define ν. Tolman defined a new coordinate time t' by $e^{\nu(t)}dt^2 = dt'^2$, which “we can necessarily reduce the line element to the form

$$ds^2 = -e^\mu(dx_i dx_i) + dt^2.\text{”} [26] \tag{22}$$

Now to show that μ has independent variables r and t, he begins by looking at the volume element

$$dV_0 = e^{\frac{3\mu}{3}} dxdydz. \tag{23}$$

By taking the partial differential in position of the partial derivative in time of logarithmic differential of the volume element and setting it equal to zero, or

$$\frac{\partial}{\partial r}\frac{\partial \ln[dV_0]}{\partial t} = \frac{3}{2}\frac{\partial^2 \mu}{\partial r \partial t} = 0, \tag{24}$$

we are able to establish that

$$\tfrac{\partial^2 \mu}{\partial r \partial t} = 0, \quad \text{or} \quad \mu = f(r) + g(t). \tag{25}$$

His function μ is now separable. Based on the form of the Einstein equations he derived, Tolman found what function would best suit his new scale factor. To satisfy the Friedman equations, and to maintain a constant ρ,

$$8\pi\rho_0 = G - 4\Lambda.[26] \tag{26}$$

G is not Newton's Gravitation constant, but the invariant spur of the tensor [26]

$$G = e^{-\mu}\left[2\frac{\partial^2 \mu}{\partial r^2} + \frac{4}{r}\frac{\partial \mu}{\partial r} + \frac{1}{2}\left(\frac{\partial \mu}{\partial r}\right)^2\right] + 3\frac{\partial \mu}{\partial t^2} + 3\left(\frac{\partial \mu}{\partial t}\right)^2. \tag{27}$$

By keeping the relationship that "the proper time is the same for all stationary observers throughout the universe, we can maintain ...[this] assumption of uniform conditions throughout the universe, only if the percentage change with the time in the proper volume enclosed by 'stationary' particles independent of its position." [26] Therefore, by equation (25), Tolman derived the spatial dependence of the metric. With the Robertson Walker Metric, there is no spacial dependence because of the cosmological scale; any spatial dependence on the metric would be obscured but the incredible scale of the universe.

To then determine the time dependence of his function had to be of the form $e^{g(t)}$, he returned back to the Einstein tensor and solved for the Einstein equation with the cosmological constant added in. This produced him the final set of equations [26]

$$\begin{aligned} 8\pi p_0 &= -\tfrac{1}{R^2}e^{-g} - \tfrac{d^2g}{dt^2} - \tfrac{3}{4}(\tfrac{dg}{dt})^2 + \Lambda, \\ 8\pi \rho_{00} &= \tfrac{3}{R^2}e^{-g} - \tfrac{3}{4}(\tfrac{dg}{dt})^2 - \Lambda, \\ 8\pi \rho_0 &= \tfrac{6}{R^2}e^{-g} + \tfrac{d^2g}{dt^2} + \tfrac{3}{4}(\tfrac{dg}{dt})^2 - 4\Lambda. \end{aligned} \tag{28}$$

He added in the cosmological constant to keep every value, well, constant. To have each quantity remain constant the cosmological constant cannot equal zero so that the derivatives are non changing. To solve these differential equations he then proposed that $g(t) = \alpha t + \beta t^2 + \gamma t^2 + ...$,[26], but only took the first order approximation, stating, [26]

> As for higher terms [t^2 and greater], we do not yet have sufficient information for their evaluation and we shall have to omit them, introducing at this point the assumption that, in the neighborhood of the time $t = 0$, the quantity g can be given with a sufficient approximation by a formula that is linear in t.

Now with the discovery of the accelerated expansion, there *is* a reason to take the second order approximation. The rest of Tolman's article is dedicated to determining the constant, α, which he found by using the data from the rate at which radiation emanated from decaying matter in nebulae. Tolman did not pursue his metric.

That is far as the similarities between our papers go. I will present a standard cosmological approach to look at inflation. I set $\Lambda = 0$ and my model assumes no radial dependence on the scale factor. Nevertheless, Tolman's paper establishes a precedent for my paper and hopefully my paper gives credit to Tolman's forgotten idea.

2.2 Defining the Metric

To begin, I will start with a completely geometrical approach that will begin like Tolman's: I will start with a metric, write down the Christoffel symbols, Ricci tensor components, and finally lead up to the Einstein equations. I will then apply those results to see how they are caused; in this paper I argue that this metric generates a linear scalar field under a *quadratic* potential. I am using dark energy in lieu of the vacuum energy because I set $\Lambda = 0$.

While the de Sitter metric is simple, it does not predict the current inflation of the universe to a very accurate degree and quickly diverges to infinity. This is why cosmologists are afraid; the de Sitter metric predicts the end of days because there is nothing in the model to stop the inflation. Introducing a perhaps more complex model may bring an alternative prediction for our universe's fate and perhaps shed some more insight into our past.

To begin, I will start with the simple line element

$$ds^2 = -e^{f(t)+g(r)}(dx_i dx_i) + dt^2. \tag{29}$$

I assume $f(t)$ to be some arbitrary function of time and $g(r)$ an arbitrary function of position.[15] I allow $g(r) \to 0$ due to the cosmological scale.[16] I put exponential in

[15] $r = \sqrt{x^2 + y^2 + z^2}$

[16] Since the exponential takes the same form as Tolman's, I can use the same argument he used involving the volume element to show that these functions are independent and separable.

this metric because it lends itself to 85 years worth of research. Every function can be approximately represented as a polynomial so

$$f(t) = \gamma + \alpha t + \beta t^2 + \delta t^3 + ... \tag{30}$$

Because this function is placed within an exponential, it can become complicated as soon as the second-order approximation is introduced. My paper will do just that. I set γ equal to zero because it can be multiplied out and will not appear in any of the following equations regarding spacetime. I will consider this metric where

$$f(t) = \alpha t + \beta t^2, \tag{31}$$

which gives me a simple $g_{\mu\nu}$. It is

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -e^{\alpha t+\beta t^2} & 0 & 0 \\ 0 & 0 & -e^{\alpha t+\beta t^2} & 0 \\ 0 & 0 & 0 & -e^{\alpha t+\beta t^2} \end{pmatrix}. \tag{32}$$

where α and β are constants; their signs are currently undefined, as there can be many different combinations of the signs yielding incredibly different results. For now I have no reason to say what sign is which; however, I do want α to be positive and β to be negative, so it can have well-behaved properties. I do not want β to be positive not only because it would lead to imaginary solutions later on, but because theoretically, if β were positive, then the current model would expand way too fast, so fast that the universe's average energy would essentially be zero right now. If the universe does exhibit Gaussian-like behavior, it means there is a possibility to have both an inflation and a future compression.[17]

2.3 From the Metric to the Einstein Equations

Now that I have the metric tensor defined, I will need to derive the Einstein equations for this metric. To begin I will calculate the Christoffel symbols which are necessary for determining the elements describing spatial curvature, then take these equations and derive the Ricci and Riemann tensors from them. The tensors describe the curvature of spacetime, which I can manipulate to receive the Einstein curvature tensor.

I can begin work on deriving the Christoffel symbols. From the formula [18], [20],

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} [\partial_b g_{cd} + \partial_c g_{db} - \partial_d g_{bc}]. \tag{33}$$

Each letter a, b, c, and d corresponds to an index ranging from 1 to 4, each referring a specific Christoffel symbol to some combination of an element in the metric and its derivatives. After the grind and confirming my answers with *Wolfram Mathematica*, I received nine nonzero answers:

$$\begin{aligned} &\Gamma^1_{22} = \tfrac{1}{2} e^{\alpha t + \beta t^2}(\alpha + 2\beta t), \quad \Gamma^1_{33} = \tfrac{1}{2} e^{\alpha t + \beta t^2}(\alpha + 2\beta t), \quad \Gamma^1_{44} = \tfrac{1}{2} e^{\alpha t + \beta t^2}(\alpha + 2\beta t), \\ &\Gamma^2_{21} = \Gamma^2_{12} = \tfrac{1}{2}(\alpha + 2\beta t), \quad \Gamma^3_{31} = \Gamma^3_{13} = \tfrac{1}{2}(\alpha + 2\beta t), \quad \Gamma^4_{41} = \Gamma^4_{14} = \tfrac{1}{2}(\alpha + 2\beta t). \end{aligned} \tag{34}$$

[17]Guth considered that the universe could, in fact, collapse back on itself once it has reached a certain average temperature. More on that in section 4 becuase it has its theoretical basis in primordial inflation.

This, after much grind, will give me the proper components for the Riemann Tensor, a tensor of rank 4 that has 256 terms:

$$R^a_{bcd} = \partial_c \Gamma^a_{bd} + \partial_d \Gamma^a_{bc} + \Gamma^a_{ce}\Gamma^e_{bd} - \Gamma^a_{df}\Gamma^f_{bc}. \tag{35}$$

The Ricci tensor, $R_{\mu\nu}$, is determined by contracting the Riemann tensor according to the formula: [18] [20],

$$R_{\mu\nu} = R^a_{\mu a\nu}. \tag{36}$$

Instead of calculating a tensor with 256 terms, I can use only 64 of the 256 and compress those 64 into 16. That leaves me with the reduced formula of:

$$R^a_{bad} = \partial_c \Gamma^a_{bd} + \partial_d \Gamma^a_{ba} + \Gamma^a_{ae}\Gamma^e_{bd} - \Gamma^a_{df}\Gamma^f_{ba}. \tag{37}$$

After working through the grind and confirming it with *Wolfram Mathematica*, I determined that

$$\begin{gathered}
R_{11} = -2\beta - \tfrac{3}{4}(\alpha + 2\beta t)^2, \\
R_{22} = e^{\alpha t + \beta t^2}(\beta + \tfrac{3}{4}(\alpha + 2\beta t)^2), \\
R_{33} = e^{\alpha t + \beta t^2}(\beta + \tfrac{3}{4}(\alpha + 2\beta t)^2), \\
R_{44} = e^{\alpha t + \beta t^2}(\beta + \tfrac{3}{4}(\alpha + 2\beta t)^2).
\end{gathered} \tag{38}$$

Without the cosmological constant the Einstein Tensor is defined as [18], [20]

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{39}$$

where R is the contracted Ricci curvature scalar [18], [20]

$$R = g^{\mu\nu} R_{\mu\nu}. \tag{40}$$

After calculating the contracted Ricci scalar for the metric, I continued to find $G_{\mu\nu}$ for the system is that

$$\begin{gathered} G_{11} = \tfrac{3}{4}(\alpha + 2\beta t)^2, \\ G_{22} = G_{33} = G_{44} = -e^{\alpha t + \beta t^2}(2\beta + \tfrac{3}{4}(\alpha + 2\beta t)^2). \end{gathered} \tag{41}$$

Since the metric follows the metric signature, (+, -, -, -), I used the form of the Stress-Energy Tensor for a general perfect fluid [18], [20]

$$T_{\mu\nu} = (\rho_e + P)u_\mu u_\nu - P g_{\mu\nu},^{18} \tag{42}$$

where $u_\mu = (1, 0, 0, 0)$. Finally after applying equation (1) and assuming the universe can be represented as an isotropic and homogeneous perfect fluid, I obtain

$$\begin{gathered} \rho = \tfrac{3}{4\kappa}(\alpha + 2\beta t)^2, \\ P = -\tfrac{2}{\kappa}\beta - \tfrac{3}{4\kappa}(\alpha + 2\beta t)^2, \end{gathered} \tag{43}$$

where $\kappa_e = \frac{8\pi G}{c^2}$, G is the gravitation constant, and c is the speed of light in a vacuum. When κ divides either α^2 or β, I receive a quantity in the correct units of energy density. What is nice is that the exponentials drop out and we are left with two beautiful parabolic relations of pressure and energy density over time. Pressure and the energy density are not quite the same, as expected in an energy dominated universe. That is good becuase the current universe still has a large ratio of mass.

[18] $\rho_e = \rho c^2$. I am using energy density which means that in the form that does not involve energy density, then ρ would simply be itself and I would divide P by c^2

Since I am using the idea that the present time is $t = 0$, I need a function that can accurately depict the current decline in energy density in addition to accurately describing its current value. The de Sitter model assumes $\beta = 0$, so then not only will I receive a constant energy density, I will also receive a constant pressure, and by current expansion data, that is simply incorrect. That would say that my universe is static.[19] I would also receive a dark energy domination due to the fact that pressure and energy density are equal and opposite.[20]

To account for the static solution, then the cosmological constant is added into the Einstein equations and we receive a ρ that changes in time. By standard evolutionary cosmology, if average energy density of the universe reaches below what is called the critical density, defined as

$$\rho_c = \frac{H^2}{\kappa}.[18], \tag{44}$$

the universe will expand so fast that the energy density will collapse to zero. The vacuum of space would rip everything into an oblivion. This is certainly the case in the cosmological constant de Sitter model because H is constant and ρ is not. In the context of my metric, the Hubble constant is a parameter defined as $\frac{\dot{a}}{a}$[21][18][9][20], so

$$H = \frac{1}{2}(\alpha + 2\beta t). \tag{45}$$

This means that the critical density will change over time, growing smaller with the

[19]The universe will not be static if $\Lambda \neq 0$ in the de Sitter model.

[20]See section 3 for an explanation.

[21]I am assuming that $a(t) = e^{\frac{1}{2}(\alpha t + 2\beta t^2)}$

changing energy density of the universe. In fact, ρ_c will always be one quarter of whatever ρ is at any time.

If we set $t = 0$ to the current time, then we have the result for the de Sitter metric for our α value. To have an expansion, yet alone inflation, H must be positive. Eventually, this parameter does become negative when $t = \frac{-\alpha}{2\beta}$, necessitating that β be negative, or else the universe would be approaching a single point, with the inflationary epoch long over. H is zero, there is no movement, and when it is negative, that means that the universe *comes back together*. If β were positive, well, the moment of zero movement in the universe would be long in the past and the doomsday theories will be correct.

2.4 The Inflationary Scalar Field Hypothesis

Now that I have my Einstein equations calculated, I can reverse-engineer a scalar field driving this inflation. Typical for quintessence models is that the potential is set up and then solved to find the field, and finally the resultant geometry. My hypothesis starts with the geometry, then obtains the field, and finally the potential. I hypothesize that I will have a resultant quadratic field potential, not only because it has been considered before in previous models, but because many forms of potential energy are some quadratic. I do not expect, however, that the resultant field should take the form of a traditional gravitational field becuase, at least for times less than $-\frac{\alpha}{2\beta}$, the universe is inflating. The exact form of this scalar field can either structurally be like Faraoni's or linear. I want this field defined for all time and to exhibit no asymptotic behavior so that this model can be used with greater flexibility than with the de Sitter model.[22] By having the Gaussian Inflation inexorably caused by the scalar field we relate this new development to a measurable physical quantity.

2.5 Reverse-Engineering the Inflationary Scalar Field

From the Lagrangian[23] of curved spacetime, one arrives at these beautiful equations that relate the equation of state to both the scalar field and its scalar potential[9]

$$\begin{aligned} \rho &= \tfrac{1}{2}\dot{\phi}^2 - V(\phi), \\ P &= \tfrac{1}{2}\dot{\phi}^2 + V(\phi). \end{aligned} \tag{46}$$

Typically, ρ and P are the unknowns, but since I am starting from my own geometrical framework, I can determine what sort of field determines its existence. Plugging in my new definitions of ρ and P, we have that

$$\begin{aligned} \tfrac{3}{4\kappa}(\alpha + 2\beta t)^2 &= \tfrac{1}{2}\dot{\phi}^2 - V(\phi), \\ -\tfrac{2}{\kappa}\beta - \tfrac{3}{4\kappa}(\alpha + 2\beta t)^2 &= \tfrac{1}{2}\dot{\phi}^2 + V(\phi). \end{aligned} \tag{47}$$

By summing the two equations, I can calculate ϕ to be

$$\dot{\phi} = \sqrt{\frac{-2\beta}{\kappa}}. \tag{48}$$

[22]It is possible to have a vector field defined by the four potential $\tilde{\mathbf{A}} = (\phi(t), A_x, A_y, A_z)$ cause the inflation, but for now I am leaving that possibility open for future research. Developing a vector potential, complete with its own four vector may be possible, but more considerations and restrictions will need to be placed such as Lorentz invariance, and it has never been done successfully. I attempted to derive such a model and failed miserably. For just a simple scalar field, $\phi(t)$, however, fits easily within the Lagrangian and adds little complication.

[23]See Dodleson [8] for a derivation.

This gives us a challenging differential equation to solve;

$$\phi = \sqrt{\frac{-2\beta}{\kappa}} t + \phi(0). \tag{49}$$

This necessitates that β is negative, or else the resultant field will be imaginary.[24] To find $V(\phi)$, I simply have to plug $\frac{\dot{\phi}}{2}$ back in equation (47) to receive that

$$V(t) = -\frac{\beta}{\kappa} - \frac{3}{4\kappa}(\alpha + 2\beta t)^2; \tag{50}$$

and, in terms of field variables,

$$V(\phi) = -\frac{\beta}{\kappa} - \frac{3}{4\kappa}(\alpha + 2\beta\sqrt{\frac{-\kappa}{2\beta}}(\phi - \phi(0)))^2. \tag{51}$$

To check that my new scalar field minimizes the action integral of the Lagrangian, I need to plug my results into the Klein-Gordon Equation. If my field does not satisfy this equation, then I will need to go back to the drawing board and conclude that this universe does not generate a viable scalar field. Using the form of the Klein-Gordon Equation derived for a Robertson Walker metric, the equation states

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0.[6][9] \tag{52}$$

After plugging in for my equations, and assuming that H is my new linear Hubble parameter, I receive the final answer equal to 0. I have a homogeneous solution that satisfies my Lagrangian for a particle moving this spacetime configuration.

[24] I *could* have an imaginary scalar field, as they generally represent charged particle fields, such as the Higgs Field. Also, linear scalar fields generally describe spin-0 particles in Quantum Field theory. However, it is also generally true to make the field complex to add more degrees of freedom; however, my determination of $V(\phi)$ is sufficient. Also, according to Dodleson, “we know too much about the Higgs of the standard model. Its interactions and properties are constrained enough for us to know that it cannot serve as the source for inflation. [8]

3 The Field's Connection to Dark Energy

3.1 Connecting α to the Hubble Parameter

After finding the field potential, the energy density function and the potential energy functions are nice parabolas. Now I need to find the corresponding physical quantities for my field parameters that way dark energy and normal baryonic matter have a connection to the expansion. By doing so, I propose that this scalar field is spawned from the balance in the equations of state, which are thus related to dark energy. To make official this connection, I can connect my scalar field to the most obvious observable quantity the Hubble parameter.

To begin, using equation (31) where I defined our scaling factor of the universe $f(t)$, I will define a new function, Δ, that is slightly different to make it compatible with the Robertson-Walker metric where

$$\Delta = e^{\frac{1}{2}(\alpha t+\beta t^2)}. \tag{53}$$

I will be looking at derivatives of Δ to determine the values for my constants.

$$\begin{aligned}\dot{\Delta} &= \tfrac{1}{2}(\alpha + 2\beta t)e^{\frac{1}{2}(\alpha t+\beta t^2)},\\ \ddot{\Delta} &= e^{\frac{1}{2}(\alpha t+\beta t^2)}(\beta + \tfrac{1}{4}(\alpha + 2\beta t)^2).\end{aligned} \tag{54}$$

The minus sign in the line element is due to the Minkowski convention and thus can be left out, as it is $-\Delta^2(dx_i dx_i) + dt^2$. Luckily, at time t=0, the exponentials and time-dependent terms drop out leaving me with

$$\begin{aligned} H_0 = \dot{\Delta}(t=0) = \tfrac{\alpha}{2}; \\ \ddot{\Delta}(t=0) = \tfrac{\alpha^2}{4} + \beta. \end{aligned} \tag{55}$$

The Hubble constant is currently measured to be $2.356 \times 10^{-18} s^{-1}$ [10] which means that $\alpha = 4.712 \times 10^{-18}$.

3.2 Connecting β to the Current Equation of State

The connection β has to metric is less obvious. Since my newly determined scalar field is derived from my field parameter, I can revise my field parameters in terms of the current equation of state,

$$w = \frac{P}{\rho}. \tag{56}$$

When directly plugging in my expressions for P and ρ,

$$w_m = \frac{-2\beta - \frac{3}{4}(\alpha + 2\beta t)^2}{\frac{3}{4}(\alpha + 2\beta t)^2}. \tag{57}$$

This means that it is possible to determine the current value for w from the current ratio of dark energy to baryonic matter. Betoule et. al.[4] recently revised Riess et al.'s measurements for the current equation of state, hoping to find a total dark energy domination, or $w = -1$. They did not find that; instead they found values ranging from -0.5 to -0.995 when comparing data collected by five major observatories.[4]

Because Betoule et. al.'s observations did not confirm that $w = -1$, that suggests my β parameter does not equal 0, and therefore that my inflationary field exists. Looking at the mathematical structure of equation (57), one can see as $t \to \pm\infty$,

$w_m \to -1$.[25] The function asymptotically approaches the constant. In addition to that, w_m blows up at $t = \frac{-\alpha}{2\beta}$ in an asymptotically positive number, suggesting that for our relevant times, that the amount of dark energy in the universe is decreasing and the resultant matter is increasing as a result.[26] This what is predicted in the current literature.[27] Eventually w approaches zero, which is a matter dominated universe; afterwards, the ratio will continue to increase up to its asymptote, soon to be a +1, implying a radiation dominated universe, and then to a universe with no dark energy. For now,

$$w_0 = \frac{-2\beta - \frac{3}{4}\alpha^2}{\frac{3}{4}\alpha^2}. \tag{58}$$

After rearranging the equation and solving for β, I receive[28]

$$\beta = -\frac{3}{8}\alpha^2(w_0 + 1). \tag{59}$$

This gives me even more reason that $w_m \neq -1$. This gives me a relationship between my two field parameters and places restraints on what w_0 values I should look for. If $w_0 < -1$ then I will receive a positive value for β and if $w_0 > -1$,

[25] Dodleson actually gives us formula for the evolution of the energy density which is $\rho_{de} = e^{(-3\int^a \frac{da'}{a}[1+w(a')])}$ [8]

[26] But then why is the energy density decreasing? While the ratio between radiation and matter is changing so is the spatial distances between the matter.

[27] For a matter dominated early universe the Robertson-Walker expansion factor is $a(t) = t_0 t^{\frac{2}{3}}$ [6].

[28] An alternative equation of state is $\dot{H} = \beta = -4\pi G \sum_i w_i \rho_i$ [6] where each index i refers to each value of w and it has the corresponding density ρ for every value of w between -1 and 1, which would also give me an appropriate value for β. However the relationship I currently have is more applicable to the data I am using.

then I am in the clear for a negative β. Luckily, I have found values that have nearly confirmed that the expansion parameter w_0 treads very finely around -1. This paper looked at three different scenarios: the first two suggested w to be much closer to -1 than the final. If w were as close to -1 as Betoule et. al. have suggested, then we would have a much more wild universe than what we have, some of which have 60 e-folds, which would imply that the universe would inflate on the order of 10^{24} times what it is today before collapsing back into a singularity at a scale of $t = 10^{19}$ seconds in the future.[29]

Looking at Betoule et. al.'s data set, although the researchers have provided relatively few data points, about 6 per various experiment, they tried to nail down this ratio. They have provided uncertainty values that are very widely ranging. Selected in Table 1 are particularly chosen values of w that give me my most favorable results and I have included the rest on a comprehensive chart in the appendix. Each point has every possible combination at the extreme $\pm$ uncertainty values for their measured Hubble constant, and their w, including the possibility of no uncertainty additions.

These data I used were from a two-point model looking at an array of two separate w_z and w_a models, looking at how the ratio of matter domination compares with dark energy domination. The point I am looking for is their w_z which is predicted to be -1, which corresponds to Betoule et. al.'s model of the cosmological constant. However, in their distribution graph, one can easily see that the center of the distribution is

[29]Looking at the w-CDM flatness model [4], I chose $w = -0.994$ and $H_0 = 67.32 \pm 1.98 \frac{\frac{km}{s}}{Mpc}$ collected by *Planck*, WP, SDSS, and SNLS. After calculating α and β, I found that at $t = -\frac{\alpha}{2\beta} = (5.100 \pm 1.5) \times 10^{19}$ and $\Delta(\frac{-\alpha}{2\beta}) = 1.341 \times 10^{24}$, which is 55.5 e-folds.

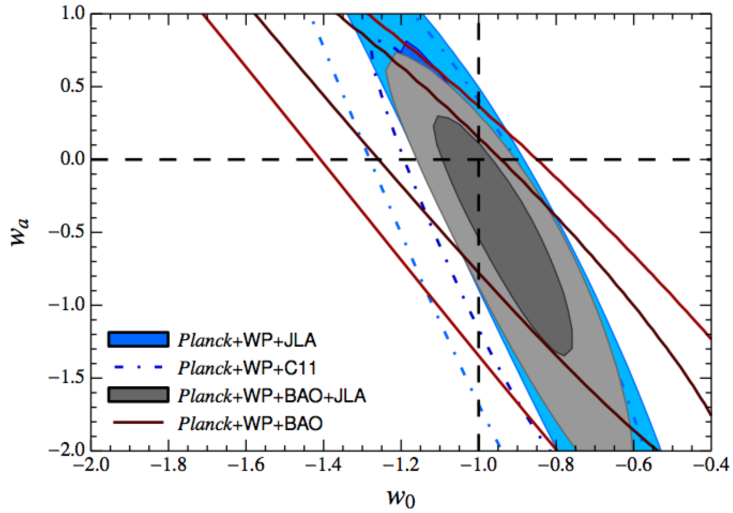

Figure 3: This diagram illustrates the "confidence contours at 68% and 95% (including systematic uncertainty) for the w and w_a cosmological parameters for the flat w-ΛCDM model." [4] If the cosmological constant were to exist, then the center of every region of possible values would be exactly along the $w_0 = -1$ line. Fortunately, the data region in the light gray is centered on $w_0 \approx -0.8$. It is possible that the true value for w_0 can be any one included in the circled or filled regions.

not near -1 on the w_z axis but between -0.7 and -0.9. This is good. I want my w to be around -0.82 because that way $\frac{-\alpha}{2\beta}$ can reasonably be around the current age of the universe. Betoule et. al. compared this model with the (-1,0) "golden" standard and computed their differences as error, despite it suggesting else.[4] The closer w goes to zero, β goes to zero and looking at the maximum value formulation for my scale factor, becomes asymptotic, and we have a runway universe. Therefore, I chose β that will only have nearly 0.375 e-folds by the time it reaches a maximum from now.

Table 1: Selected measured values for w_0 and H_0

Planck, WP, BAO, SDSS[4]	w	H in $\frac{\frac{km}{s}}{Mpc}$	H in s^{-1}	α	β
raw	-0.848 [4]	67.31 [4]	2.178×10^{-18}	4.357×10^{-18}	-1.081×10^{-36}
$+H_0$ uncertainty	-0.848 [4]	69.35	2.244×10^{-18}	4.489×10^{-18}	-1.148×10^{-36}
$-H_0$ uncertainty	-0.848 [4]	65.27	2.112×10^{-18}	4.225×10^{-18}	-1.017×10^{-36}
$+w$ uncertainty	-0.828	67.31 [4]	2.178×10^{-18}	4.357×10^{-18}	-1.224×10^{-36}
$+w + H_0$ uncertainty	-0.828	69.35	2.244×10^{-18}	4.489×10^{-18}	-1.300×10^{-36}
$+w - H_0$ uncertainty	-0.828	65.27	2.112×10^{-18}	4.225×10^{-18}	-1.151×10^{-36}
$-w$ uncertainty	-0.868	67.31 [4]	2.178×10^{-18}	4.357×10^{-18}	-9.395×10^{-37}
$-w + H_0$ uncertainty	-0.868	69.35	2.244×10^{-18}	4.489×10^{-18}	-9.973×10^{-37}
$-w - H_0$ uncertainty	-0.868	65.27	2.112×10^{-18}	4.225×10^{-18}	-8.834×10^{-37}

These data suggest that when the universe reaches its maximum, not in about another 13.7 billion years, but rather on the order of 50 billion years in the future. The universe will be reasonably behaved despite its prolonged expansion with the e-folds[30] it will undergo will be anywhere from 1 to 7.75, which suggests that the universe will not become less dense than its critical density. Here is what I calculated for this data set.

[30]the N e-folds is the number of times an exponentially growing factor increases by e. For a RW metric, it is defined as $N = \int_{t_i}^{t_e} H dt$. For my metric and from $(0, t, \frac{-\alpha}{2\beta})$, $N = \frac{-\alpha}{8\beta} = \frac{1}{3(w_0+1)}$.

Table 2: Various Quantities Derived from Table 1

Planck, WP, BAO, SDSS	Time of peak [4]	N e-folds at peak	$\ddot{\Delta}(0) = \frac{\alpha^2}{4} + \beta$
raw	2.014×10^{18}	2.193	3.66319×10^{-36}
$+H_0$ uncertainty	1.954×10^{18}	2.193	3.889×10^{-36}
$-H_0$ uncertainty	2.076×10^{18}	2.193	3.445×10^{-36}
$+w$ uncertainty	1.779×10^{18}	1.938	3.521×10^{-36}
$+w + H_0$ uncertainty	1.727×10^{18}	1.938	3.738×10^{-36}
$+w - H_0$ uncertainty	1.835×10^{18}	1.938	3.311×10^{-36}
$-w$ uncertainty	2.319×10^{18}	2.525	3.806×10^{-36}
$-w + H_0$ uncertainty	2.250×10^{18}	2.525	4.040×10^{-36}
$-w - H_0$ uncertainty	2.391×10^{18}	2.525	3.584×10^{-36}

Since all of these results are close in value, I could theoretically choose any one I please. Every combination of α and β, regardless of the model, has suggested that the scale factor's second derivative are positive at $t = 0$. This means that there is inflation without the cosmological constant needed. These data also show that the universe will go under a period of inflation followed by the universe collapsing in on itself back into a single point at a scale of 10^{18} seconds later. $10^{18}s$, is about a five times the current age of the universe.[31] This means that any sort of big move on the universe is much later to come. It will be interesting to see what this means for calculating the potentials and what the energy density will be given these values for α and β.

After averaging the data set, my final values for α and β are as follows:

$$w = -0.830, \quad \alpha = 4.441 \times 10^{-18} s^{-1}, \quad \text{and} \beta = -1.257 \times 10^{-36} s^{-2}. \tag{60}$$

With these constants, not only have I confirmed my hypothesis, but it gives me a relevant time frame to place everything.

[31] Assuming we are at $t \approx 4.35 \times 10^{17}$ or 13.7 billion years.

3.3 Analyzing the Choice in Field Parameters

After establishing the expansion parameters, I will apply my new results to my derived expressions for the potential $V(t)$ and $\rho(t)$. I chose the averaged values for α and β for these graphs.

For the majority of time, $V(t)$ is negative. For our relevant time frame, it will be negative and increasing. However at around five times the current age of the universe *more*, then do we see action starting to happen and the potential turns positive, and will stay positive for the $\Delta' s$ Full Width at Half Maximum.[32].

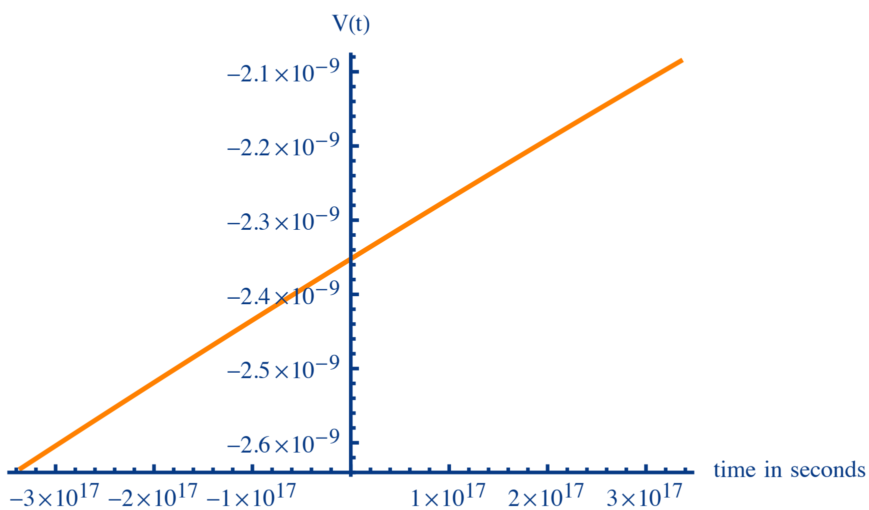

Figure 4: $V(t)$ plotted at our relevant scale

As with $V(t)$, these values for α and β give me a fairly accurate result for the current energy density of the universe, and gives me a beautifully and concave up evolution. Its parabolic development is only visible at the scale of $t = 10^{19}$. For calculating ρ and V I assumed $\kappa \approx 6.22 \times 10^{-27} \frac{\text{meters}}{\text{kg}}$. [2] Figure(5) gives us a clearer

[32]see appendix

picture that the energy density is now.

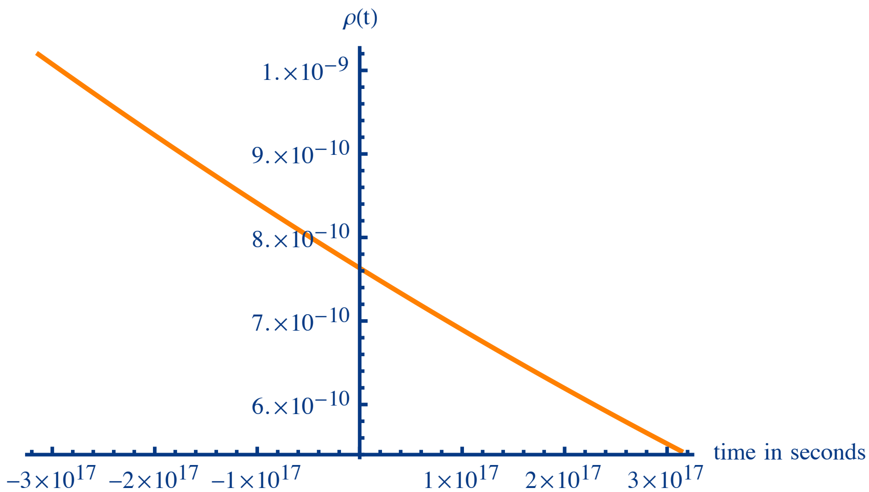

Figure 5: $\rho(t)$ plotted at our relevant scale

We receive a curve that looks frighteningly similar to a line. For our relevant time, our energy density is decreasing linearly. This gives me hope I have the correct values energy density at our current time, and as predicted, will continue to decline for another several billion years. However, since this is a parabola it will eventually increase again as the universe starts coming back on itself about five times the current age of the universe from now. My predicted value for the energy density is 15% too small.[33] I also have a negative value for my potential energy. This new metric is looking very hopeful, however, my work is still not done.

[33]The current mass density is $9.9 \times 10^{-27} kgm^{-3}$[19][3] I then multiplied that number by c^2 to receive the energy density.

4 Possible Application to Primordial Inflation

4.1 The Standard Model for Primordial Inflation

Since the scalar inflationary field exists throughout time, I can say that it was present around the time of the Big Bang was said to happen 13.7 billion years ago. The development of quintessence was originally designed to describe primordial inflation and these models do not describe the inflation today. Until now, both problems have been treated as different and unrelated phenomena. Every paper that I have cited involving primordial inflation calculated what scalar field was in the early universe. They mention nothing about what the scalar field would be like today other than deriving the scalar field in terms of the ratio of the contents of the universe.

To make a proper comparison with the early inflation model, I will look first how theoretical contemporary inflation models are structured. They rest on probabilities for the determination for w and the various Ωs and literally consider every possibility of its value. I could theoretically adopt the anthropic principle and consider every possible Ω from the infinite Big Bangs, but that is inefficient. Limiting possibilities in this case will be advantageous. I do not consider the anthropic principle, as this model assumes that this universe is the only universe we have. My paper seeks a more efficient solution for this problem given to me by the scalar field.

I will analyze it in the next section in detail and then derive what roll-over potential I need to use to describe the temperature fluctuations in the Cosmic Microwave Background (CMB). If my model can accurately fit the CMB temperature fluctuations, then the inflationary epoch would be a gradual growth over time of the universe. From $t \to -\infty$, the universe was in state of dark energy domination and therefore compressed into a single point. Because the universe was so small then, quantum mechanics could have taken hold, producing the temperature fluctuations in the CMB we see today. The CMB radiation is roughly 300,000 years younger than the Big Bang. Around that time, both my universe and the standard matter-dominated universe looked very similar[34]. The CMB is currently the earliest observed radiation in our universe and so if information about this infinitesimally small universe were to exist at all, it would not be visible to us yet and the average temperature of the tiny universe would mirror that of the Big Bang.

Around the time of the Big Bang, the universe was said to be matter dominated and evolved over time in a specific way. Guth used an exponentially expanding universe for his metric to solve both the flatness and the horizon problems. If, however, my model accurately predicts fluctuations in the CMB, then Big Bang model needs to be revised.

To begin I will use Carroll's metric, on page 367 [6], the established early model for the early matter dominated universe. It uses the Robertson Walker convention, where

$$a(t) = t_0^{\frac{-2}{3}}(t^{\frac{2}{3}}), \tag{61}$$

where t_0 is the current age of the universe. That way the scale factor can be modeled to be equal to one today. When I plot my scale factor over the same time period[35], to receive this plot, illustrated in Figure 6.

The blue line is my scale factor, the blue represents the standard model scale

[34] see Figure 6

[35] I shifted my scale factor to the right by $t = 4.35135 \times 10^{17}$ seconds

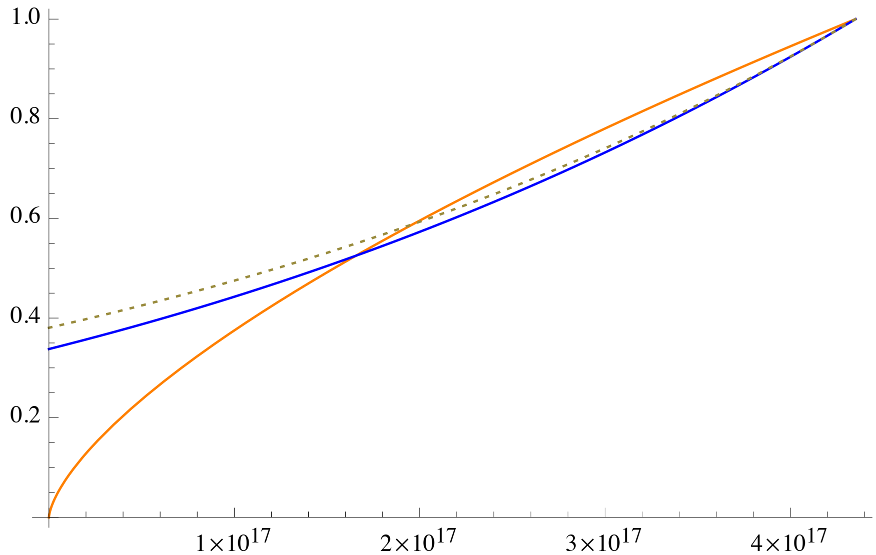

Figure 6: The Gaussian Scale factor, Δ, plotted in blue, versus the matter dominated expansion, $a(t)$, plotted in orange and the de Sitter metric as the dotted line. Despite the differences in concavity, both functions match exactly at two times and between the two intersections, both functions are closely related in value. However for times closer to zero, then the models diverge significantly. According to my model, the universe was only about a third of the size it is today at the time that would be the big bang. Despite these differences, I can say my metric is a fairly close estimate to the standard model.

factor, and the dotted line represents the de Sitter scale factor. The established metric goes to zero as t approached zero and does not continue into the negative values of time. My scale factor, on the other hand eventually goes to zero but a much, much greater time in the past. What is interesting is how close their values approach each other as time approaches the current day at $4.35135 \times 10^{17} seconds$. With this, my scale factor is a compatible alternative model for previous times up to at least 7 billion years, unless that my new scale factor is telling me that something else other than the matter is contributing to the universe's expansion. The second derivative for the matter-dominated universe model is always negative, while mine (at least for this time) is positive. In fact, according to Carroll [6]

$$H = \frac{2}{3}t^{-1} = a^{\frac{-3}{2}}H_0, \tag{62}$$

which always concave down.

4.2 Quintessence and Slow Roll Potentials

To fit the model to the early universe, Carroll set up initial parameters based on the Klein-Gordon Equation and the Friedman equations to set up the conditions for inflation. He determined that $\dot{\phi}^2 << V(\phi)$[6] and that $|\ddot{\phi}| << |3H\dot{\phi}|, |V'|$[6] so that when the "evolution of the field is sufficiently gradual that the potential energy dominates the kinetic energy, and the second derivative of ϕ is small enough to allow this state of affairs to be maintained for a sufficient period." Carroll was looking for a deceleration, which he did receive. It is interesting to note that my metric satisfies this condition except when $V(\phi)$ reaches its maximum at $\phi = \frac{-\alpha}{\sqrt{2\beta\kappa}}$ when $V(\phi)_{max} = \dot{\phi}^2$. At that point the universe stops inflating and begins to progress back into a single point. The second condition is satisfied for times until, again $t \geq \frac{-\alpha}{2\beta}$.

As does mine, Carroll's solution requires slow roll potential to be added to give the universe the "juice" or extra potential energy to cause a decelerated inflation, given to us by [6] [14]

$$\begin{aligned} \epsilon &= \tfrac{1}{2}\bar{m}_p^2 \left(\tfrac{V'}{V}\right)^2, \\ \eta &= \bar{m}_p^2 \tfrac{V''}{V}. \end{aligned} \tag{63}$$

Carroll makes a notes "these definitions are not universal; some people like to define them in terms of the Hubble parameter rather than the potential. Our choices determine whether a field has the chance to roll slowly for a while; the description in terms of the Hubble parameter describes whether actually the field is rolling slowly." These terms tell me whether or not the scalar field described by this metric can change to represent inflation correctly.

To check if this field satisfies these conditions, then, Carroll [6] mathematically rearranges his two parameters, by defining a general $V(\phi) = \frac{1}{2}m\phi^2$ as

$$\epsilon = \eta = \frac{\kappa c^2}{\phi^2}. \tag{64}$$

That potential describes a universe acting like a harmonic oscillator. Unfortunately I did not receive a harmonic oscillator universe, I received quadratic potential that has differing properties. Comparing the potential of the form in Linde's and Carroll's, mine is also of the form of a quadratic multiplied by my constants of α, β, and κ.With that in mind, I used equation (26) with the potential I determined, and plugged in values in and I received a very complicated expression for η and ϵ.

$$\epsilon = \frac{1}{2}\kappa c^2 \left[\frac{-\frac{3}{2}\sqrt{\frac{-2\beta}{\kappa}}(\alpha + \sqrt{-2\beta\kappa}(\phi - \phi_0))\dot{\phi}}{\frac{-\beta}{\kappa} - \frac{3}{4\kappa}(\alpha + \sqrt{-2\beta\kappa}(\phi - \phi_0))^2} \right]^2 << 1, \tag{65}$$

and

$$\eta = \kappa c^2 \frac{\frac{3}{2}\sqrt{\frac{-2\beta}{\kappa}}\dot{\phi}(\sqrt{-2\beta\kappa}\dot{\phi})}{\frac{-\beta}{\kappa} - \frac{3}{4\kappa}(\alpha + \sqrt{-2\beta\kappa}\phi)^2} = \kappa c^2 \frac{3\beta\sqrt{\kappa}\dot{\phi}^2}{\frac{-\beta}{\kappa} - \frac{3}{4\kappa}(\alpha + \sqrt{-2\beta\kappa}\phi)^2} << 1. \tag{66}$$

The first derivative of the scalar field is just a scalar, so if I set the proper ϕ_0, then I can an inflationary field where $\eta = \epsilon$. An inflationary scalar field under a harmonic potential has the initial condition $V(\phi = 0) = 0$, and shown by my potential, it is not the case because it is shifted up and to the right. To account for the shifted field, I can set my ϕ_0 to a value such that $V(\phi = 0) = 0$. I will set ϕ_0 to $\frac{\alpha}{\sqrt{-2\beta\kappa}} \pm \frac{2}{3}\sqrt{\frac{3}{2\kappa}}$. Now I receive a new potential

$$V(\phi) = -\frac{3}{4\kappa}\left(\sqrt{-2\beta\kappa}\phi\right)^2 = \frac{3}{2}\beta\phi^2. \tag{67}$$

I have a quadratic potential but not an inflationary scalar field. Since the classical harmonic potential $V_{\text{harm}} = \frac{1}{2}kr^2$, where r is the displacement and k is the spring constant, one easily sees that this inflationary potential is not harmonic. It is negative like the gravitational potential, which is attractive. This makes sense since the field is doing positive work on the universe that is already expanding. During earlier times in the universe, the field was small enough so that it simply was not dominant enough to push the universe back together. The time at which $\phi = 0$ is now $t = \frac{\alpha}{2\beta} \mp \sqrt{\frac{1}{-3\beta}}$, which is anywhere from 3 to 6 times the age of the universe in the past. The field's value is very slowly developing, and thus the field dominates in our current age.

Since I have defined the causal mechanism for the inflation as this scalar field, then it brings reasonable challenge to the prevailing quantum model. However, details into that are for future research. I will strive to set up an experimental basis for this work in the Cosmic Microwave Background (CMB), since my model can be potentially used to explain primordial inflation. If my results can work with the CMB, then I have discovered something powerful. To begin fitting my model to the CMB data, Carroll predicted the inflationary potential required to make the temperature fluctuations as they are today is given by the general formula

$$V_{\text{inflation}}^{\frac{1}{4}} \approx \epsilon^{\frac{1}{4}} 10^{16}\text{GeV.}[6] \tag{68}$$

Rewriting the temperature as a function of pressure would make make comparison final as in the following equation.

$$H^2 = \kappa\rho_R \approx 0.1 g_* \frac{T^4}{\bar{m}_p^2}[6], \tag{69}$$

where g_* is the effective number of relativistic degrees of freedom[36] and $\bar{m}_p$ is the reduced Planck mass. Plugging in for H^2, I receive

$$T \approx \frac{1}{2}\left(\frac{\bar{m}_p^2}{0.1 g_*}\right)^{\frac{1}{4}} \sqrt{|(\alpha + 2\beta t)|}. \tag{70}$$

Unfortunately there is a point in which the average temperature goes to zero as the universe expands when $t = \frac{-\alpha}{2\beta}$, but this is average temperature due to ambient radiation, at which point the universe's $w = \infty$. This is only the beginning of

[36] For the purposes of generality, these are the sum of the degrees of freedom for both fermions and bosons in their natural ratios determined by Quantum Chromodynamics[6].

perhaps a more solid model. The temperature will logically increase as the universe recompresses.

Guth predicted that it is possible that the universe will collapse back on itself due to the lack of thermal energy when the average temperature of the universe is at its minimum, which is consistent with my model. He states that

$$T_{\min}^4 = 30\frac{\rho_0}{\pi^2}\left[\frac{\epsilon}{\epsilon_0} + \left(\frac{\epsilon^2}{\epsilon_0} - 1\right)^{\frac{1}{2}}\right]^2, [12] \tag{71}$$

where ϵ_0 is the roll over at the time of the Big Bang. He placed the rollover in his expression connect the energy of the Big Bang to the average energy density of the universe now. That way there is a finite amount of energy in the universe and that its density is supposed to decrease with time, slowing the growth of the expansion. Unfortunately, in my model Guth's connection is instead tied to my scalar field doing work on the universe. If everything works correctly in my metric, the temperature here would reach the minimum at $t = -\frac{\alpha}{2\beta}$. This is normally placed to explain the lack of inflation that the universe experienced after the Big Bang. However, in my model, since the two scale factors are so close to each other, then the Gaussian behavior is suggestive of imperfect matter, radiation, and dark energy domination. Since my equation of state changes over time, then perhaps the universe would not neatly fit in with the $t^{\frac{2}{3}}$ development. With Gaussian cosmology, we are in a completely new ball game where anything is possible.

5 Conclusion

5.1 Comparison of the Scale Factors

For quick comparison on the different interpretations of the universe's development, Figure 7 illustrates the differences in development between the matter dominated model, the Gaussian universe model, and the de Sitter universe model.

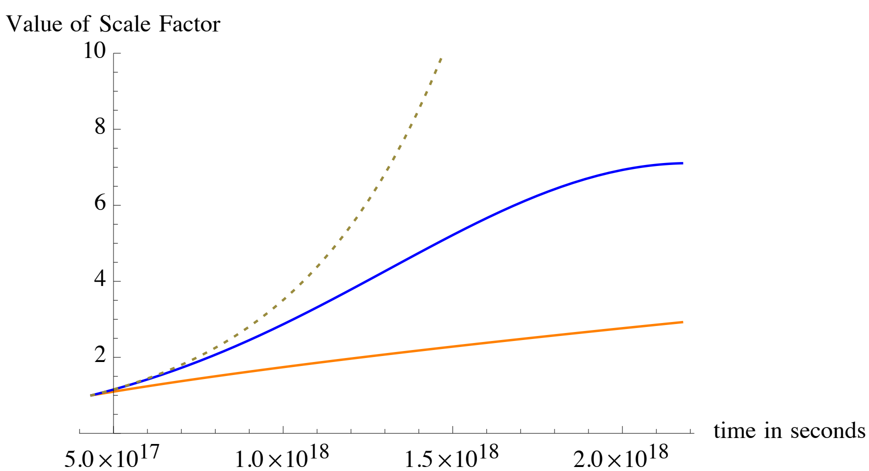

Figure 7: The Gaussian Scale factor, plotted in blue, versus the matter dominated expansion, plotted in orange, and the de Sitter metric as the dotted line for times after the current day up to five times the age of the universe. It is clear that the de Sitter universe diverges, the matter dominated universe peters out, and the Gaussian universe develops smoothly over time.

Despite their little difference in the past, their behaviors show as time continues on that they wildly differ. Only time will tell which model is correct. It is certainly possible for the universe to follow none of the three, and there has yet to be determined a more definitive solution.

5.2 A Final Look at the Scalar Field

To see how the field directly affects my metric, I will re-parameterize it in terms of field parameters α, β and the equation of state w_0:

$$\ln\Delta = \frac{\sqrt{\kappa}(\phi-\phi_0)}{2}\left(\frac{4}{3\sqrt{(w_0+1)}} - \frac{\sqrt{\kappa}}{2}(\phi-\phi_0)\right). \tag{72}$$

Section three went in to much detail explaining the development of dark energy of this universe. An oddity of this function is that for both early times and much, much later times the dark energy dominates. If the universe today was completely dominated, that is if $w_0 = -1$, then equation (72) becomes

$$\ln\Delta = \infty - \frac{\kappa}{4}(\phi_{w_0=-1} - \phi_0)^2 \approx \infty. \tag{73}$$

Due to equation (49), then the field's would be zero (in addition to being negligible) since β also depends on the existence of dark energy in accordance to equation (59). If, on the other hand, say $w = \infty$ where there is *no dark energy at all*, then

$$\ln\Delta \approx -\frac{\kappa}{4}(\phi_{w_0\to\infty} - \phi_0)^2 = -\infty, \tag{74}$$

and we are left with just the inflationary field pushing everything back together.

It would also be interesting to see how the cosmological constant can interact in the framework of my metric. I noticed that it has the same units as my β constant and is at a similar scale. The measured value of the cosmological constant, as observed is $2.036 \times 10^{-35} s^{-2}$ [5]. If I were to make a connection between my model and the cosmological constant, I would have to rewrite the Einstein equations and figure out, if at all, I can bridge a connection between it and my scalar expansion parameters.

5.3 Questions for Future Research

Since this paper merely presents a beginning for what results could come of this metric, I would like to propose questions for other researchers and me to tackle using a universe dominated by Gaussian Inflation:

- What impact will this metric have on calculating the redshift of supernovae and other astronomical phenomena?
- Can gravity waves exist in such a metric, and where would we find them?
- How would reintroducing the cosmological constant affect this model and can it exist simultaneously with my scalar field?
- Is the field I derived the only scalar inflationary field, and if so, is it possible to create another one to prevent the universe from collapsing back into a single point for all time?
- How accurately can this model predict the CMB fluctuations?

Some of these questions, and likewise their answers, vary in scope in their ability to be solved and observationally verified. For now, however, I will have to leave these questions open for future research.

5.4 Final Remarks

Typically, most papers in cosmology set up a potential and a scalar field to see what geometry falls out as a result (e.g., Linde [14]). Like Robertson [25] or Einstein[21], I proposed a metric and hypothesized its consequences. I hypothesized that Gaussian

Inflation would be driven a scalar field driven by dark energy, but I did not know its specific form. Working backwards has allowed me to predict my results that I would have a quadratic potential.

Apart from Faraoni's contribution, a quadratic exponential was only recently considered for primordial inflation and it has not been considered for 85 years for the contemporary expansion of the universe. In solving for primordial inflation, Faraoni was still married to the cosmological constant and Linde was married to quantum perturbations. By assuming both a classical model and $\Lambda = 0$, I was able to make the connection between both periods of the universe via the scalar field. This is not rewriting 86 years of cosmology, but merely building on what has been done with scalar fields, and both contemporary and primordial inflation. In fact, this paper calls for more thorough research on the benefits of using this metric. This metric is extraordinarily flexible and can be repurposed for more precise and varied astrophysical problems.

The connection I have to the scalar field would not have been possible in the de Sitter solution. To receive an expanding solution, I would have needed to place the cosmological constant in the field equations. However with the Gaussian model, I now have an inflationary model that has a dark energy scalar field directly derived from the field equations. If Tolman were alive now, he probably would have made this connection. Nevertheless, it comes as a great surprise how such a simple change in the metric could produce such dramatic results in quintessence. The dark energy scalar field that I have discovered is begging to be fleshed out more thoroughly. These initial attempts at understanding it inspire me to pursue future research in all areas of astronomy and cosmology.

A Mathematics of Gaussian Functions

The error function, defined by the integral of the Gaussian, is necessary for the calculating the distance between redshifts of co-moving objects in interstellar space. In Reiss's[24] paper used probability distributions to determine the most likely ratio dark matter to matter. Gaussian functions are used in probably analysis for their ability to match normal distributions by defining

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}.[17] \tag{75}$$

σ is the standard deviation, μ is the mean, and x is the variate. The function has no antiderivative in terms of elementary functions. It does, however, have an antiderivative of the $\mathrm{erf}(x)$ and can be solved analytically as a definite integral. In fact,

$$\int_0^x P(x) = \frac{1}{\sigma\sqrt{2\pi}} \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right).[15] \tag{76}$$

$\frac{1}{\sigma\sqrt{2\pi}}$ is multiplied it to normalize the function, which means that

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} P(x) = 1. \tag{77}$$

As beautiful as statistics is, when I compare Δ with $P(x)$, they appear to be nothing alike since

$$\ln \Delta = \frac{1}{2}(\alpha t + \beta t^2). \tag{78}$$

To make the Gaussian more directly integrable, I can complete the square in the exponential. If I have an arbitrary Gaussian of the form $e^{a+\frac{(t-b)^2}{2c^2}}$, where a, b, and c

are undetermined constants, I can define various combinations of α and β in terms of them. Since Δ is shifted to peak when $t = \frac{\alpha}{2\beta}$, then it becomes obvious that $b = \frac{-\alpha}{2\beta}$. To finish completing the square, then $2c^2 = \frac{1}{-\beta}$, which implies that $c = \sqrt{\frac{1}{-4\beta}}$. To get rid of the third term, then a becomes $-\frac{\alpha^2}{8\beta}$. Finally Δ can be rewritten as

$$\Delta = e^{-\frac{\alpha^2}{8\beta}+\frac{\beta}{2}(t+\frac{\alpha}{2\beta})^2}. \tag{79}$$

This gives me promise since the c value is fairly simple, despite the rest of the function being so complicated.

The above expression can give me an idea how long the second derivative will be negative, which is pretty useful. When integrated out to an arbitrary time t and starting with the current time, we have nothing pretty.

$$\int_0^t \Delta(t)dt = \sqrt{\frac{\pi}{-2\beta}} e^{\frac{\alpha^2}{8\beta}} \operatorname{erf}\left(\frac{-2\beta t - \alpha}{2\sqrt{-2\beta}}\right). \tag{80}$$

By rewriting scale factor, I have made it easier for those who need to integrate the scale factor over a discrete time frame. The next quantity is defined as the Full Width at Half Maximum which tells us how long $\ddot{\Delta}$ will be less than or equal to 0 and how long $V(t)$ is positive. To calculate the Full Width at Half Maximum I will use the following formula and plug in the necessary quantities:

$$2\sqrt{2\ln 2}c = 2\sqrt{\frac{-\ln 2}{2\beta}}.[16] \tag{81}$$

Perhaps the universe's expansion might have relevance into a statistical mechanical explanation for inflation. This would be an excellent topic for future research.

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